1.- **Resolver:** $\sqrt{3x-6} + \sqrt{2x+6} = \sqrt{9x+4}$

2.- **Resolver:** $\log 8 + (x^2 - 5x + 7) \cdot \log 3 = \log 24$

3.- **Resolver:** $\begin{cases} 2^x + 5^y = 9 \\ 2^{x+2} + 5^{y+1} = 41 \end{cases}$

4.- a) **Si** $\log A = \frac{1}{2} \cdot \log a + 3 \log b - \frac{1}{3} \cdot \log c - 2$ **calcular el valor de A**

b) **Calcular:** $-\log_2 (\log_2 \sqrt{\sqrt{\sqrt{2}}}) =$

5.- **Resolver:** $\begin{cases} x - y = 4 \\ x^2 + y^2 = 58 \end{cases}$

① $\sqrt{3x-6} + \sqrt{2x+6} = \sqrt{9x+4} \Rightarrow \left(\sqrt{3x-6} + \sqrt{2x+6}\right)^2 = \left(\sqrt{9x+4}\right)^2 \Rightarrow$

$\Rightarrow 3x - \cancel{6} + 2 \cdot \sqrt{3x-6} \cdot \sqrt{2x+6} + 2x + \cancel{6} = 9x+4 \Rightarrow 5x + 2 \cdot \sqrt{(3x-6)(2x+6)} = 9x+4$

$\Rightarrow 2\sqrt{6x^2+18x-12x-36} = 9x+4-5x \Rightarrow 2\sqrt{6x^2+6x-36} = 4x+4 \Rightarrow$

$\Rightarrow \left(2 \cdot \sqrt{6x^2+6x-36}\right)^2 = (4x+4)^2 \Rightarrow 4 \cdot (6x^2+6x-36) = 16x^2+32x+16 \Rightarrow$

$\Rightarrow 24x^2 + 24x - 144 = 16x^2 + 32x + 16 \Rightarrow 24x^2 - 16x^2 + 24x - 32x - 144 - 16 = 0 \Rightarrow$

$\Rightarrow 8x^2 - 8x - 160 = 0 \Rightarrow x^2 - x - 20 = 0 \left.\begin{matrix} \\ \end{matrix}\right\} \begin{matrix} \underline{x_1 = 5} \\ \underline{x_2 = -4} \end{matrix}$ (Cardano)

$\oplus\ 1$

$\odot\ -20$

Comprobación:

Si $\boxed{x=5} \Rightarrow \sqrt{9} + \sqrt{16} = \sqrt{49} \Rightarrow 3+4 = 7 \Rightarrow \boxed{7=7}$ Sí es solución

Si $\cancel{x=-4} \Rightarrow \sqrt{-18} + \sqrt{-2} = \sqrt{-32} \Rightarrow \nexists$ soluc. real $\Rightarrow$ NO es solución

② $\log 8 + (x^2-5x+7) \cdot \log 3 = \log 24 \Rightarrow \log 8 + \log 3^{x^2-5x+7} = \log 24 \Rightarrow$

$\Rightarrow \log 8 \cdot 3^{x^2-5x+7} = \log 24 \Rightarrow 8 \cdot 3^{x^2-5x+7} = 24 \Rightarrow 3^{x^2-5x+7} = \frac{24}{8} \Rightarrow$

$\Rightarrow 3^{x^2-5x+7} = 3^1 \Rightarrow x^2-5x+7 = 1 \Rightarrow x^2-5x+7-1 = 0 \Rightarrow$

$\Rightarrow x^2 - 5x + 6 = 0 \left.\begin{matrix} \\ \end{matrix}\right\} \begin{matrix} \underline{x_1 = 3} \\ \underline{x_2 = 2} \end{matrix}$

$\oplus\ 5$

$\odot\ 6$

Comprobación:

Si $\boxed{x=3} \Rightarrow \underbrace{\log 8 + 1 \cdot \log 3}_{\log(8 \cdot 3)} = \log 24 = \log 24 \Rightarrow$ CIERTO

Si $\boxed{x=2} \Rightarrow \underbrace{\log 8 + 1 \cdot \log 3}_{\log(8 \cdot 3)} = \log 24 = \log 24 \Rightarrow$ CIERTO

③ $\begin{cases} 2^x + 5^y = 9 \\ 2^{x+2} + 5^{y+1} = 41 \end{cases} \Rightarrow \begin{cases} 2^x + 5^y = 9 \\ 2^2 \cdot 2^x + 5 \cdot 5^y = 41 \end{cases} \Rightarrow \begin{cases} 2^x = L \\ 5^y = M \end{cases} \Rightarrow \begin{cases} L + M = 9 \\ 4L + 5M = 41 \end{cases}$

4Ea $\Rightarrow \begin{cases} 4L + 4M = 36 \\ 4L + 5M = 41 \end{cases}$

$\ominus$ / $-M = -5 \Rightarrow \underline{M = 5}$

$5^y = 5^1 \Rightarrow \underline{y = 1}$

$L + 5 = 9 \Rightarrow L = 9 - 5 \Rightarrow$

$\underline{L = 4}$

$2^x = 2^2 \Rightarrow \underline{x = 2}$

$\boxed{S(2,1)}$

④ a) Si $\log A = \frac{1}{2} \cdot \log a + 3 \log b - \frac{1}{3} \cdot \log c - 2$ ¿A?

$$\log A = \log a^{1/2} + \log b^3 - \log c^{1/3} - \log 100 =$$
$$= \log a^{1/2} + \log b^3 - (\log c^{1/3} + \log 100) =$$
$$= \log (a^{1/2} \cdot b^3) - \log (c^{1/3} \cdot 100) = \log \frac{a^{1/2} \cdot b^3}{c^{1/3} \cdot 100}$$

$$\boxed{A = \frac{\sqrt{a} \cdot b^3}{100 \cdot \sqrt[3]{c}}}$$

b) $$-\log_2 \left(\log_2 \sqrt{\sqrt{\sqrt{2}}}\right) = -\log_2 \left(\log_2 \sqrt[8]{2}\right) = -\log_2 \underbrace{\left(\log_2 2^{1/8}\right)}_{\frac{1}{8} \text{ por definición de logaritmo}} =$$
$$= -\log_2 \frac{1}{8} = -\log_2 \frac{1}{2^3} = -\underbrace{\log_2 2^{-3}}_{-3 \text{ por definición de logaritmo}} = -(-3) = \boxed{3}$$

⑤ $$\begin{cases} x - y = 4 \Rightarrow x = 4 + y \\ x^2 + y^2 = 58 \Rightarrow (4+y)^2 + y^2 = 58 \Rightarrow 16 + 8y + y^2 + y^2 = 58 \end{cases}$$

$\Rightarrow 2y^2 + 8y + 16 - 58 = 0 \Rightarrow 2y^2 + 8y - 42 = 0$ (Divido entre 2)

$\Rightarrow y^2 + 4y - 21 = 0$

$+\ -4$

$\cdot\ 21$

$y_1 = -7 \Rightarrow x_1 = 4 + (-7) = -3$

$y_2 = 3 \Rightarrow x_2 = 4 + 3 = 7$

$\boxed{S_1 (-3, -7)}$ $\boxed{S_2 = (7, 3)}$