1.- (2 puntos)

a) Resolver la ecuación: $\cos 8x + \cos 6x = 2 \cdot \cos 210° \cdot \cos x$

b) Demostrar: $\dfrac{sen\,(2\alpha)}{1+\cos\,(2\alpha)} = tg\alpha$

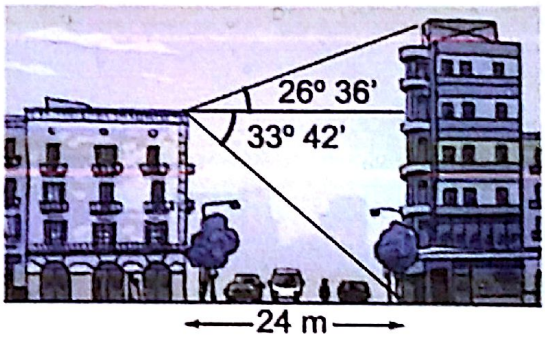
2.- (2 puntos)

Sabiendo que sen α = - 0,2 y cos β = - 0,5 siendo $\frac{3\pi}{2} < \alpha < 2\pi$ y que $\pi < \beta < \frac{3\pi}{2}$ calcular: a) $\cos(\alpha+\beta)$ b) $tg\left(\frac{\beta}{2}\right)$

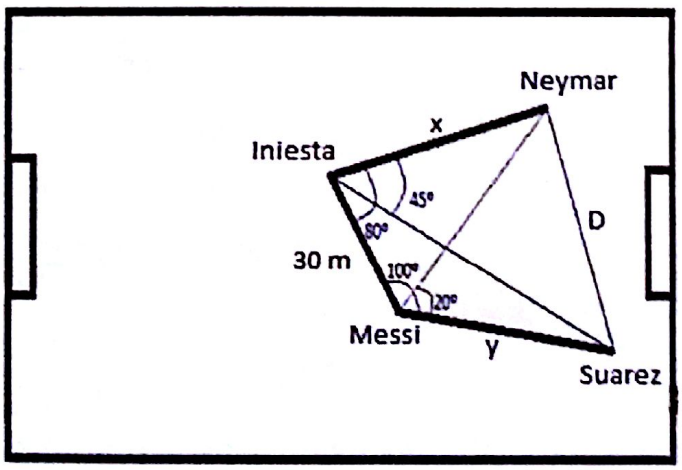
3.- (3 puntos)

a) Calcular la altura de los dos edificios del dibujo:

b) Resolver el sistema en el intervalo $[0°, 360°]$ $\begin{cases} tg\,x + tg\,y = 2 \\ x - y = \pi \end{cases}$



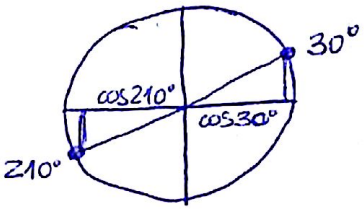
4.- (3 puntos)



Ataca el Barcelona, ya le gana 0-3 al Real Madrid, y el periodista Manolo Lama quiere saber, con los datos que veis en el dibujo, las distancias siguientes:

a) Distancia entre Iniesta y Neymar (x)
b) Distancia entre Messi y Suarez (y)
c) Distancia entre los delanteros Neymar y Suarez (D)

① a) $\cos 8x + \cos 6x = 2\cos 210° \cdot \cos x$



$$2\cos\frac{8x+6x}{2}\cdot\cos\frac{8x-6x}{2} = 2(-\cos 30°)\cdot\cos x$$

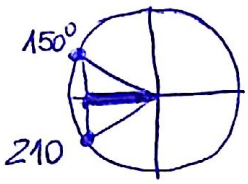
$$2\cos 7x\cdot\cos x = 2\cdot\left(-\frac{\sqrt{3}}{2}\right)\cdot\cos x$$

$$2\cos 7x\cdot\cos x + \sqrt{3}\cos x = 0$$

$$\cos x\cdot(2\cos 7x + \sqrt{3}) = 0 \Rightarrow \begin{cases} 1)\ \cos x = 0 \Rightarrow \begin{cases} x = 90° + 2\pi K \\ x = 270° + 2\pi K \end{cases} \Rightarrow \\ \Rightarrow \boxed{x = 90° + \pi K}\ K\in\mathbb{Z} \\ 2)\ 2\cos 7x + \sqrt{3} = 0 \Rightarrow \end{cases}$$

$$\Rightarrow 2\cos 7x = -\sqrt{3} \Rightarrow \cos 7x = -\frac{\sqrt{3}}{2} \Rightarrow \begin{cases} 7x = 150° + 2\pi K \Rightarrow x = \frac{150°}{7} + \frac{2\pi}{7}K \\ 7x = 210° + 2\pi K \Rightarrow x = 30° + \frac{2\pi}{7}K \end{cases}$$

$K\in\mathbb{Z}$



b) $\dfrac{\operatorname{sen} 2\alpha}{1+\cos 2\alpha} = \operatorname{tg}\alpha$

$$\frac{\operatorname{sen} 2\alpha}{1+\cos 2\alpha} = \frac{2\operatorname{sen}\alpha\cos\alpha}{1+\cos^2\alpha-\operatorname{sen}^2\alpha} = \frac{2\operatorname{sen}\alpha\cos\alpha}{1-\operatorname{sen}^2\alpha+\cos^2\alpha} = \frac{2\operatorname{sen}\alpha\cos\alpha}{\cos^2\alpha+\cos^2\alpha}$$

$$= \frac{2\operatorname{sen}\alpha\cos\alpha}{2\cos^2\alpha} = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \operatorname{tg}\alpha$$

② $\operatorname{sen}\alpha = -0{,}2$, $\cos\beta = -0{,}5$, $\alpha\in$ IV y $\beta\in$ III

a) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta = 0{,}98\cdot(-0{,}5) - (-0{,}2)\cdot(-0{,}87) = 0{,}664$

$\cos^2\alpha + \operatorname{sen}^2\alpha = 1 \Rightarrow \cos^2\alpha + (-0{,}2)^2 = 1 \Rightarrow \cos^2\alpha + 0{,}04 = 1 \Rightarrow \cos^2\alpha = 1 - 0{,}04 \Rightarrow$

$\Rightarrow \cos^2\alpha = 0{,}96 \Rightarrow \underset{\alpha\in IV}{\cos\alpha} = \pm\sqrt{0{,}96} = +0{,}98$

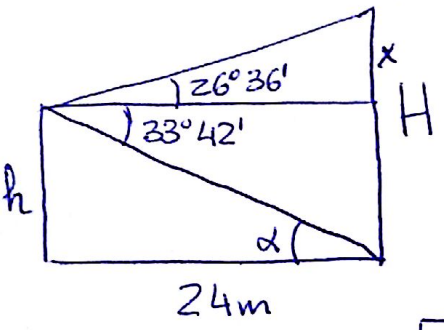
$\operatorname{sen}^2\beta + \cos^2\beta = 1 \Rightarrow \operatorname{sen}^2\beta + (-0{,}5)^2 = 1 \Rightarrow \operatorname{sen}^2\beta + 0{,}25 = 1 \Rightarrow \operatorname{sen}^2\beta = 1 - 0{,}25 \Rightarrow$

$\Rightarrow \operatorname{sen}^2\beta = 0{,}75 \Rightarrow \underset{\beta\in III}{\operatorname{sen}\beta} = \pm\sqrt{0{,}75} = -0{,}87$

b) $\boxed{tg\left(\frac{\beta}{2}\right) = \pm\sqrt{\frac{1-\cos\beta}{1+\cos\beta}}} = -\sqrt{\frac{1-\cos\beta}{1+\cos\beta}} = -\sqrt{\frac{1-(-0{,}5)}{1+(-0{,}5)}} =$

$\beta \in III \Rightarrow \beta/2 \in II$ // $= -\sqrt{\frac{1{,}5}{0{,}5}} = \boxed{-\sqrt{3}}$

③



a) $\alpha = 33°42'$ por ser ángulos alternos internos.

$tg\ 33°42' = \frac{h}{24} \Rightarrow h = 24 \cdot tg\ 33°42' \Rightarrow$

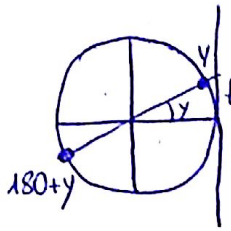
$\boxed{h = 16\,m}$

$H = h + x$

$tg\ 26°36' = \frac{x}{24} \Rightarrow x = 24 \cdot tg\ 26°36' = 12{,}02\,m$

$\boxed{H = 16 + 12{,}02 = 28{,}02\,m}$

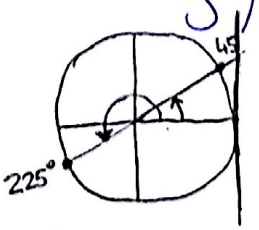
b) $\begin{cases} tg\,x + tg\,y = 2 \\ x - y = \pi \Rightarrow x = 180° + y \end{cases}$ $x, y \in (0, 360)$



$tg(180+y) = tg\,y$

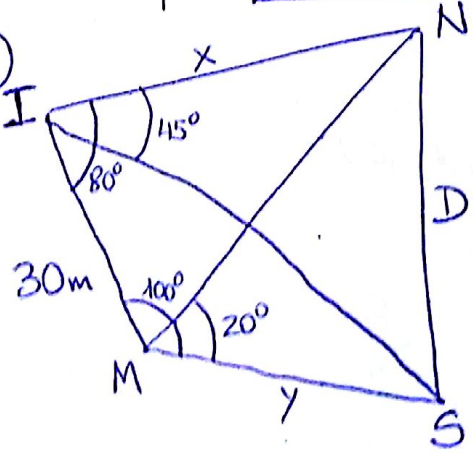
$tg(180+y) + tg\,y = 2 \Rightarrow tg\,y + tg\,y = 2 \Rightarrow 2\,tg\,y = 2 \Rightarrow$

$\Rightarrow tg\,y = 1 \Rightarrow \begin{cases} y_1 = 45° \Rightarrow x_1 = 180° + 45° = 225° \\ y_2 = 225° \Rightarrow y_2 = 180° + 225° = 405° \notin (0, 360°) \end{cases}$



$\boxed{S_1\,(225°,\ 45°)}$

④



a) En $\hat{IMN}$ $\begin{cases} I = 80° \\ M = 100° - 20° = 80° \ (\text{isósceles}) \end{cases} \Rightarrow N = 180° - 80° - 80° = 20°$

$\frac{30}{sen\,20°} = \frac{x}{sen\,80°} \Rightarrow \boxed{x = \frac{30 \cdot sen\,80°}{sen\,20°} = 86{,}38\,m}$

b) En $\hat{IMS}$ $\begin{cases} M = 100° \\ I = 80° - 45° = 35° \end{cases} \Rightarrow S = 180° - 100° - 35° = 45°$

$\frac{30}{sen\,45°} = \frac{y}{sen\,35°} \Rightarrow \boxed{y = \frac{30 \cdot sen\,35°}{sen\,45°} = 24{,}33\,m}$

c) $\overline{MN} = \overline{IN} = 86{,}38\,m$ (isósceles IMN)

En $\hat{MSN}$ conozco $\overline{MN}$, $\overline{MS}$ (y) y el ángulo opuesto a D $\Rightarrow$ Aplico Tª COSENO

$D^2 = \overline{MN}^2 + \overline{MS}^2 - 2\,\overline{MN}\,\overline{MS} \cdot \cos 20° = 86{,}38^2 + 24{,}33^2 - 2 \cdot 86{,}38 \cdot 24{,}33 \cdot \cos 20° = 4103{,}69$

$\boxed{D = \sqrt{4103{,}69} = 64{,}06\,m}$