

1.- (3 puntos)

a) Desarrollar la expresión de $\operatorname{tg} 3a$ dejándola en función de $\operatorname{tg} a$

b) Sea a un ángulo del II cuadrante y $\operatorname{sen} a = 3/5$, calcular las razones de $\frac{a}{2}$
(seno, coseno y tangente)

2.- (2 puntos)

a) Resolver la ecuación: $2 \operatorname{sen}^2 x + \cos 2x = 4 \cos^2 x$

b) Calcular en función de h la $\sec 203^\circ$, siendo $\cotg 67^\circ = h$

3.- (3 puntos)

a) Resuelve el triángulo y calcula su área si se conocen $A = 80^\circ$, $a = 10$ m y $b = 5$ m

b) Calcular $\sqrt[5]{\frac{-1-\sqrt{3}i}{-3+\sqrt{3}i}}$

4.- (2 puntos)

a) Dado el vector $u = (-10, 12)$, hallar sus coordenadas respecto de la base $B = \{v, w\}$ donde $v = (3, -4)$ y $w = (1, -1)$

b) Dados los complejos $z_1 = 2_{60^\circ}$, $z_2 = -1 + i$ y $z_3 = 2(\cos 210^\circ + i \operatorname{sen} 210^\circ)$
calcular

$$\frac{z_1 \cdot \bar{z}_2}{z_3} \text{ en forma binómica}$$

$$\begin{aligned}
 \textcircled{1} \text{ a) } \operatorname{tg} 3a &= \operatorname{tg} (2a+a) = \frac{\operatorname{tg} 2a + \operatorname{tg} a}{1 + \operatorname{tg} 2a \cdot \operatorname{tg} a} = \frac{\frac{2 \operatorname{tg} a}{1 + \operatorname{tg}^2 a} + \operatorname{tg} a}{1 + \frac{2 \operatorname{tg} a}{1 + \operatorname{tg}^2 a} \cdot \operatorname{tg} a} = \\
 &= \frac{\frac{2 \operatorname{tg} a + \operatorname{tg} a + \operatorname{tg}^3 a}{1 + \operatorname{tg}^2 a}}{\frac{1 + \operatorname{tg}^2 a + 2 \operatorname{tg}^2 a}{1 + \operatorname{tg}^2 a}} = \boxed{\frac{3 \operatorname{tg} a + \operatorname{tg}^3 a}{1 + 3 \operatorname{tg}^2 a}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \alpha \in \text{II}, \quad \operatorname{sen} \alpha &= \frac{3}{5} \Rightarrow \operatorname{sen}^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \frac{9}{25} + \cos^2 \alpha = 1 \Rightarrow \\
 \cos^2 \alpha &= 1 - \frac{9}{25} \Rightarrow \cos^2 \alpha = \frac{16}{25} \Rightarrow \cos \alpha = \pm \sqrt{\frac{16}{25}} \left. \vphantom{\cos \alpha} \right\} \alpha \in \text{II} \quad \cos \alpha = -\frac{4}{5}
 \end{aligned}$$

$$\frac{\alpha}{2} \in \text{I}:$$

$$\operatorname{sen} \frac{\alpha}{2} = + \sqrt{\frac{1 - \cos \alpha}{2}} = \sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{9}{10}} = \boxed{\frac{3}{\sqrt{10}}}$$

$$\cos \frac{\alpha}{2} = + \sqrt{\frac{1 + \cos \alpha}{2}} = \sqrt{\frac{1 - \frac{4}{5}}{2}} = \sqrt{\frac{\frac{1}{5}}{2}} = \sqrt{\frac{1}{10}} = \boxed{\frac{1}{\sqrt{10}}}$$

$$\operatorname{tg} \frac{\alpha}{2} = \frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}} = \boxed{3}$$

$$\textcircled{2} \text{ a) } 2 \operatorname{sen}^2 x + \cos 2x = 4 \cos^2 x$$

$$2 \operatorname{sen}^2 x + \cos^2 x - \operatorname{sen}^2 x = 4 \cos^2 x$$

$$\operatorname{sen}^2 x - 3 \cos^2 x = 0 \Rightarrow \operatorname{sen}^2 x - 3(1 - \operatorname{sen}^2 x) = 0 \Rightarrow$$

$$\Rightarrow \operatorname{sen}^2 x - 3 + 3 \operatorname{sen}^2 x = 0 \Rightarrow 4 \operatorname{sen}^2 x - 3 = 0 \Rightarrow 4 \operatorname{sen}^2 x = 3 \Rightarrow$$

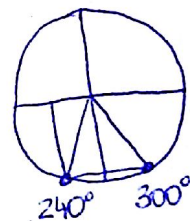
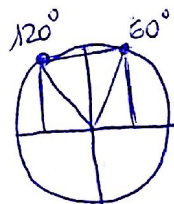
$$\Rightarrow \operatorname{sen}^2 x = \frac{3}{4} \Rightarrow \operatorname{sen} x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

$$\operatorname{sen} x = \frac{\sqrt{3}}{2} \Rightarrow$$

$$x = \begin{cases} 60^\circ + 2\pi K \\ 120^\circ + 2\pi K \end{cases}$$

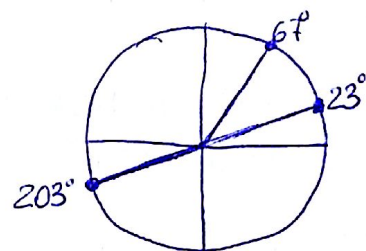
$$\operatorname{sen} x = -\frac{\sqrt{3}}{2} \Rightarrow$$

$$x = \begin{cases} 240^\circ + 2\pi K \\ 300^\circ + 2\pi K \end{cases}$$



② b) $\sec 203^\circ$, si $\cotg 67^\circ = h$

$h = \cotg 67^\circ = \tg 23^\circ$ por ser complementarios los ángulos.



$$\sec^2 203^\circ = 1 + \tg^2 203^\circ \Rightarrow \sec^2 203^\circ = 1 + h^2 \Rightarrow \boxed{\sec 203^\circ = -\sqrt{1+h^2}}$$

$203^\circ \in \text{III}$

Como $\tg 23^\circ = \tg 203^\circ = h$

③ a) $A = 80^\circ$, $a = 10\text{m}$ y $b = 5\text{m}$

$$\frac{a}{\sen A} = \frac{b}{\sen B} \Rightarrow \sen B = \frac{b \cdot \sen A}{a} = \frac{5 \cdot \sen 80^\circ}{10} = \frac{\sen 80^\circ}{2} = 0,4924$$

$$B = \arc \sen 0,4924 = \boxed{29,49^\circ}$$

$+150,5^\circ$ NO VALE ($B = 80^\circ$)

$$\boxed{C = 180^\circ - 80^\circ - 29,49^\circ = 70,51^\circ}$$

$$\frac{a}{\sen A} = \frac{c}{\sen C} \Rightarrow \frac{10}{\sen 80^\circ} = \frac{c}{\sen 70,51^\circ} \Rightarrow \boxed{C = \frac{10 \cdot \sen 70,51^\circ}{\sen 80^\circ} = 9,57\text{m}}$$

$$A = \frac{1}{2} a \cdot b \cdot \sen C = \frac{1}{2} \cdot 10 \cdot 5 \cdot \sen 70,51^\circ = 25 \cdot \sen 70,51^\circ = \boxed{23,57\text{m}^2}$$

b) $\sqrt[5]{\frac{-1-\sqrt{3}i}{-3+\sqrt{3}i}}$

$$\begin{aligned} -1-\sqrt{3}i &\Rightarrow |-1-\sqrt{3}i| = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2 \\ \alpha &= \arc \tg \frac{-\sqrt{3}}{-1} = \arc \tg \sqrt{3} = \left. \begin{aligned} &\cancel{60^\circ} \text{ Afijo } (-1, -\sqrt{3}) \in \text{III} \\ &240^\circ \end{aligned} \right\} 2_{340^\circ} \end{aligned}$$

$A \in \text{III}$

$$\begin{aligned} -3+\sqrt{3}i &\Rightarrow |-3+\sqrt{3}i| = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3} \\ \beta &= \arc \tg \frac{\sqrt{3}}{-3} = \left. \begin{aligned} &-30^\circ \rightarrow \cancel{330^\circ} \text{ Afijo } (-3, \sqrt{3}) \in \text{II} \\ &150^\circ \end{aligned} \right\} 2\sqrt{3}_{150^\circ} \end{aligned}$$

$B \in \text{II}$

$$\begin{aligned} \sqrt[5]{\frac{2_{240^\circ}}{2\sqrt{3}_{150^\circ}}} &= \sqrt[5]{\left(\frac{2}{2\sqrt{3}}\right)_{90^\circ}} = \sqrt[5]{\left(\frac{1}{\sqrt{3}}\right)_{90^\circ}} = \left(\sqrt[5]{\frac{1}{\sqrt{3}}}\right)_{\frac{90^\circ + 2\pi K}{5}} \\ &= \left\{ \begin{aligned} \text{Si } K=0 &\Rightarrow \left(\sqrt[5]{\frac{1}{\sqrt{3}}}\right)_{18^\circ} \\ \text{Si } K=1 &\Rightarrow \left(\sqrt[5]{\frac{1}{\sqrt{3}}}\right)_{90^\circ} \\ \text{Si } K=2 &\Rightarrow \left(\sqrt[5]{\frac{1}{\sqrt{3}}}\right)_{162^\circ} \\ \text{Si } K=3 &\Rightarrow \left(\sqrt[5]{\frac{1}{\sqrt{3}}}\right)_{234^\circ} \\ \text{Si } K=4 &\Rightarrow \left(\sqrt[5]{\frac{1}{\sqrt{3}}}\right)_{306^\circ} \end{aligned} \right. \end{aligned}$$

$18^\circ + 72^\circ K \quad \forall K=0,1,2,3,4$

(4)

a) $\vec{u} = (-10, 12)$ $B = \{(3, -4), (1, -1)\}$

$$(-10, 12) = \alpha(3, -4) + \beta(1, -1) = (3\alpha + \beta, -4\alpha - \beta)$$

$$\begin{cases} 3\alpha + \beta = -10 \\ -4\alpha - \beta = 12 \end{cases} \xrightarrow{\quad} 3 \cdot (-2) + \beta = -10 \Rightarrow -6 + \beta = -10 \Rightarrow \beta = -10 + 6 \Rightarrow \beta = -4$$

$$\textcircled{+} -\alpha = 2 \Rightarrow \alpha = -2$$

$$\boxed{\vec{u} = (-2, -4)} \text{ respecto de } B$$

b) $z_1 = 2_{60^\circ}$ $z_2 = -1 + i \Rightarrow \bar{z}_2 = -1 - i$, $z_3 = 2(\cos 210^\circ + i \sin 210^\circ) = 2_{210^\circ}$

$$\bar{z}_2 = \sqrt{2}_{225^\circ} \begin{cases} |\bar{z}_2| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2} \\ \alpha = \arctg \frac{-1}{-1} = \arctg 1 = \begin{cases} 45^\circ \\ 225^\circ \end{cases} \text{ A} \in \text{III} \\ A(-1, -1) \in \text{III} \end{cases}$$

$$\frac{z_1 \cdot \bar{z}_2}{z_3} = \frac{2_{60^\circ} \cdot \sqrt{2}_{225^\circ}}{2_{210^\circ}} = \left(\frac{\cancel{2}\sqrt{2}}{\cancel{2}} \right)_{60^\circ + 225^\circ - 210^\circ} = \sqrt{2}_{75^\circ} =$$

$$= \sqrt{2} (\cos 75^\circ + i \sin 75^\circ) = \boxed{\sqrt{2} \cos 75^\circ + \sqrt{2} \sin 75^\circ \cdot i}$$