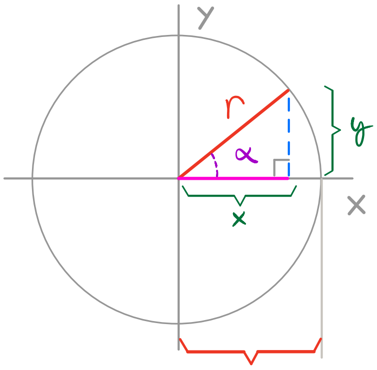
FÓRMULAS DE TRIGONOMETRÍA

Razones (divisiones) trigonométricas de una circunferencia. Definiciones.

Definimos las razones a partir de un triángulo rectángulo inscrito en la circunferencia.



$$\text{seno} \equiv \frac{\text{cateto opuesto}}{\text{hipotenusa}} \Rightarrow \operatorname{sen}\alpha = \frac{y}{r}$$

$$\text{coseno} \equiv \frac{\text{cateto contiguo}}{\text{hipotenusa}} \Rightarrow \cos\alpha = \frac{x}{r}$$

$$\text{tangente} \equiv \frac{\text{cateto opuesto}}{\text{cateto contiguo}} \Rightarrow \operatorname{tg}\alpha = \frac{y}{x}$$

$$\operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha} = \frac{r}{y}$$

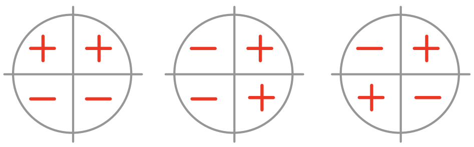
$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{r}{x}$$

$$\operatorname{cotg}\alpha = \frac{1}{\operatorname{tg}\alpha} = \frac{x}{y}$$

r = radio = hipotenusa

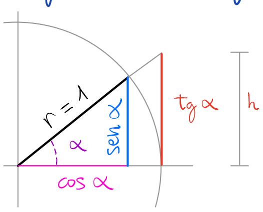
Dominio: $\operatorname{sen}\alpha, \cos\alpha \in [-1,1]$ $\quad \operatorname{tg}\alpha \in [-\infty,+\infty] = \mathbb{R}$

El coseno es la "base" y el seno la "altura" del triángulo. La tangente es la "pendiente".



seno $\quad$ coseno $\quad$ tangente

Signos por cuadrante



$$\boxed{\pi \text{ rad} = 180^\circ}$$

Factor de conversión radianes – grados

$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} \; ; \quad \operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha} \; ; \quad \sec\alpha = \frac{1}{\cos\alpha} \; ; \quad \operatorname{cotg}\alpha = \frac{1}{\operatorname{tg}\alpha}$$

$$\cos^2\alpha + \operatorname{sen}^2\alpha = 1$$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \operatorname{sen}\alpha\cdot\operatorname{sen}\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \operatorname{sen}\alpha\cdot\operatorname{sen}\beta$$

$$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cdot\cos\beta + \cos\alpha\cdot\operatorname{sen}\beta$$

$$\operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha\cdot\cos\beta - \cos\alpha\cdot\operatorname{sen}\beta$$

$$\operatorname{sen}2\alpha = 2\operatorname{sen}\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$$

$$\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

TRIÁNGULOS

Teorema del Coseno

$$\boxed{a^2 = b^2 + c^2 - 2\cdot b\cdot c\cdot\cos\hat{A}}$$

$$\boxed{b^2 = a^2 + c^2 - 2\cdot a\cdot c\cdot\cos\hat{B}}$$

$$\boxed{c^2 = a^2 + b^2 - 2\cdot a\cdot b\cdot\cos\hat{C}}$$

$$\boxed{\frac{a}{\operatorname{sen}\hat{A}} = \frac{b}{\operatorname{sen}\hat{B}} = \frac{c}{\operatorname{sen}\hat{C}}}$$

Teorema de los senos

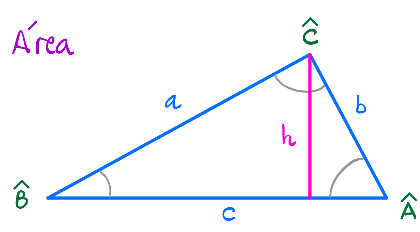
$$\boxed{A = \frac{1}{2}\cdot c\cdot b\cdot\operatorname{sen}\hat{A}}$$

$$\boxed{A = \frac{1}{2}\cdot c\cdot a\cdot\operatorname{sen}\hat{B}}$$

$$\boxed{A = \frac{1}{2}\cdot a\cdot b\cdot\operatorname{sen}\hat{C}}$$

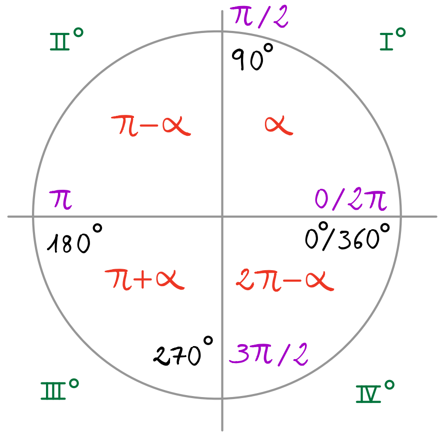
$$\boxed{\hat{A}+\hat{B}+\hat{C} = 180^\circ}$$

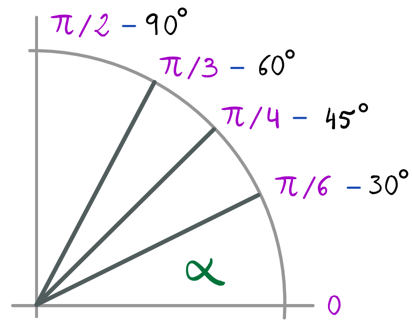
Área



Circunferencia goniométrica (r=1)
Cuadrantes y reducción al 1er cuadrante

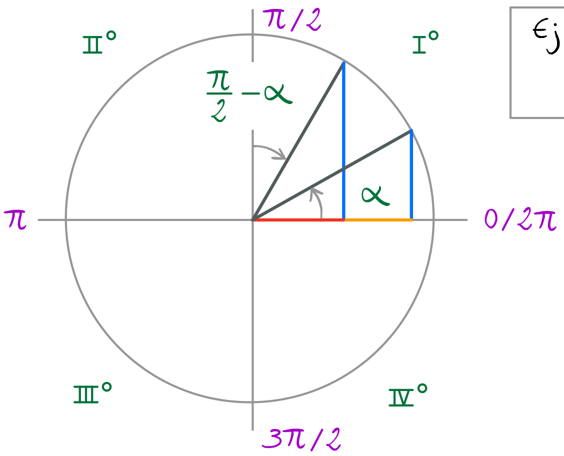
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sen α	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos α	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg α	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞





I° Ángulos complementarios $\boxed{\alpha\,,\ \frac{\pi}{2}-\alpha}$

Ej: $90^\circ-30^\circ=60^\circ$
$\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$

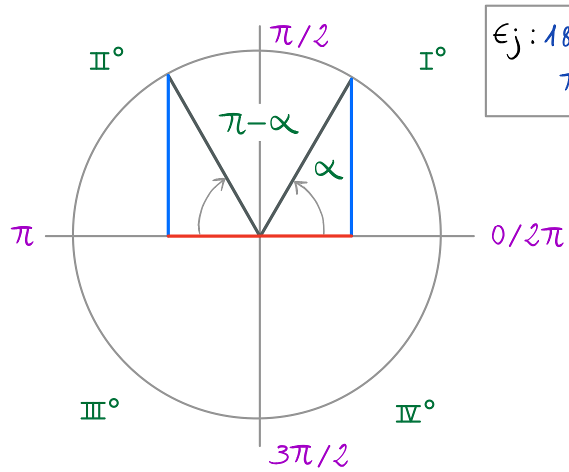


$$\operatorname{sen}\alpha=\cos\left(\frac{\pi}{2}-\alpha\right)\ ;\ \cos\alpha=\operatorname{sen}\left(\frac{\pi}{2}-\alpha\right)$$

$$\operatorname{tg}\alpha=\frac{1}{\operatorname{tg}\left(\frac{\pi}{2}-\alpha\right)}$$

II° Ángulos suplementarios $\boxed{\alpha\,,\ \pi-\alpha}\ \boxed{\text{I}^\circ\ \ \pi-\alpha}$

Ej: $180^\circ-30^\circ=150^\circ$
$\pi-\frac{\pi}{6}=\frac{5\pi}{6}$

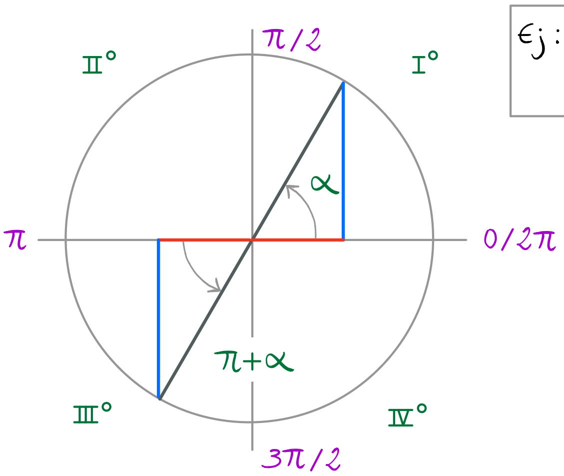


$$\operatorname{sen}\alpha=\operatorname{sen}(\pi-\alpha)\ ;\ \cos\alpha=-\cos(\pi-\alpha)$$

$$\operatorname{tg}\alpha=-\operatorname{tg}(\pi-\alpha)$$

III° Ángulos del 3er cuadrante $\boxed{\alpha\,,\ \pi+\alpha}\ \boxed{\text{I}^\circ\ \ \alpha-\pi}$

Ej: $180^\circ+30^\circ=210^\circ$
$\pi+\frac{\pi}{6}=\frac{7\pi}{6}$

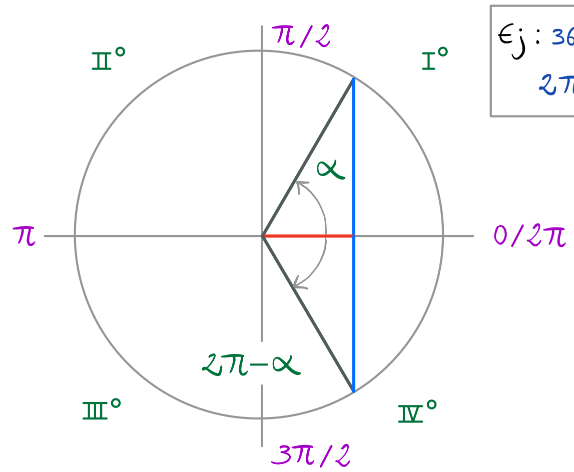


$$\operatorname{sen}\alpha=-\operatorname{sen}(\pi+\alpha)\ ;\ \cos\alpha=-\cos(\pi+\alpha)$$

$$\operatorname{tg}\alpha=\operatorname{tg}(\pi+\alpha)$$

IV° Ángulos opuestos $\boxed{\alpha\,,\ 2\pi-\alpha}\ \boxed{\text{I}^\circ\ 2\pi-\alpha}$

Ej: $360^\circ-30^\circ=330^\circ$
$2\pi-\frac{\pi}{6}=\frac{11\pi}{6}$



$$\operatorname{sen}\alpha=-\operatorname{sen}(2\pi-\alpha)\ ;\ \cos\alpha=\cos(2\pi-\alpha)$$

$$\operatorname{tg}\alpha=-\operatorname{tg}(2\pi-\alpha)$$