Ejercicio 1. **(Puntuación máxima 3.75 puntos)**

a) Sabiendo que $\cos\alpha = -0.5$ y $sen\beta = -0,2$, que $\pi < \alpha < \frac{3\pi}{2}$ y $\frac{3\pi}{2} < \beta < 2\pi$, calcula $\cos(\alpha - 2\beta)$

III IV

b) Calcular cosec 280° sabiendo que $tg(10^\circ) = h$.

c) Resolver la ecuación: $\cos 2x + 5\cos x + 3 = 0$

Ejercicio 2. **(Puntuación máxima 6.25 puntos)**

a) Resolver por el método de Gauss: $\begin{cases} 2x + 5y - 2z = 10 \\ x + 2y - 2z = 4 \\ 4x + 9y - 6z = 18 \end{cases}$

b) Desarrollar la expresión: $3 \cdot |x + 1| - |2x - 6|$

c) Resolver: $9^x - 2 \cdot 3^{x+2} + 81 = 0$

d) Desarrollar por el binomio de Newton: $\left(\frac{3x^2}{y^3} - \frac{2y^5}{x^4}\right)^4 =$

e) Calcular x en los siguientes casos:

$log_{\frac{5}{3}}\left(\sqrt{\frac{27}{125}}\right) = x$ $\qquad$ $log_x\left(\frac{2}{5}\right) = -2$

OBSERVACIÓN: **Cada apartado de los dos ejercicios vale 1,25 puntos**

① a) $\cos\alpha = -0{,}5 \quad \operatorname{sen}\beta = -0{,}2 \quad \alpha \in \text{III} \text{ y } \beta \in \text{IV}$

$$\boxed{\cos(\alpha - 2\beta)} = \cos\alpha \cdot \cos 2\beta + \operatorname{sen}\alpha \cdot \operatorname{sen} 2\beta = \cos\alpha(\cos^2\beta - \operatorname{sen}^2\beta) + \operatorname{sen}\alpha \cdot 2\operatorname{sen}\beta\cos\beta$$

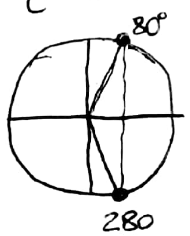
$$= \cos\alpha \cos^2\beta - \cos\alpha \operatorname{sen}^2\beta + 2\operatorname{sen}\alpha \operatorname{sen}\beta \cos\beta \underset{(*)}{=} -0{,}5 \cdot 0{,}96 - (-0{,}5)\cdot(-0{,}2)^2 + 2\cdot(-0{,}866)\cdot(-0{,}2)\cdot 0{,}9798$$

$(*)$ $\operatorname{sen}^2\alpha = 1 - \cos^2\alpha = 1 - (-0{,}5)^2 = 1 - 0{,}25 = 0{,}75 \Rightarrow \left.\begin{array}{l}\operatorname{sen}\alpha = \pm\sqrt{0{,}75} \\ \alpha \in \text{III}\end{array}\right\} \operatorname{sen}\alpha = -0{,}866$

$\cos^2\beta = 1 - \operatorname{sen}^2\beta = 1 - (-0{,}2)^2 = 1 - 0{,}04 = 0{,}96 \Rightarrow \left.\begin{array}{l}\cos\beta = \pm\sqrt{0{,}96} \\ \beta \in \text{IV}\end{array}\right\} \cos\beta = 0{,}9798$

$$= \boxed{-0{,}1206.}$$

b) ¿cosec 280°? conocida $\operatorname{tg} 10° = h$



$$\left.\begin{array}{l}\operatorname{cosec} 280° = -\operatorname{cosec} 80° = -\dfrac{1}{\operatorname{sen} 80°} = -\dfrac{1}{\cos 10°} = -\sec 10° \\ 1 + \operatorname{tg}^2 10 = \sec^2 10 \Rightarrow 1 + h^2 = \sec^2 10 \Rightarrow \sec 10 = \sqrt{1+h^2} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 10 \in \text{I}\end{array}\right\}$$

$$\Rightarrow \boxed{\operatorname{cosec} 280° = -\sqrt{1+h^2}}$$

c) $\cos 2x + 5\cos x + 3 = 0$

$\cos^2 x - \operatorname{sen}^2 x + 5\cos x + 3 = 0 \Rightarrow \cos^2 x - (1 - \cos^2 x) + 5\cos x + 3 = 0$

$\cos^2 x - 1 + \cos^2 x + 5\cos x + 3 = 0 \Rightarrow 2\cos^2 x + 5\cos x + 2 = 0$

$$\cos x = \frac{-5 \pm \sqrt{25 - 4\cdot 2\cdot 2}}{2\cdot 2} = \frac{-5 \pm 3}{4} \begin{cases} -\dfrac{2}{4} = -\dfrac{1}{2} \\ -\dfrac{8}{4} = -2 \text{ NO VALE!!} \end{cases}$$

$$\cos x = -\frac{1}{2} \Rightarrow x = \arccos\left(-\frac{1}{2}\right) = \boxed{\begin{cases} 120° + 2\pi K \\ 240° + 2\pi K \end{cases} \quad \forall K \in \mathbb{Z}}$$

2) a) $\begin{cases} 2x+5y-2z=10 \\ x+2y-2z=4 \\ 4x+9y-6z=18 \end{cases} \xrightarrow{E_2 \leftrightarrow E_1} \begin{cases} x+2y-2z=4 \\ 2x+5y-2z=10 \\ 4x+9y-6z=18 \end{cases} \xrightarrow[E_3-4E_1]{E_2-2E_1} \begin{cases} x+2y-2z=4 \\ / \quad y+2z=2 \\ / \quad y+2z=2 \end{cases}$

$\xrightarrow{E_3=E_2} \begin{cases} x+2y-2z=4 \\ y+2z=2 \end{cases} \xrightarrow{z=t} \begin{array}{l} x+2y=4+2t \\ \underline{y=2-2t} \end{array} \Rightarrow \begin{array}{l} x+2(2-2t)=4+2t \\ x+\cancel{4}-4t=\cancel{4}+2t \\ \underline{x=6t} \end{array}$

$$\boxed{S:(6t,\,2-2t,\,t)\ \forall t\in\mathbb{R}}$$

b) $3\cdot|x+1|-|2x-6| = \begin{cases} 3\cdot(-x-1)-(-2x+6) & \text{si } x<-1 \\ 3(x+1)-(-2x+6) & \text{si } -1\le x<3 \\ 3(x+1)-(2x-6) & \text{si } x\ge 3 \end{cases} \Rightarrow \boxed{\begin{cases} -x-9 & \text{si } x<-1 \\ 5x-3 & \text{si } -1\le x<3 \\ x+9 & \text{si } x\ge 3 \end{cases}}$

$|x+1| = \begin{cases} x+1 & \text{si } x+1\ge 0 \Rightarrow x\ge -1 \\ -x-1 & \text{si } x+1<0 \Rightarrow x<-1 \end{cases}$ $\qquad |2x-6| = \begin{cases} 2x-6 & \text{si } 2x-6\ge 0 \Rightarrow 2x\ge 6 \Rightarrow x\ge 3 \\ -2x+6 & \text{si } 2x-6<0 \Rightarrow 2x<6 \Rightarrow x<3 \end{cases}$

c) $9^x-2\cdot 3^{x+2}+81=0 \Rightarrow (3^2)^x-2\cdot 3^2\cdot 3^x+81=0 \Rightarrow (3^x)^2-18\cdot 3^x+81=0$

$3^x=L \Rightarrow 3^{2x}=L^2 \Rightarrow L^2-18\cdot L+81=0 \Rightarrow \begin{cases} L_1=9 \\ L_2=9 \end{cases}$ (Doble)

$3^x=9=3^2 \Rightarrow \boxed{x=2}$ (Doble)

d) $\left(\frac{3x^2}{y^3}-\frac{2y^5}{x^4}\right)^4=\left(\frac{3x^2}{y^3}\right)^4-4\cdot\left(\frac{3x^2}{y^3}\right)^3\cdot\frac{2y^5}{x^4}+6\cdot\left(\frac{3x^2}{y^3}\right)^2\cdot\left(\frac{2y^5}{x^4}\right)^2-4\left(\frac{3x^2}{y^3}\right)\cdot\left(\frac{2y^5}{x^4}\right)^3+\left(\frac{2y^5}{x^4}\right)^4=$

$=\frac{81x^8}{y^{12}}-4\cdot\frac{27x^6}{y^9}\cdot\frac{2y^5}{x^4}+6\cdot\frac{9x^4}{y^6}\cdot\frac{4y^{10}}{x^8}-4\cdot\frac{3x^2}{y^3}\cdot\frac{8y^{15}}{x^{12}}+\frac{16y^{20}}{x^{16}}=$

$=\boxed{\frac{81x^8}{y^{12}}-\frac{216x^2}{y^4}+\frac{216y^4}{x^4}-\frac{96y^{12}}{x^{10}}+\frac{16y^{20}}{x^{16}}}$

e) a) $\log_{\frac{5}{3}}\left(\sqrt{\frac{27}{125}}\right)=x \Rightarrow \underline{\left(\frac{5}{3}\right)^x}=\sqrt{\frac{27}{125}}=\sqrt{\left(\frac{3}{5}\right)^3}=\left(\frac{3}{5}\right)^{3/2}=\underline{\left(\frac{5}{3}\right)^{-\frac{3}{2}}}$

$$\boxed{x=-\frac{3}{2}}$$

b) $\log_x\left(\frac{2}{5}\right)=-2 \Rightarrow x^{-2}=\frac{2}{5} \Rightarrow x^2=\frac{5}{2} \Rightarrow x=\pm\sqrt{\frac{5}{2}} \Rightarrow$

$\boxed{x=+\sqrt{\frac{5}{2}}}$ (la base debe ser positiva)