1.- a) **(1,25 puntos)** Desarrollar por el binomio de Newton: $\left(3x^2y^3 - \frac{2x^3}{y^2}\right)^5 =$

b) **(1,25 puntos)** Hallar el lugar que ocupa el término de grado 84 en el desarrollo del binomio $(3x^3 + 2x^5)^{20}$

2.- a) **(1,25 puntos)** Resolver la ecuación: $\binom{x}{3} = 2\binom{x}{1}$

b) **(1,25 puntos)** Calcular, dejando el resultado final en forma de "entorno":

$$\{x \in R \,/\, |x+3| \leq 2\} \quad \cap \quad E[2\ ,5]$$

3.- a) **(1,25 puntos)** Dada $A = \frac{81 \cdot \sqrt[7]{x^5}}{y^3 \cdot \sqrt[4]{27z^3}}$ calcular $\log_3 A$

b) **(1,25 puntos)** Sabiendo que $log_2\ V = m$ calcular:

$$\log_2 \frac{V}{8} - \log_2 \frac{16}{\sqrt[4]{V^3}} + \log_2 (32V)$$

4.- a) **(1,25 puntos)** Desarrollar la expresión: $3 - |x+2| - |2-3x| =$

b) a) **(1,25 puntos)** Simplificar la expresión: $\frac{x^2 - xy}{x^3y - xy^3} : \frac{y}{x^2 + 2xy + y^2} =$

1 a) $\left(3x^2y^3 - \frac{2x^3}{y^2}\right)^5 = \binom{5}{0}(3x^2y^3)^5 - \binom{5}{1}(3x^2y^3)^4 \cdot \frac{2x^3}{y^2} +$

$+\binom{5}{2}(3x^2y^3)^3 \cdot \left(\frac{2x^3}{y^2}\right)^2 - \binom{5}{3}(3x^2y^3)^2 \cdot \left(\frac{2x^3}{y^2}\right)^3 +$

$+\binom{5}{4} 3x^2y^3 \cdot \left(\frac{2x^3}{y^2}\right)^4 - \binom{5}{5}\left(\frac{2x^3}{y^2}\right)^5 =$

$= 1 \cdot 3^5 \cdot x^{10} \cdot y^{15} - 5 \cdot 3^4 \cdot x^8 y^{12} \cdot \frac{2x^3}{y^2} + 10 \cdot 3^3 x^6 y^9 \cdot \frac{2^2 x^6}{y^4}$

$-10 \cdot 3^2 \cdot x^4 \cdot y^6 \cdot \frac{2^3 \cdot x^9}{y^6} + 5 \cdot 3x^2y^3 \cdot \frac{2^4 x^{12}}{y^8} - 1 \cdot \frac{2^5 x^{15}}{y^{10}} =$

$= \boxed{243x^{10}y^{15} - 810x^{11}y^{10} + 1080x^{12}y^5 - 720x^{13} + 240\frac{x^{14}}{y^5} - 32\frac{x^{15}}{y^{10}}}$

b) T_k de grado 84 ¿k? $(3x^3+2x^5)^{20}$

$T_k = \binom{20}{k-1}(3x^3)^{20-(k-1)} \cdot (2x^5)^{k-1} =$

$= \binom{20}{k-1} 3^{21-k} (x^3)^{21-k} \cdot 2^{k-1} \cdot (x^5)^{k-1} \Rightarrow$

$\Rightarrow x^{63-3k} \cdot x^{5k-5} = x^{84} \Rightarrow$

$\Rightarrow x^{58+2k} = x^{84} \Rightarrow 58+2k = 84 \Rightarrow$

$\Rightarrow 2k = 84-58 \Rightarrow 2k = 26 \Rightarrow \boxed{k=13}$

2 a) $\binom{x}{3} = 2\binom{x}{1} \Rightarrow \dfrac{x!}{3!\,(x-3)!} = 2\cdot\dfrac{x!}{1!\,(x-1)!} \Rightarrow$

$$\Rightarrow \frac{x\cdot(x-1)(x-2)\cdot\cancel{(x-3)!}}{3\cdot2\cdot1\cdot\cancel{(x-3)!}} = \frac{2\cdot x\cdot\cancel{(x-1)!}}{1\cdot\cancel{(x-1)!}}$$

Multiplico en cruz igualando:

$$x\cdot(x-1)\cdot(x-2) = 12x$$

$$x\cdot(x-1)\cdot(x-2) - 12x = 0$$

$$x\cdot\big((x-1)\cdot(x-2) - 12\big) = 0$$

$$x\cdot(x^2-3x+2-12) = 0$$

$$x\cdot(x^2-3x-10) = 0$$

$$\begin{cases} \underline{x=0} \\ x^2-3x-10=0 \left\{ \begin{array}{l} + \; 3 \\ \cdot \; -10 \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \underline{x=5} \\ \underline{x=-2} \end{array} \right. \end{cases}$$

Por tener $\binom{x}{3} \Rightarrow \underline{x \geq 3} \Rightarrow \begin{cases} x=0 \\ x=-2 \end{cases}$ NO VALEN

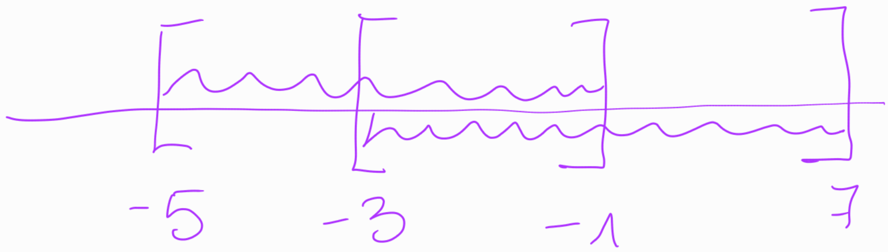
Solución: $\boxed{x=5}$

b) $\{x\in\mathbb{R} \,/\, |x+3|\leq 2\} \cap E[2,5]$

$|x+3| \leq 2 \Leftrightarrow E[-3,2] = [-3-2, -3+2] =$

$= [-5, -1]$

$E[2,5] = [2-5, 2+5] = [-3, 7]$



$[-5,-1] \cap [-3,7] = [-3,-1]$

$\text{Centro} = \dfrac{-3+(-1)}{2} = \dfrac{-4}{2} = -2$

$\text{Radio} = \dfrac{(-1)-(-3)}{2} = \dfrac{-1+3}{2} = \dfrac{2}{2} = 1$

Solución: $\boxed{E[-2,1]}$

3 a) $A = \dfrac{81 \cdot \sqrt[7]{x^5}}{y^3 \cdot \sqrt[4]{27z^3}}$ ¿$\log_3 A$?

$\log_3 A = \log_3 \left(\dfrac{81 \cdot x^{5/7}}{y^3 \cdot 3^{3/4} \cdot z^{3/4}} \right) =$

$= \log_3 (81 \cdot x^{5/7}) - \log_3 (y^3 \cdot 3^{3/4} \cdot z^{3/4}) =$

$= \log_3 81 + \log_3 x^{5/7} - [\log_3 y^3 + \log_3 3^{3/4} + \log_3 z^{3/4}] =$

$= \log_3 3^4 + \frac{5}{7} \log_3 x - 3 \log_3 y - \frac{3}{4} \log_3 3 - \frac{3}{4} \log_3 z =$

$= 4 \log_3 3 + \frac{5}{7} \log_3 x - 3 \log_3 y - \frac{3}{4} \cdot 1 - \frac{3}{4} \log_3 z =$

$= 4 \cdot 1 + \frac{5}{7} \log_3 x - 3 \log_3 y - \frac{3}{4} - \frac{3}{4} \log_3 z =$

$$= \boxed{\frac{13}{4} + \frac{5}{7} \log_3 x - 3 \log_3 y - \frac{3}{4} \log_3 z}$$

b) Si $\log_2 V = m$

$\log_2 \frac{V}{8} - \log_2 \frac{16}{\sqrt[4]{V^3}} + \log_2 (32V) =$

$= \log_2 V - \log_2 8 - (\log_2 16 - \log_2 V^{3/4}) + \log_2 32 + \log_2 V$

$\overset{(*)}{=} m - 3 - (4 - \frac{3}{4} \log_2 V) + 5 + m =$

$= m - 3 - 4 + \frac{3}{4} \cdot m + 5 + m = 2m + \frac{3}{4} m - 2 =$

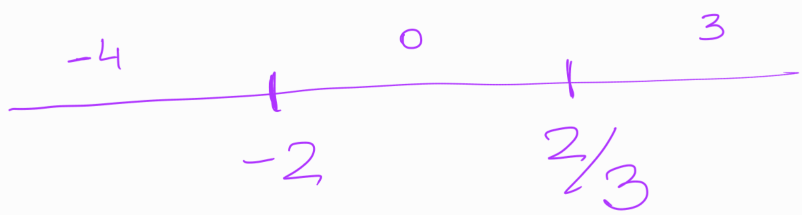
$$= \boxed{\frac{11}{4}m - 2}$$

$$* \begin{cases} \log_2 8 = \log_2 2^3 = 3\log_2 2 = 3\cdot 1 = 3 \\ \log_2 16 = \log_2 2^4 = 4\log_2 2 = 4\cdot 1 = 4 \\ \log_2 32 = \log_2 2^5 = 5\log_2 2 = 5\cdot 1 = 5 \end{cases}$$

4 a) $3 - |x+2| - |2-3x| = (*)$

$$|x+2| = \begin{cases} x+2 \text{ si } x+2 \geq 0 \Rightarrow x \geq -2 \\ -x-2 \text{ si } x+2 < 0 \Rightarrow x < -2 \end{cases}$$

$$|2-3x| = \begin{cases} 2-3x \text{ si } 2-3x \geq 0 \Rightarrow 2 \geq 3x \Rightarrow x \leq \frac{2}{3} \\ -2+3x \text{ si } 2-3x < 0 \Rightarrow 2 < 3x \Rightarrow x > \frac{2}{3} \end{cases}$$



$$\overset{*}{=} \begin{cases} 3-(-x-2)-(2-3x) \text{ si } x < -2 \\ 3-(x+2)-(2-3x) \text{ si } -2 \leq x \leq \frac{2}{3} \\ 3-(x+2)-(-2+3x) \text{ si } x > \frac{2}{3} \end{cases} =$$

$$= \boxed{\begin{cases} 3+4x \text{ si } x < -2 \\ -1+2x \text{ si } -2 \leq x \leq \frac{2}{3} \\ 3-4x \text{ si } x > \frac{2}{3} \end{cases}}$$

b) $\frac{x^2-xy}{x^3y-xy^3} \div \frac{y}{x^2+2xy+y^2} = \frac{x(x-y)}{xy(x^2-y^2)} \div \frac{y}{(x+y)^2}$

$= \frac{\cancel{x} \cdot \cancel{(x-y)} \cdot (x+y)^{\cancel{2}}}{\cancel{x} \cdot y \cdot \underbrace{\cancel{(x+y)}\cancel{(x-y)}}_{(x^2-y^2)} \cdot y} = \boxed{\frac{x+y}{y^2}}$