1.- a) **(1,25 ptos)** Racionalizar: $\dfrac{4}{\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}}$

b) **(1.25 ptos)** Calcular el valor de:

$$\log\left(\sqrt[5]{0{,}001}\right) + Ln\,\frac{1}{e} + \log_{\frac{1}{3}}\left(\sqrt{\sqrt{3}}\cdot\sqrt[5]{\sqrt{3}:\sqrt[3]{27}}\right)$$

2.- a) **(1,25 ptos)** Dada $A = \dfrac{16\cdot\sqrt[7]{x^5}}{y^3\cdot\sqrt[4]{8z^3}}$ calcular $log_2 A$

b) **(1,25 ptos)** Sabiendo que $log_3\ M = a$ calcular:

$$log_3\frac{M}{27} - log_3\frac{81}{\sqrt[4]{M^3}} + log_3\ (243M)$$

3.- a) **(1,25 ptos)** Desarrollar la expresión: $3 - |x+1| + |5-2x| =$

b) **(1,25 ptos)** Representa en la recta real $\sqrt{40}$

4.- a) **(1,25 ptos)** Simplifica: $\dfrac{(-20)^4\cdot 21^{-2}\cdot 14}{(-12)^{-3}\cdot 100^2} =$

b) **(1,25 ptos)** Calcular: $\sqrt{\dfrac{a}{2b}}\cdot\left(\sqrt[3]{\dfrac{4\,b^2}{a^2}} : \sqrt{\dfrac{2b}{a}}\right)$

1.- a) (1,25 ptos) Racionalizar: $\dfrac{4}{\sqrt{4+2\sqrt{3}}-\sqrt{4-2\sqrt{3}}}$

$$\frac{4}{\sqrt{4+2\sqrt{3}}-\sqrt{4-2\sqrt{3}}}=\frac{4\left(\sqrt{4+2\sqrt{3}}+\sqrt{4-2\sqrt{3}}\right)}{\left(\sqrt{4+2\sqrt{3}}-\sqrt{4-2\sqrt{3}}\right)\left(\sqrt{4+2\sqrt{3}}+\sqrt{4-2\sqrt{3}}\right)}=$$

$$=\frac{4\left(\sqrt{4+2\sqrt{3}}+\sqrt{4-2\sqrt{3}}\right)}{\left(\cancel{\sqrt{4+2\sqrt{3}}}\right)^{\cancel{2}}-\left(\cancel{\sqrt{4-2\sqrt{3}}}\right)^{\cancel{2}}}=\frac{\cancel{4}\cdot\left(\sqrt{4+2\sqrt{3}}+\sqrt{4-2\sqrt{3}}\right)}{\cancel{4}\cdot\sqrt{3}}=$$

$$=\frac{\sqrt{3}\left(\sqrt{4+2\sqrt{3}}+\sqrt{4-2\sqrt{3}}\right)}{\sqrt{3}\cdot\sqrt{3}}=\boxed{\frac{\sqrt{12+6\sqrt{3}}+\sqrt{12-6\sqrt{3}}}{3}}$$

b) (1.25 ptos) Calcular el valor de:

$$\underbrace{\log\left(\sqrt[5]{0{,}001}\right)}_{(1)}+\underbrace{Ln\frac{1}{e}}_{(2)}+\underbrace{\log_{\frac{1}{3}}\left(\sqrt{\sqrt{3}}\cdot\sqrt[5]{\sqrt{3}:\sqrt[3]{27}}\right)}_{(3)}=(*)$$

(1) $\log\left(\sqrt[5]{0{,}001}\right)=\log\sqrt[5]{10^{-3}}=\log 10^{-3/5}=-\dfrac{3}{5}$

(2) $Ln\dfrac{1}{e}=Ln\, e^{-1}=-1$

(3) $\log_{1/3}\left(\sqrt{\sqrt{3}}\cdot\sqrt[5]{\sqrt{3}:\sqrt[3]{27}}\right)=\log_{1/3}\left(\sqrt[4]{3}\cdot\sqrt[5]{\sqrt{3}:\cancel{\sqrt[3]{3^3}}}\right)$

$$=\log_{\frac{1}{3}}\left(3^{1/4}\cdot\sqrt[5]{3^{1/2}:3}\right)=\log_{1/3}\left(3^{1/4}\cdot\left(3^{-1/2}\right)^{1/5}\right)=$$

$$= \log_{\frac{1}{3}} \left(3^{1/4} \cdot 3^{-\frac{1}{10}}\right) = \log_{\frac{1}{3}} 3^{\frac{3}{20}} = \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-\frac{3}{20}} = -\frac{3}{20}$$

$$\underline{\underline{(*)}} \quad -\frac{3}{5} + (-1) + \left(-\frac{3}{20}\right) = \frac{-12-20-3}{20} = \frac{-35}{20} = \boxed{\frac{-7}{4}}$$

2.- a) **(1,25 ptos)** Dada $A = \dfrac{16 \cdot \sqrt[7]{x^5}}{y^3 \cdot \sqrt[4]{8z^3}}$ calcular $\log_2 A$

$$\log_2 A = \log_2 \frac{16 \cdot \sqrt[7]{x^5}}{y^3 \cdot \sqrt[4]{8z^3}} = \log_2 \left(16 \cdot x^{5/7}\right) - \log_2 \left(y^3 \cdot (2^3 \cdot z^3)^{1/4}\right)$$

$$= \log_2 2^4 + \log_2 x^{5/7} - \left(\log_2 y^3 + \log_2 (2^3 \cdot z^3)^{1/4}\right) =$$

$$= 4 + \frac{5}{7} \log_2 x - 3 \log_2 y - \frac{1}{4} \log_2 (2^3 \cdot z^3) =$$

$$= 4 + \frac{5}{7} \log_2 x - 3 \log_2 y - \frac{1}{4} \left(\log_2 2^3 + \log_2 z^3\right) =$$

$$= 4 + \frac{5}{7} \log_2 x - 3 \log_2 y - \frac{1}{4} \left(3 + 3 \log_2 z\right) =$$

$$= 4 + \frac{5}{7} \log_2 x - 3 \log_2 y - \frac{3}{4} - \frac{3}{4} \log_2 z =$$

$$= \boxed{\frac{13}{4} + \frac{5}{7} \log_2 x - 3 \log_2 y - \frac{3}{4} \log_2 z}$$

b) **(1,25 ptos)** Sabiendo que $log_3 \ M = a$ calcular:

$$\underset{(1)}{log_3 \frac{M}{27}} - \underset{(2)}{log_3 \frac{81}{\sqrt[4]{M^3}}} + \underset{(3)}{log_3 \ (243M)} = (*)$$

$$(1) \ \log_3 \frac{M}{27} = \log_3 M - \log_3 3^3 = a - 3$$

$$(2) \ \log_3 \frac{81}{\sqrt[4]{M^3}} = \log_3 3^4 - \log_3 M^{3/4} = 4 - \frac{3}{4}\log_3 M =$$

$$= 4 - \frac{3}{4} a$$

$$(3) \ \log_3 (243M) = \log_3 3^5 + \log_3 M = 5 + a$$

$$(*) \quad a - 3 - \left(4 - \frac{3}{4}a\right) + 5 + a =$$

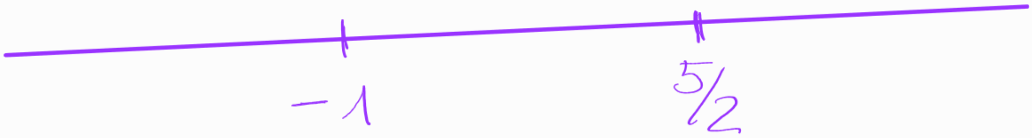
$$= a - 3 - 4 + \frac{3}{4}a + 5 + a =$$

$$= \boxed{-2 + \frac{11}{4}a}$$

3.- a) **(1,25 ptos)** Desarrollar la expresión: 3 - |x + 1| + |5 - 2x| = (*)

$$|x+1| = \begin{cases} \underline{x+1} \ \text{ si } \ x+1 \geq 0 \longrightarrow \underline{x \geq -1} \\ \underline{-x-1} \ \text{ si } \ x+1 < 0 \longrightarrow \underline{x < -1} \end{cases}$$

$$|5-2x| = \begin{cases} 5-2x & \text{si } 5-2x \geq 0 \Rightarrow 5 \geq 2x \Rightarrow \frac{5}{2} \geq x \Rightarrow x \leq \frac{5}{2} \\ -5+2x & \text{si } 5-2x < 0 \Rightarrow 5 < 2x \Rightarrow \frac{5}{2} < x \Rightarrow x > \frac{5}{2} \end{cases}$$



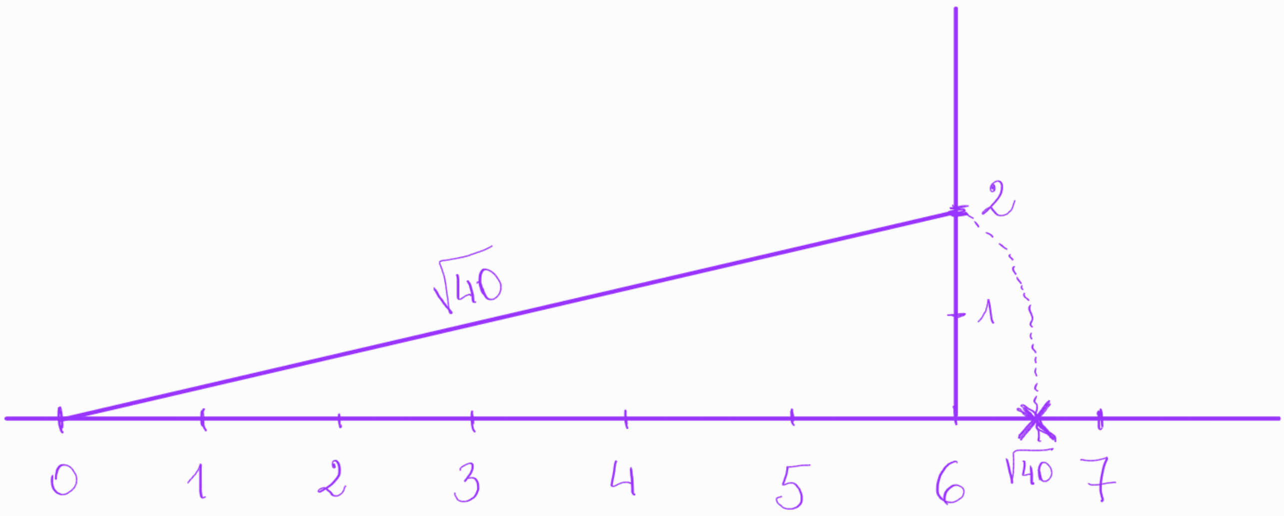
$$(*) = \begin{cases} \text{Si } x < -1 : & 3-(-x-1)+(5-2x) \\ \text{Si } -1 \leq x \leq \frac{5}{2} : & 3-(x+1)+(5-2x) \\ \text{Si } x > \frac{5}{2} : & 3-(x+1)+(-5+2x) \end{cases} =$$

$$= \begin{cases} \text{Si } x < -1 : & 3+x+1+5-2x \\ \text{Si } -1 \leq x \leq \frac{5}{2} : & 3-x-1+5-2x \\ \text{Si } x > \frac{5}{2} : & 3-x-1-5+2x \end{cases} =$$

$$= \boxed{\begin{cases} -x+9 & \text{si } x < -1 \\ -3x+7 & \text{si } -1 \leq x \leq \frac{5}{2} \\ x-3 & \text{si } x > \frac{5}{2} \end{cases}}$$

b) (1,25 ptos) Representa en la recta real $\sqrt{40}$

$(\sqrt{40})^2 = 6^2 + 2^2$ (Tª Pitágoras) $\quad 6 \leq \sqrt{40} \leq 7$



4.- a) (1,25 ptos) Simplifica: $\dfrac{(-20)^4\cdot 21^{-2}\cdot 14}{(-12)^{-3}\cdot 100^2} = (*)$

$(-20)^4 = 20^4$ (exponente par)

$(-12)^3 = -12^3$ (exponente impar)

$$(*) = \frac{20^4\cdot 14\cdot(-12)^3}{21^2\cdot 100^2} = \frac{(2^2\cdot 5)^4\cdot 2\cdot 7\cdot\left(-(2^2\cdot 3)^3\right)}{(3\cdot 7)^2\cdot(2^2\cdot 5^2)^2} =$$

$$= -\frac{2^8\cdot\cancel{5^4}\cdot 2\cdot 7\cdot 2^6\cdot 3^3}{3^2\cdot 7^2\cdot 2^4\cdot\cancel{5^4}} = -\frac{2^{15}\cdot 7\cdot 3^3}{3^2\cdot 7^2\cdot 2^4} =$$

$$= \boxed{-2^{11}\cdot 7^{-1}\cdot 3} = \boxed{-\frac{2^{11}\cdot 3}{7}}$$

b) (1,25 ptos) Calcular: $\sqrt{\dfrac{a}{2b}}\cdot\left(\sqrt[3]{\dfrac{4\,b^2}{a^2}} : \sqrt{\dfrac{2b}{a}}\right) =$

$$= \sqrt{\frac{a}{2b}} \cdot \left(\sqrt[6]{\frac{4^2 \cdot b^4}{a^4}} : \sqrt[6]{\frac{2^3 b^3}{a^3}} \right) = \sqrt{\frac{a}{2b}} \cdot \sqrt[6]{\frac{4^2 \cdot b^4 \cdot a^3}{a^4 \cdot 2^3 \cdot b^3}} =$$

$$= \sqrt[6]{\frac{a^3}{2^3 \cdot b^3}} \cdot \sqrt[6]{\frac{2^4 \cdot b^4 \cdot a^3}{a^4 \cdot 2^3 \cdot b^3}} = \sqrt[6]{\frac{a^3 \cdot 2^4 \cdot b^4 \cdot a^3}{2^3 \cdot b^3 \cdot a^4 \cdot 2^3 \cdot b^3}} =$$

$$= \sqrt[6]{\frac{2^4 \cdot a^6 \cdot b^4}{2^6 \cdot a^4 \cdot b^6}} = \sqrt[6]{\frac{a^2}{2^2 \cdot b^2}} = \sqrt[6]{\left(\frac{a}{2b}\right)^2} = \sqrt[3]{\frac{a}{2b}} =$$

$$= \frac{\sqrt[3]{a}}{\sqrt[3]{2b}} = \frac{\sqrt[3]{a}\,\sqrt[3]{2^2 b^2}}{\sqrt[3]{2b} \cdot \sqrt[3]{2^2 \cdot b^2}} = \frac{\sqrt[3]{4ab^2}}{\sqrt[3]{2^3 \cdot b^3}} = \boxed{\frac{\sqrt[3]{4ab^2}}{2b}}$$