

REPASO DERIVADAS

Calcula as derivadas das seguintes funcións, e se se pode simplifica:

$$a)y = \ln \sqrt{\frac{1+x}{1-x}}$$

$$b)y = \frac{x}{x+1} \operatorname{tag} x$$

$$c)y = \frac{\operatorname{sen}^2 x \cos x^2 - \arccos \sqrt{x}}{3}$$

$$d)y = \ln (\cos x)^2 - \sqrt{4x^2 - 5x} + 3 \cdot 5^{6-3x^3}$$

$$e)y = \left(\frac{\operatorname{sen} x}{x}\right)^x$$

$$a) y = \ln \sqrt{\frac{1+x}{1-x}}$$

Aplico propiedades dos logaritmos:

$$\begin{aligned} y &= \ln \sqrt{\frac{1+x}{1-x}} = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) = \\ &= \frac{1}{2} [\ln(1+x) - \ln(1-x)] \end{aligned}$$

Derivo agora:

$$y' = \frac{1}{2} \cdot \left[\frac{1}{1+x} - \frac{(-1)}{1-x} \right] = \frac{1}{2} \left[\frac{1}{1+x} + \frac{1}{1-x} \right] \Rightarrow$$

$$y' = \frac{1}{2} \left[\frac{1-\cancel{x} + 1+\cancel{x}}{1-x^2} \right] = \frac{1}{1-x^2}$$

$$b) y = \underbrace{\frac{x}{x+1}}_f \cdot \underbrace{\tan x}_g$$

$$y' = \frac{1 \cdot (x+1) - x \cdot 1}{(x+1)^2} \cdot \tan x + \frac{x}{x+1} \cdot \frac{1}{\cos^2 x}$$

$$= \frac{1}{(x+1)^2} \tan x + \frac{x}{x+1} \cdot \frac{1}{\cos^2 x}$$

$$c) y = \frac{\sin^2 x \cos^2 x - \arccos \sqrt{x}}{3} =$$

$$= \frac{1}{3} (\sin^2 x \cos^2 x - \arccos \sqrt{x})$$

$$y' = \frac{1}{3} \left[2 \sin x \cos x \cos^2 x + \sin^2 x 2 \cos x \cdot (-\sin x) + \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}} \right]$$

$$y' = \frac{1}{3} \left[2 \sin x \cos^3 x - 2 \cos x \sin^3 x + \frac{1}{2\sqrt{x} \cdot \sqrt{1-x}} \right]$$

$$= \frac{1}{3} \left[\underbrace{(\cos^2 x - \sin^2 x)}_{\cos 2x} \underbrace{2 \sin x \cos x}_{\sin 2x} + \frac{1}{2\sqrt{x} \sqrt{1-x}} \right] =$$

$$= \frac{1}{3} \left[\cos 2x \cdot \sin 2x + \frac{1}{2\sqrt{x(1-x)}} \right]$$

$$d) y = \ln(\cos x)^2 - \sqrt{4x^2 - 5x} + 3 \cdot 5^{6-3x^3}$$

$$y = 2 \ln(\cos x) - \sqrt{4x^2 - 5x} + 3 \cdot 5^{6-3x^3}$$

$$y' = 2 \frac{(-\sin x)}{\cos x} - \frac{8x-5}{2\sqrt{4x^2-5x}} + 3 \cdot 5^{6-3x^3} \cdot \ln 5 \cdot (-9x^2)$$

$$y' = \frac{-2\sin x}{\cos x} - \frac{8x-5}{2\sqrt{4x^2-5x}} - 27x^2 \ln 5 \cdot 5^{6-3x^3}$$

$$e) y = \left(\frac{\sin x}{x}\right)^x$$

Derivació Logarítmica

$$\ln y = x \cdot \ln\left(\frac{\sin x}{x}\right)$$

$$\frac{y'}{y} = 1 \cdot \ln\left(\frac{\sin x}{x}\right) + \cancel{x} \cdot \frac{1}{\cancel{\frac{\sin x}{x}}} \cdot \frac{\cos x \cdot \cancel{x} - \sin x}{\cancel{x}^2}$$

$$y' = \left(\frac{\sin x}{x}\right)^x \cdot \left[\ln\left(\frac{\sin x}{x}\right) + \frac{x \cos x - \sin x}{\sin x} \right]$$