RELACIÓN DE EJERCICIOS DE INTEGRALES.

1. $\int x^5 dx$

$$\int x^5 dx = \frac{x^{5+1}}{5+1} + k = \frac{x^6}{6} + k$$

2. $\int (x + \sqrt{x})dx$

$$\int (x + \sqrt{x})dx = \int (x + x^{1/2})dx = \frac{x^{1+1}}{1+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + k = \frac{x^2}{2} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{x^2}{2} + \frac{2\sqrt{x^3}}{3} + k =$$

$$= \frac{x^2}{2} + \frac{2x\sqrt{x}}{3} + k$$

3. $\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4} \right) dx$ Sol: $6\sqrt{x} - \frac{1}{10} x^2 \sqrt{x} + k$

$$\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4} \right) dx = \int (3x^{-\frac{1}{2}} - \frac{1}{4} x^{\frac{3}{2}})dx = 3 \cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - \frac{1}{4} \cdot \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + k = 3 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{4} \cdot \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + k =$$

$$= 6\sqrt{x} - \frac{1}{2} \frac{\sqrt{x^5}}{5} + k = 6\sqrt{x} - \frac{1}{10} x^2 \sqrt{x} + k$$

4. $\int \frac{x^2 dx}{\sqrt{x}}$ Sol: $\frac{2}{5} x^2 \sqrt{x} + k$

$$\int \frac{x^2 dx}{\sqrt{x}} = \int \frac{x^2}{x^{\frac{1}{2}}} dx = \int x^{\frac{3}{2}} dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + k = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + k = \frac{2\sqrt{x^5}}{5} + k = \frac{2x^2\sqrt{x}}{5} + k$$

5. $\int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2 \right) dx$ Sol: $-\frac{1}{x} - \frac{8}{\sqrt{x}} + 2x + k$

$$\int \left(\frac{1}{x^2} + \frac{4}{x\sqrt{x}} + 2 \right) dx = \int (x^{-2} + 4x^{-\frac{3}{2}} + 2)dx = \frac{x^{-2+1}}{-2+1} + 4 \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + 2x + k =$$

$$= \frac{x^{-1}}{-1} + 4 \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 2x + k = -\frac{1}{x} - 8 \cdot \frac{1}{x^{\frac{1}{2}}} + 2x + k = -\frac{1}{x} - \frac{8}{\sqrt{x}} + 2x + k$$

6. $\displaystyle\int \frac{dx}{\sqrt[4]{x}}$ Sol: $\dfrac{4}{3}\sqrt[4]{x^3}+k$

$$\int \frac{dx}{\sqrt[4]{x}}=\int x^{-\frac{1}{4}}dx=\frac{x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1}+k=\frac{x^{\frac{3}{4}}}{\frac{3}{4}}+k=\frac{4x^{\frac{3}{4}}}{3}+k=\frac{4}{3}\sqrt[4]{x^3}+k$$

7. $\displaystyle\int\left(x^2+\frac{1}{\sqrt[3]{x}}\right)^2dx$ Sol: $\dfrac{x^5}{5}+\dfrac{3}{4}x^2\sqrt[3]{x^2}+3\sqrt[3]{x}+k$

$$\int\left(x^2+\frac{1}{\sqrt[3]{x}}\right)^2dx=\int\left(x^2+x^{-\frac{1}{3}}\right)^2dx=\int(x^4+2x^{\frac{5}{3}}+x^{-\frac{2}{3}}).dx=\frac{x^5}{5}+2\cdot\frac{x^{\frac{8}{3}}}{\frac{8}{3}}+\frac{x^{\frac{1}{3}}}{\frac{1}{3}}+K=$$

$$=\frac{1}{5}\cdot x^5+\frac{3}{4}\cdot x^{\frac{8}{3}}+3.x^{\frac{1}{3}}+K=\frac{1}{5}\cdot x^5+\frac{3}{4}\cdot\sqrt[3]{x^8}+3.\sqrt[3]{x}+K=$$

$$=\frac{1}{5}\cdot x^5+\frac{3}{4}\cdot x^2.\sqrt[3]{x^2}+3.\sqrt[3]{x}+K$$

8. $\displaystyle\int\frac{L\,x}{x}\cdot dx$ Sol: $\dfrac{1}{2}L^2x+k$

$$\int\frac{\mathrm{Ln}(x)}{x}\cdot dx=\int\mathrm{Ln}(x)\cdot\frac{1}{x}\cdot dx=\left\{\int f^{\alpha}\cdot f'dx\right\}=\frac{(\mathrm{Ln}(x))^2}{2}+k$$

9. $\displaystyle\int \mathrm{tg}\,x\cdot\sec^2 x\cdot dx$ Sol: $\dfrac{1}{2}\cdot\mathrm{tg}^2\,x+k$

$$\int\mathrm{tg}\,x\cdot\sec^2 x\cdot dx=\left\{\int f^1\cdot f'dx\right\}=\frac{(\mathrm{tg}\,x)^2}{2}+k$$

10. $\displaystyle\int \mathrm{sen}^2\,x\cdot\cos x\cdot dx$ Sol: $\dfrac{\mathrm{sen}^3\,x}{3}+k$

$$\int\mathrm{sen}^2\,x\cdot\cos x\cdot dx==\left\{\int f^2\cdot f'dx\right\}=\frac{\mathrm{sen}^3\,x}{3}+k$$

11. $\displaystyle\int \cos^3 x\cdot\mathrm{sen}\,x\cdot dx$ Sol: $-\dfrac{\cos^4 x}{4}+k$

$$\int\cos^3 x\cdot\mathrm{sen}\,x\cdot dx=-\int\underbrace{\cos^3 x}_{f^3}\cdot\underbrace{(-\mathrm{sen}\,x)}_{f'}\cdot dx=\frac{\cos^4 x}{4}+k$$

12. $\int x\sqrt{x^2+1}\cdot dx$ Sol: $\frac{1}{3}\sqrt{(x^2+1)^3}+k$

$$\int x\sqrt{x^2+1}\cdot dx=\frac{1}{2}\int \underbrace{2x}_{f'}\cdot\underbrace{(x^2+1)^{\frac{1}{2}}}_{f^{1/2}}dx=\frac{1}{2}\cdot\frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}}+k=\frac{\sqrt{(x^2+1)^3}}{3}+k$$

13. $\int \frac{xdx}{\sqrt{2x^2+3}}$ Sol: $\frac{1}{2}\sqrt{2x^2+3}+k$

$$\int \frac{xdx}{\sqrt{2x^2+3}}=\frac{1}{4}\int \underbrace{4x}_{f'}\cdot\underbrace{(2x^2+3)^{-\frac{1}{2}}}_{f^{-1/2}}dx=\frac{1}{4}\cdot\frac{(2x^2+3)^{\frac{1}{2}}}{\frac{1}{2}}+k=\frac{1}{2}\sqrt{2x^2+3}+k \qquad \text{o}$$

$$\int \frac{xdx}{\sqrt{2x^2+3}}=\frac{2}{4}\int\frac{4xdx}{2\sqrt{2x^2+3}}=\left\{\int\frac{f'}{2\sqrt{f}}dx=\sqrt{f}+k\right\}=\frac{1}{2}\sqrt{2x^2+3}+k$$

14. $\int \frac{x^2dx}{\sqrt{x^3+1}}$ Sol: $\frac{2}{3}\sqrt{x^3+1}+k$

$$\int \frac{x^2dx}{\sqrt{x^3+1}}=\int x^2(x^3+1)^{-\frac{1}{2}}dx=\frac{1}{3}\int 3x^2(x^3+1)^{-\frac{1}{2}}dx=\frac{1}{3}\cdot\frac{(x^3+1)^{\frac{1}{2}}}{\frac{1}{2}}+k=\frac{2}{3}\sqrt{x^3+1}+k$$

$$\int \frac{x^2dx}{\sqrt{x^3+1}}=\frac{2}{3}\int\frac{3x^2dx}{2\sqrt{x^3+1}}=\left\{\int\frac{f'}{2\sqrt{f}}dx=\sqrt{f}+k\right\}=\frac{2}{3}\sqrt{x^3+1}+k$$

15. $\int \frac{\cos x}{\operatorname{sen}^2 x}dx$ Sol: $-\frac{1}{\operatorname{sen} x}+k$

$$\int \frac{\cos x}{\operatorname{sen}^2 x}dx=\int \underbrace{\cos x}_{f'}\cdot\underbrace{\operatorname{sen}^{-2} x}_{f^{-2}}dx=\frac{\operatorname{sen}^{-1} x}{-1}+k=-\frac{1}{\operatorname{sen} x}+k$$

16. $\int x\cdot(x^2+1)^4dx$ Sol: $\frac{(x^2+1)^5}{10}+k$

$$\int x\cdot(x^2+1)^4dx=\frac{1}{2}\int \underbrace{2x}_{f'}\cdot\underbrace{(x^2+1)^4}_{f^4}dx=\frac{1}{2}\cdot\frac{(x^2+1)^5}{5}+k=\frac{(x^2+1)^5}{10}+k$$

17. $\int \frac{\operatorname{sen} x}{\cos^3 x}\cdot dx$ Sol: $\frac{1}{2\cos^2 x}+k$

$$\int \frac{\operatorname{sen} x}{\cos^3 x}\cdot dx=-\int \underbrace{-\operatorname{sen} x}_{f'}\cdot\underbrace{\cos^{-3} x}_{f^{-3}}dx=-\frac{\cos^{-2} x}{-2}+k=\frac{1}{2\cos^2 x}+k$$

18. $\int \frac{\operatorname{tg} x}{\cos^2 x} \cdot dx$ Sol: $\frac{\operatorname{tg}^2 x}{2} + k$

$$\int \frac{\operatorname{tg} x}{\cos^2 x} \cdot dx = \int \underbrace{\operatorname{tg} x}_{f^1} \cdot \underbrace{\frac{1}{\cos^2 x}}_{f'} \cdot dx = \frac{\operatorname{tg}^2 x}{2} + k$$

19. $\int \frac{\operatorname{cotg} x}{\operatorname{sen}^2 x} \cdot dx$ Sol: $-\frac{\operatorname{cotg}^2 x}{2} + k$

$$\int \frac{\operatorname{cotg} x}{\operatorname{sen}^2 x} \cdot dx = -\int \operatorname{cotg} x \cdot \frac{-1}{\operatorname{sen}^2 x} \cdot dx = -\frac{\operatorname{cotg}^2 x}{2} + k$$

20. $\int \frac{1}{\cos^2 x \sqrt{\operatorname{tg} x - 1}} \cdot dx$ Sol: $2\sqrt{\operatorname{tg} x - 1} + k$

$$\int \frac{1}{\cos^2 x \sqrt{\operatorname{tg} x - 1}} \cdot dx = \int \frac{1}{\cos^2 x} \cdot (\operatorname{tg} x - 1)^{-\frac{1}{2}} \cdot dx = \frac{(\operatorname{tg} x - 1)^{\frac{1}{2}}}{\frac{1}{2}} + k = 2\sqrt{\operatorname{tg} x - 1} + k$$

21. $\int \frac{\mathrm{L}\,(x+1)}{x+1} dx$ Sol: $\frac{\mathrm{L}^2\,(x+1)}{2} + k$

$$\int \frac{\mathrm{L}\,(x+1)}{x+1} dx = \int \underbrace{\mathrm{L}\,(x+1)}_{f^1} \cdot \underbrace{\frac{1}{x+1}}_{f'} dx = \frac{\mathrm{L}^2\,(x+1)}{2} + k$$

22. $\int \frac{\cos x}{\sqrt{2\operatorname{sen} x + 1}} dx$ Sol: $\sqrt{2\operatorname{sen} x + 1} + k$

$$\int \frac{\cos x}{\sqrt{2\operatorname{sen} x + 1}} dx = \frac{1}{2} \int \underbrace{2\cos x}_{f'} \underbrace{(2\operatorname{sen} x + 1)^{-\frac{1}{2}}}_{f^{-1/2}} dx = \frac{1}{2} \cdot \frac{(2\operatorname{sen} x + 1)^{\frac{1}{2}}}{\frac{1}{2}} + k = \sqrt{2\operatorname{sen} x + 1} + k$$

23. $\int \frac{\operatorname{sen} 2x}{(1+\cos 2x)^2} dx$ Sol: $\frac{1}{2(1+\cos 2x)} + k$

$$\int \frac{\operatorname{sen} 2x}{(1+\cos 2x)^2} dx = -\frac{1}{2} \int -2\operatorname{sen} 2x \cdot (1+\cos 2x)^{-2} dx = -\frac{1}{2} \cdot \frac{(1+\cos 2x)^{-1}}{-1} + k =$$

$$= \frac{1}{2(1+\cos 2x)} + k$$

24. $\int \frac{\text{sen}2x}{\sqrt{1+\text{sen}^2 x}}dx$ Sol: $2\sqrt{1+\text{sen}^2 x}+k$

$$\int \frac{\text{sen}2x}{\sqrt{1+\text{sen}^2 x}}dx = \int \text{sen}2x(1+\text{sen}^2 x)^{-\frac{1}{2}}dx = \int \underbrace{2\text{sen}x\cdot\cos x}_{f'}\underbrace{(1+\text{sen}^2 x)^{-\frac{1}{2}}}_{f^{-1/2}}dx =$$

$$= \frac{(1+\text{sen}^2 x)^{\frac{1}{2}}}{\frac{1}{2}}+k = 2\sqrt{1+\text{sen}^2 x}+k$$

25. $\int \frac{\sqrt{\text{tg}\,x+1}}{\cos^2 x}\cdot dx$ Sol: $\frac{2}{3}\sqrt{(\text{tg}\,x+1)^3}+k$

$$\int \frac{\sqrt{\text{tg}\,x+1}}{\cos^2 x}\cdot dx = \int (\text{tg}\,x+1)^{\frac{1}{2}}\cdot\frac{1}{\cos^2 x}\cdot dx = \frac{(\text{tg}\,x+1)^{\frac{3}{2}}}{\frac{3}{2}}+k = \frac{2}{3}\sqrt{(\text{tg}\,x+1)^3}+k$$

26. $\int \frac{\cos 2x}{(2+3\text{sen}\,2x)^3}dx$ Sol: $-\frac{1}{12}\frac{1}{(2+3\text{sen}\,2x)^2}+k$

$$\int \frac{\cos 2x}{(2+3\text{sen}\,2x)^3}dx = \frac{1}{6}\int 6\cos 2x\cdot(2+3\text{sen}\,2x)^{-3}dx = \frac{1}{6}\cdot\frac{(2+3\text{sen}\,2x)^{-2}}{-2}+k =$$

$$= -\frac{1}{12}\frac{1}{(2+3\text{sen}\,2x)^2}+k$$

27. $\int \frac{\text{sen}\,3x}{\sqrt[3]{\cos^4 3x}}dx$ Sol: $\frac{1}{\sqrt[3]{\cos 3x}}+k$

$$\int \frac{\text{sen}\,3x}{\sqrt[3]{\cos^4 3x}}dx = -\frac{1}{3}\int \underbrace{-3\text{sen}\,3x}_{f'}\underbrace{\cos^{-\frac{4}{3}} 3x}_{f^{-4/3}}dx = -\frac{1}{3}\cdot\frac{\cos^{-\frac{1}{3}} 3x}{-\frac{1}{3}}+k = \frac{1}{\sqrt[3]{\cos 3x}}+k$$

28. $\int \frac{\text{Ln}^2 x\ \ dx}{x}$ Sol: $\frac{\text{Ln}^3 x}{3}+k$

$$\int \frac{\text{Ln}^2 x\ \ dx}{x} = \int \text{Ln}^2 x\ \ \frac{1}{x}dx = \frac{\text{Ln}^3 x}{3}+k$$

29. $\int \frac{\operatorname{arc\,sen} x\, dx}{\sqrt{1-x^2}}$ Sol: $\frac{\operatorname{arc\,sen}^2 x}{2} + k$

$$\int \frac{\operatorname{arc\,sen} x\, dx}{\sqrt{1-x^2}} = \int \operatorname{arc\,sen} x \frac{1}{\sqrt{1-x^2}}\, dx = \frac{\operatorname{arc\,sen}^2 x}{2} + k$$

30. $\int \frac{\operatorname{arc\,cos}^2 x\, dx}{\sqrt{1-x^2}}$ Sol: $-\frac{\operatorname{arc\,cos}^3 x}{3} + k$

$$\int \frac{\operatorname{arc\,cos}^2 x\, dx}{\sqrt{1-x^2}} = -\int \operatorname{arc\,cos}^2 x \frac{-1}{\sqrt{1-x^2}}\, dx = -\frac{\operatorname{arc\,cos}^3 x}{3} + k$$

31. $\int \frac{\operatorname{arc\,tg} x}{1+x^2}\, dx$ Sol: $\frac{\operatorname{arc\,tg}^2 x}{2} + k$

$$\int \frac{\operatorname{arc\,tg} x}{1+x^2}\, dx = \int \operatorname{arc\,tg} x \cdot \frac{1}{1+x^2}\, dx = \frac{\operatorname{arc\,tg}^2 x}{2} + k$$

32. $\int \frac{\operatorname{arc\,ctg} x}{1+x^2}\, dx$ Sol: $-\frac{\operatorname{arc\,ctg}^2 x}{2} + k$

$$\int \frac{\operatorname{arc\,ctg} x}{1+x^2}\, dx = -\int \operatorname{arc\,ctg} x \cdot \frac{-1}{1+x^2}\, dx = -\frac{\operatorname{arc\,ctg}^2 x}{2} + k$$

33. $\int \frac{x}{x^2+1}\, dx$ Sol: $\frac{1}{2}\operatorname{Ln}(x^2+1) + k$

$$\int \frac{x}{x^2+1}\, dx = \frac{1}{2}\int \frac{2x}{x^2+1}\, dx = \left\{ \int \frac{f'}{f}\, dx = \operatorname{Ln}|f| + k \right\} = \frac{1}{2}\operatorname{Ln}(x^2+1) + k$$

34. $\int \frac{dx}{1-x}$ Sol: $-\operatorname{Ln}|1-x| + k$

$$\int \frac{dx}{1-x} = -\int \frac{-1}{1-x} \cdot dx = \left\{ \int \frac{f'}{f}\, dx = \operatorname{Ln}|f| + k \right\} = -\operatorname{Ln}|1-x| + k$$

35. $\int \frac{dx}{3x-7}$ Sol: $\frac{1}{3}\operatorname{L}|3x-7| + k$

$$\int \frac{dx}{3x-7} = \frac{1}{3}\int \frac{3}{3x-7} \cdot dx = \left\{ \int \frac{f'}{f}\, dx = \operatorname{Ln}|f| + k \right\} = \frac{1}{3}\operatorname{Ln}|3x-7| + k$$

36. $\displaystyle\int \frac{dx}{5-2x}$ Sol: $-\frac{1}{2}\text{L}\,|5-2x|+k$

$$\int \frac{dx}{5-2x} = -\frac{1}{2}\int \frac{-2}{5-2x}\cdot dx = \left\{\int \frac{f'}{f}dx = \text{Ln}|f|+k\right\} = -\frac{1}{2}\text{Ln}\,|5-2x|+k$$

37. $\displaystyle\int \frac{x+1}{x^2+2x+3}dx$ Sol: $\frac{1}{2}\text{L}\,|\,x^2+2x+3\,|+k$

$$\int \frac{x+1}{x^2+2x+3}dx = \frac{1}{2}\int \frac{2(x+1)}{x^2+2x+3}dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x+3}dx = \frac{1}{2}\text{L}\,|\,x^2+2x+3\,|+k$$

38. $\displaystyle\int \frac{dx}{x\cdot \text{Ln}\,x}$ Sol: $\text{Ln}\,|\,\text{Ln}\,x\,|+k$

$$\int \frac{dx}{x\cdot \text{Ln}\,x} = \int \frac{\frac{1}{x}dx}{\text{Ln}\,x} = \left\{\int \frac{f'}{f}dx = \text{Ln}\,|\,f\,|+k\right\} = \text{Ln}\,|\,\text{Ln}\,x\,|+k$$

39. $\displaystyle\int \text{tg}\,x\,dx$ Sol: $-\text{Ln}\,|\cos x|+k$

$$\int \text{tg}\,x\cdot dx = \int \frac{\text{sen}\,x}{\cos x}\cdot dx = -\int \frac{-\text{sen}\,x}{\cos x}\cdot dx = -\text{Ln}\,|\cos x|+K$$

40. $\displaystyle\int \text{tg}\,2x\,dx$ Sol: $-\frac{1}{2}\text{L}\,|\cos 2x|+k$

$$\int \text{tg}\,2x\cdot dx = \int \frac{\text{sen}\,2x}{\cos 2x}\cdot dx = -\frac{1}{2}\int \frac{-2\text{sen}\,2x}{\cos 2x}\cdot dx = -\frac{1}{2}\text{Ln}\,|\cos 2x|+K$$

41. $\displaystyle\int \text{ctg}\,x\,dx$ Sol: $\text{Ln}\,|\text{sen}\,x|+k$

$$\int \text{ctg}\,x\cdot dx = \int \frac{\cos x}{\text{sen}\,x}\cdot dx = \text{Ln}\,|\text{sen}\,x|+K$$

42. $\displaystyle\int \text{ctg}\,(5x-7)\,dx$ Sol: $\frac{1}{5}\text{L}\,|\text{sen}\,(5x-7)|+k$

$$\int \text{ctg}\,(5x-7)\cdot dx = \int \frac{\cos(5x-7)}{\text{sen}\,(5x-7)}\cdot dx = \frac{1}{5}\int \frac{5\cos(5x-7)}{\text{sen}\,(5x-7)}\cdot dx = \frac{1}{5}\text{Ln}\,|\text{sen}\,(5x-7)|+K$$

43. $\displaystyle\int \frac{dx}{\text{ctg}\,3x}$ Sol: $-\frac{1}{3}\text{L}\,|\,\cos 3x\,|+k$

$$\int \frac{dx}{\text{ctg}\,3x} = \int \text{tg}\,3x\cdot dx = \int \frac{\text{sen}\,3x}{\cos 3x}\cdot dx = -\frac{1}{3}\int \frac{-3\text{sen}\,3x}{\cos 3x}\cdot dx = -\frac{1}{3}\text{Ln}\,|\cos 3x|+K$$

44. $\int \mathbf{ctg}\,\frac{x}{3}\,dx$ Sol: $3\mathrm{L}\left|\,\mathbf{sen}\,\frac{x}{3}\right| + k$

$$\int \mathbf{ctg}\,\frac{x}{3}\cdot dx = \int \frac{\cos\frac{x}{3}}{\mathrm{sen}\,\frac{x}{3}}\cdot dx = 3\int \frac{\frac{1}{3}\cos\frac{x}{3}}{\mathrm{sen}\,\frac{x}{3}}\cdot dx = 3\cdot \mathrm{Ln}\left|\mathrm{sen}\,\frac{x}{3}\right| + K$$

45. $\int (\mathbf{ctg}\,e^x)e^x\,dx$ Sol: $\mathrm{L}\left|\,\mathrm{sen}\,e^x\,\right| + k$

$$\int (\mathrm{ctg}\,e^x)e^x\cdot dx = \int \frac{(\cos e^x)e^x}{\mathrm{sen}\,e^x}\cdot dx = \mathrm{Ln}\left|\mathrm{sen}\,e^x\right| + K$$

46. $\int\left(\mathrm{tg}\,4x - \mathrm{ctg}\,\frac{x}{4}\right)dx$ Sol: $\mathbf{Sol:} -\frac{1}{4}\mathrm{Ln}\left|\,\cos 4x\,\right| - 4\mathrm{Ln}\left|\,\mathrm{sen}\,\frac{x}{4}\right| + k$

$$\int\left(\mathrm{tg}\,4x - \mathrm{ctg}\,\frac{x}{4}\right)dx = \int\left(\frac{\mathrm{sen}\,4x}{\cos 4x} - \frac{\cos\frac{x}{4}}{\mathrm{sen}\,\frac{x}{4}}\right)dx = \int \frac{\mathrm{sen}\,4x}{\cos 4x}dx - \int \frac{\cos\frac{x}{4}}{\mathrm{sen}\,\frac{x}{4}}dx =$$

$$= -\frac{1}{4}\int \frac{-4\mathrm{sen}\,4x}{\cos 4x}dx - 4\int \frac{\frac{1}{4}\cos\frac{x}{4}}{\mathrm{sen}\,\frac{x}{4}}dx = -\frac{1}{4}\mathrm{Ln}\left|\cos 4x\right| - 4\,\mathrm{Ln}\left|\,\mathrm{sen}\,\frac{x}{4}\right| + k$$

47. $\int \frac{\cos x}{2\mathrm{sen}\,x + 3}dx$ Sol: $\frac{1}{2}\mathrm{Ln}\,(2\mathrm{sen}\,x + 3) + k$

$$\int \frac{\cos x}{2\mathrm{sen}\,x + 3}dx = \frac{1}{2}\int \frac{2\cos x}{2\mathrm{sen}\,x + 3}dx = \frac{1}{2}\mathrm{Ln}\,(2\mathrm{sen}\,x + 3) + k$$

48. $\int \frac{dx}{(1 + x^2)\,\mathrm{arc\,tg}\,x}$ Sol: $\mathrm{Ln}\left|\,\mathrm{arc\,tg}\,x\,\right| + k$

$$\int \frac{dx}{(1 + x^2)\,\mathrm{arc\,tg}\,x} = \int \frac{\frac{1}{1 + x^2}}{\mathrm{arc\,tg}\,x}dx = \mathrm{Ln}\left|\,\mathrm{arc\,tg}\,x\,\right| + k$$

49. $\int \frac{dx}{\cos^2 x\,(3\mathrm{tg}\,x + 1)}$ Sol: $\frac{1}{3}\mathrm{Ln}\,(3\mathrm{tg}\,x + 1) + k$

$$\int \frac{dx}{\cos^2 x\,(3\mathrm{tg}\,x + 1)} = \int \frac{\frac{1}{\cos^2 x}}{3\mathrm{tg}\,x + 1}dx = \frac{1}{3}\int \frac{\frac{3}{\cos^2 x}}{3\mathrm{tg}\,x + 1}dx = \frac{1}{3}\mathrm{Ln}\,(3\mathrm{tg}\,x + 1) + k$$

50. $\int \frac{dx}{\sqrt{1-x^2}\,\operatorname{arcsen} x}$ Sol: $\operatorname{Ln}\left|\operatorname{arcsen} x\right| + k$

$$\int \frac{dx}{\sqrt{1-x^2}\,\operatorname{arcsen} x} = \int \frac{\frac{1}{\sqrt{1-x^2}}}{\operatorname{arcsen} x}dx = \operatorname{Ln}\left|\operatorname{arcsen} x\right| + k$$

51. $\int \frac{\cos 2x}{2+3\operatorname{sen} 2x}dx$ Sol: $\frac{1}{6}\operatorname{Ln}\left|2+3\operatorname{sen} 2x\right| + k$

$$\int \frac{\cos 2x}{2+3\operatorname{sen} 2x}dx = \frac{1}{6}\int \frac{6\cos 2x}{2+3\operatorname{sen} 2x}dx = \frac{1}{6}\operatorname{Ln}\left|2+3\operatorname{sen} 2x\right| + k$$

52. $\int e^{2x}dx$ Sol: $\frac{1}{2}e^{2x} + k$

$$\int e^{2x}dx = \frac{1}{2}\int 2e^{2x}dx = \left\{\int f' e^f dx = e^f + k\right\} = \frac{1}{2}e^{2x} + k$$

53. $\int e^{\frac{x}{2}}dx$ Sol: $2e^{\frac{x}{2}} + k$

$$\int e^{\frac{x}{2}}dx = 2\int \frac{1}{2}e^{\frac{x}{2}}dx = \left\{\int f' e^f dx = e^f + k\right\} = 2e^{\frac{x}{2}} + k$$

54. $\int e^{\operatorname{sen} x}\cos x\,dx$ Sol: $e^{\operatorname{sen} x} + k$

$$\int e^{\operatorname{sen} x}\cos x\,dx = \left\{\int f' e^f dx = e^f + k\right\} = e^{\operatorname{sen} x} + k$$

55. $\int a^{x^2}\cdot x\cdot dx$ Sol: $\frac{a^{x^2}}{2\operatorname{L} a} + k$

$$\int a^{x^2} x\cdot dx = \frac{1}{2\operatorname{Ln} a}\int \underbrace{a^{x^2}\,2x\cdot \operatorname{Ln} a}_{D(a^{x^2})}\,dx = \frac{1}{2\operatorname{Ln} a}a^{x^2} + k$$

56. $\int e^{\frac{x}{a}}dx$ Sol: $ae^{\frac{x}{a}} + k$

$$\int e^{\frac{x}{a}}dx = a\int \frac{1}{a}e^{\frac{x}{a}}dx = ae^{\frac{x}{a}} + k$$

57. $\int \left(e^{2x}\right)^2 dx$ Sol: $\frac{1}{4}e^{4x}+k$

$$\int \left(e^{2x}\right)^2 dx = \int e^{4x}dx = \frac{1}{4}\int 4e^{4x}dx = \frac{1}{4}e^{4x}+k$$

58. $\int e^{-3x}dx$ Sol: $-\frac{1}{3}e^{-3x}+k$

$$\int e^{-3x}dx = -\frac{1}{3}\int (-3)e^{-3x}dx = -\frac{1}{3}e^{-3x}+k$$

59. $\int 5^x e^x dx$ Sol: $\frac{5^x e^x}{\text{Ln}\,5+1}+k$

$$\int 5^x e^x dx = \int (5e)^x dx = \frac{1}{\text{Ln}(5e)}\int (5e)^x\,\text{Ln}(5e)dx = \frac{1}{\text{Ln}(5e)}(5e)^x+k = \frac{5^x e^x}{\text{Ln}\,5+1}+k$$

60. $\int \left(e^{5x}+a^{5x}\right)dx$ Sol: $\frac{1}{5}\left(e^{5x}+\frac{a^{5x}}{\text{L}\,a}\right)+k$

$$\int \left(e^{5x}+a^{5x}\right)dx = \frac{1}{5}\int 5e^{5x}dx + \frac{1}{5\,\text{Ln}\,a}\int 5a^{5x}\,\text{Ln}\,a\,dx = \frac{1}{5}e^{5x}+\frac{1}{5\,\text{Ln}\,a}\cdot a^{5x}+k =$$

$$= \frac{1}{5}\left(e^{5x}+\frac{a^{5x}}{\text{L}\,a}\right)+k$$

61. $\int e^{x^2+4x+3}(x+2)\,dx$ Sol: $\frac{1}{2}e^{x^2+4x+3}+k$

$$\int e^{x^2+4x+3}(x+2)\,dx = \frac{1}{2}\int e^{x^2+4x+3}\,2(x+2)\,dx = \frac{1}{2}e^{x^2+4x+3}+k$$

62. $\int \frac{(a^x-b^x)^2}{a^x b^x}dx$ Sol: $\frac{\left(\frac{a}{b}\right)^x-\left(\frac{b}{a}\right)^x}{\text{L}\,a-\text{L}\,b}-2x+k$

$$\int \frac{(a^x-b^x)^2}{a^x b^x}dx = \int \frac{a^{2x}-2a^x b^x+b^{2x}}{a^x b^x}dx = \int\left(\frac{a^{2x}}{a^x b^x}+\frac{b^{2x}}{a^x b^x}-2\right)dx = \int\left(\frac{a^x}{b^x}+\frac{b^x}{a^x}-2\right)dx =$$

$$= \int\left[\left(\frac{a}{b}\right)^x+\left(\frac{b}{a}\right)^x-2\right]dx = \frac{1}{\text{Ln}\left(\frac{a}{b}\right)}\cdot\left(\frac{a}{b}\right)^x+\frac{1}{\text{Ln}\left(\frac{b}{a}\right)}\cdot\left(\frac{b}{a}\right)^x-2x+k =$$

$$= \frac{1}{\text{Ln}\, a - \text{Ln}\, b} \cdot \left(\frac{a}{b}\right)^x + \frac{1}{\text{Ln}\, b - \text{Ln}\, a} \cdot \left(\frac{b}{a}\right)^x - 2x + k =$$

$$= \frac{\left(\frac{a}{b}\right)^x}{\text{Ln}\, a - \text{Ln}\, b} - \frac{\left(\frac{b}{a}\right)^x}{\text{Ln}\, a - \text{Ln}\, b} - 2x + k = \frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\text{L}\, a - \text{L}\, b} - 2x + k$$

63. $\int \frac{e^x}{3+4e^x}dx$ Sol: $\frac{1}{4}\text{Ln}\,(3+4e^x)+k$

$$\int \frac{e^x}{3+4e^x}dx = \frac{1}{4}\int \frac{4e^x}{3+4e^x}dx = \frac{1}{4}\text{Ln}\,(3+4e^x)+k$$

64. $\int \cos 5x dx$ Sol: $\frac{1}{5}\text{sen}\, 5x + k$

$$\int \cos 5x dx = \frac{1}{5}\int 5\cos 5x dx = \left\{\int f'(x)\cdot \cos f(x)dx = \text{sen}\, f(x) + k\right\} = \frac{1}{5}\text{sen}\, 5x + k$$

65. $\int \text{sen}\frac{x}{3}dx$ Sol: $-3\cos\frac{x}{3}+k$

$$\int \text{sen}\frac{x}{3}dx = 3\int \frac{1}{3}\text{sen}\frac{x}{3}dx = \left\{\int f'(x)\cdot \text{sen}\, f(x)dx = -\cos f(x) + k\right\} = 3(-\cos\frac{x}{3}) + k =$$

$$= -3\cos\frac{x}{3}+k$$

66. $\int \sec^2(7x+2)dx$ Sol: $\frac{1}{7}\text{tg}\,(7x+2)+k$

$$\int \sec^2(7x+2)dx = \frac{1}{7}\int 7\sec^2(7x+2)dx = \left\{\int f'(x)\sec^2 f(x)dx = \text{tg}\, f(x) + k\right\} =$$

$$= \frac{1}{7}\text{tg}\,(7x+2)+k$$

67. $\int x\cos 3x^2 dx$ Sol: $\frac{1}{6}\text{sen}\, 3x^2 + k$

$$\int x\cos(3x^2)dx = \frac{1}{6}\int 6x\cos(3x^2)dx = \frac{1}{6}\text{sen}\,(3x^2)+k$$

68. $\int \mathbf{tg^2\, x\, dx}$ Sol: $\mathbf{tg\, x - x + k}$

Por trigonometría sabemos que $\mathbf{tg^2 x + 1 = sec^2\, x} \quad \Rightarrow \quad \mathbf{tg^2 x = sec^2\, x - 1,}$ entonces

$$\int \mathbf{tg^2\, x\, dx} = \int \mathbf{(sec^2\, x - 1) dx} = \int \mathbf{sec^2\, x\, dx} - \int \mathbf{dx = tg\, x - x + k}$$

69. $\int \frac{\cos(\mathrm{Ln}(x))}{x} dx$ Sol: $\mathrm{sen}\ (\mathrm{Ln}(x)) + k$

$$\int \frac{\cos(\mathrm{Ln}(x))}{x} dx = \int \cos(\mathrm{Ln}(x)) \frac{1}{x} dx = \mathrm{sen}\ (\mathrm{Ln}(x)) + k$$

70. $\int \mathbf{tg^3\, x\, dx}$ Sol: $\mathbf{\frac{tg^2\, x}{2} + Ln\, |cos\, x| + k}$

$$\int \mathbf{tg^3\, x\, dx} = \int \mathbf{tg\, x \cdot tg^2 x\, dx} = \int \mathbf{tg\, x \cdot (sec^2\, x - 1)\, dx} = \int \underbrace{\mathbf{tg\, x}}_{f^1} \cdot \underbrace{\mathbf{sec^2\, x}}_{f} \mathbf{dx} - \int \mathbf{tg\, x\, dx} =$$

$$= \mathbf{\frac{tg^2\, x}{2}} - \int \mathbf{\frac{sen\, x}{cos x} dx} = \mathbf{\frac{tg^2\, x}{2}} + \int \mathbf{\frac{-sen\, x}{cos x} dx} = \mathbf{\frac{tg^2\, x}{2} + Ln\, |cos\, x| + k}$$

71. $\int \mathbf{cos \sqrt{x}\, \frac{dx}{\sqrt{x}}}$ Sol: $\mathbf{2 sen \sqrt{x} + k}$

$$\int \mathbf{cos \sqrt{x}\, \frac{dx}{\sqrt{x}}} = \int \mathbf{cos \sqrt{x}\, \frac{1}{\sqrt{x}} dx} = \mathbf{2} \int \mathbf{cos \sqrt{x}\, \frac{1}{2\sqrt{x}} dx = 2 sen \sqrt{x} + k}$$

72. $\int \mathbf{\frac{x}{\sqrt{1 - x^4}} dx}$ Sol: $\mathbf{\frac{1}{2} arc\, sen\, x^2 + k}$

$$\int \mathbf{\frac{x}{\sqrt{1 - x^4}} dx} = \int \mathbf{\frac{x}{\sqrt{1 - (x^2)^2}} dx} = \left\{ \int \mathbf{\frac{f'(x)}{\sqrt{1 - (f(x))^2}} dx} = \begin{cases} \mathbf{arc\, sen\, f(x) + k} \\ \mathbf{-arccos f(x) + k} \end{cases} \right\} =$$

$$= \mathbf{\frac{1}{2}} \int \mathbf{\frac{2x}{\sqrt{1 - (x^2)^2}} dx} = \left\{ \int \mathbf{\frac{f'(x) dx}{\sqrt{1 - (f(x))^2}}} \right\} = \mathbf{\frac{1}{2} arc\, sen\, (x^2) + k}$$

73. $\int \mathbf{\frac{dx}{\sqrt{1 - 4x^2}}}$ Sol: $\mathbf{\frac{1}{2} arc\, sen\, (2x) + k}$

$$\int \mathbf{\frac{dx}{\sqrt{1 - 4x^2}}} = \int \mathbf{\frac{dx}{\sqrt{1 - (2x)^2}}} = \mathbf{\frac{1}{2}} \int \mathbf{\frac{2dx}{\sqrt{1 - (2x)^2}}} = \left\{ \int \mathbf{\frac{f'(x) dx}{\sqrt{1 - (f(x))^2}}} \right\} = \mathbf{\frac{1}{2} arc\, sen\, (2x) + k}$$

74. $\int \frac{dx}{\sqrt{9-4x^2}}$ Sol: $\frac{1}{2}\text{arc sen}\left(\frac{2x}{3}\right)+k$

$$\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{\sqrt{9(1-\frac{4x^2}{9})}} = \int \frac{dx}{3\sqrt{1-\left(\frac{2x}{3}\right)^2}} = \frac{1}{3}\int \frac{dx}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} = \frac{1}{3}\cdot\frac{3}{2}\int \frac{\frac{2}{3}dx}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} =$$

$$= \frac{1}{2}\int \frac{\frac{2}{3}dx}{\sqrt{1-\left(\frac{2x}{3}\right)^2}} = \left\{\int \frac{f'(x)dx}{\sqrt{1-(f(x))^2}}\right\} = \frac{1}{2}\text{arc sen}\left(\frac{2x}{3}\right)+k$$

75. $\int \frac{dx}{\sqrt{a^2-b^2x^2}}$ Sol: $\frac{1}{b}\text{arc sen}\left(\frac{bx}{a}\right)+k$

$$\int \frac{dx}{\sqrt{a^2-b^2x^2}} = \int \frac{dx}{\sqrt{a^2(1-\frac{b^2x^2}{a^2})}} = \int \frac{dx}{a\sqrt{1-\left(\frac{bx}{a}\right)^2}} = \frac{1}{a}\int \frac{dx}{\sqrt{1-\left(\frac{bx}{a}\right)^2}} = \frac{1}{a}\cdot\frac{a}{b}\int \frac{\frac{b}{a}dx}{\sqrt{1-\left(\frac{bx}{a}\right)^2}} =$$

$$= \frac{1}{b}\int \frac{\frac{b}{a}dx}{\sqrt{1-\left(\frac{bx}{a}\right)^2}} = \left\{\int \frac{f'(x)dx}{\sqrt{1-(f(x))^2}}\right\} = \frac{1}{b}\text{arc sen}\left(\frac{bx}{a}\right)+k$$

76. $\int \frac{e^x}{3+4e^x}dx$ Sol: $\frac{1}{4}\text{Ln}\,(3+4e^x)+k$

$$\int \frac{e^x}{3+4e^x}dx = \frac{1}{4}\int \frac{4e^x}{3+4e^x}dx = \left\{\int \frac{f'(x)}{f(x)}dx\right\} = \frac{1}{4}\text{Ln}\,(3+4e^x)+k$$

77. $\int \frac{e^{2x}}{2+e^{2x}}dx$ Sol: $\frac{1}{2}\text{Ln}\,(2+e^{2x})+k$

$$\int \frac{e^{2x}}{2+e^{2x}}dx = \frac{1}{2}\int \frac{2e^{2x}}{2+e^{2x}}dx = \left\{\int \frac{f'(x)}{f(x)}dx\right\} = \frac{1}{2}\text{Ln}\,(2+e^{2x})+k$$

78. $\int \frac{e^x}{1+e^{2x}}dx$ Sol: $\text{arc tg}\,(e^x)+k$

$$\int \frac{e^x}{1+e^{2x}}dx = \int \frac{e^x}{1+(e^x)^2}dx = \left\{\int \frac{f'(x)}{1+(f(x))^2}dx = \text{arc tg}\,f(x)+k\right\} = \text{arc tg}\,(e^x)+k$$

79. $\int \frac{dx}{1+2x^2}$ Sol: $\frac{1}{\sqrt{2}} \operatorname{arc\,tg}(\sqrt{2}x)+k$

$$\int \frac{dx}{1+2x^2} = \int \frac{dx}{1+(\sqrt{2}x)^2} = \frac{1}{\sqrt{2}} \int \frac{\sqrt{2}\,dx}{1+(\sqrt{2}x)^2} = \left\{ \int \frac{f'(x)}{1+(f(x))^2} dx \right\} = \frac{1}{\sqrt{2}} \operatorname{arc\,tg}(\sqrt{2}x)+k$$

80. $\int \frac{dx}{4+x^2}$ Sol: $\frac{1}{2} \operatorname{arc\,tg}(\frac{x}{2})+k$

$$\int \frac{dx}{4+x^2} = \int \frac{dx}{4(1+\frac{x^2}{4})} = \frac{1}{4} \int \frac{dx}{1+\left(\frac{x}{2}\right)^2} = \frac{1}{4}\cdot 2 \int \frac{\frac{1}{2}dx}{1+\left(\frac{x}{2}\right)^2} = \frac{1}{2} \operatorname{arc\,tg}(\frac{x}{2})+k$$

81. $\int \frac{xdx}{x^4+a^4}$ Sol: $\frac{1}{2a^2} \operatorname{arc\,tg}(\frac{x^2}{a^2})+k$

$$\int \frac{xdx}{x^4+a^4} = \int \frac{xdx}{a^4(\frac{x^4}{a^4}+1)} = \frac{1}{a^4} \int \frac{xdx}{1+\frac{x^4}{a^4}} = \frac{1}{a^4} \int \frac{xdx}{1+\left(\frac{x^2}{a^2}\right)^2} = \frac{1}{a^4}\cdot \frac{a^2}{2} \int \frac{\frac{2x}{a^2}dx}{1+\left(\frac{x^2}{a^2}\right)^2} =$$

$$= \frac{1}{2a^2} \int \frac{\frac{2x}{a^2}dx}{1+\left(\frac{x^2}{a^2}\right)^2} = \frac{1}{2a^2} \operatorname{arc\,tg}(\frac{x^2}{a^2})+k$$

82. $\int \frac{\cos x dx}{a^2+\operatorname{sen}^2 x}$ Sol: $\frac{1}{a} \operatorname{arc\,tg}(\frac{\operatorname{sen} x}{a})+k$

$$\int \frac{\cos x dx}{a^2+\operatorname{sen}^2 x} = \int \frac{\cos x dx}{a^2(1+\frac{\operatorname{sen}^2 x}{a^2})} = \frac{1}{a^2} \int \frac{\cos x dx}{1+\frac{\operatorname{sen}^2 x}{a^2}} = \frac{1}{a^2} \int \frac{\cos x dx}{1+\left(\frac{\operatorname{sen} x}{a}\right)^2} =$$

$$= \frac{1}{a^2}\cdot a \int \frac{\frac{1}{a}\cos x dx}{1+\left(\frac{\operatorname{sen} x}{a}\right)^2} = \frac{1}{a} \int \frac{\frac{1}{a}\cos x dx}{1+\left(\frac{\operatorname{sen} x}{a}\right)^2} = \frac{1}{a} \operatorname{arc\,tg}(\frac{\operatorname{sen} x}{a})+k$$

83. $\int \frac{dx}{x\sqrt{1-\mathrm{Ln}^2(x)}}$ Sol: $\arcsin(\mathrm{Ln}(x))+k$

$$\int \frac{dx}{x\sqrt{1-\mathrm{Ln}^2(x)}} = \int \frac{\frac{1}{x}dx}{\sqrt{1-(\mathrm{Ln}(x))^2}} = \left\{\int \frac{f'(x)dx}{\sqrt{1-(f(x))^2}}\right\} = \arcsin(\mathrm{Ln}(x))+k$$

84. $\int \frac{\arccos x - x}{\sqrt{1-x^2}}dx$ Sol: $-\frac{1}{2}(\arccos(x))^2+\sqrt{1-x^2}+k$

$$\int \frac{\arccos x - x}{\sqrt{1-x^2}}dx = \int \frac{\arccos x}{\sqrt{1-x^2}}dx + \int \frac{-x}{\sqrt{1-x^2}}dx = -\int \underbrace{\arccos x}_{f^1}\underbrace{\frac{-1}{\sqrt{1-x^2}}}_{f'}dx + \int \frac{-2x}{2\sqrt{1-x^2}}dx =$$

$$= \left\{\int f^1 \cdot f'dx + \int \frac{f'}{2\sqrt{f}}dx\right\} = -\frac{1}{2}(\arccos(x))^2 + \sqrt{1-x^2}+k$$

85. $\int \frac{x-\mathrm{arctg}\, x}{1+x^2}dx$ Sol: $\frac{1}{2}\mathrm{Ln}(1+x^2)-\frac{1}{2}(\mathrm{arctg}\, x)^2+k$

$$\int \frac{x-\mathrm{arctg}\, x}{1+x^2}dx = \int \frac{x}{1+x^2}dx - \int \frac{\mathrm{arctg}\, x}{1+x^2}dx = \frac{1}{2}\int \frac{2x}{1+x^2}dx - \int \mathrm{arctg}\, x \frac{1}{1+x^2}dx =$$

$$= \left\{\int \frac{f'(x)}{f(x)}dx - \int f^1 \cdot f'dx\right\} = \frac{1}{2}\mathrm{Ln}(1+x^2) - \frac{1}{2}(\mathrm{arctg}\, x)^2 + k$$

86. $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx$ Sol: $\frac{4}{3}\sqrt{(1+\sqrt{x})^3}+k$

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = \int \frac{1}{\sqrt{x}}\sqrt{1+\sqrt{x}}\,dx = 2\int \frac{1}{2\sqrt{x}}\left(1+\sqrt{x}\right)^{\frac{1}{2}}dx = \left\{\int f^{\frac{1}{2}} \cdot f'dx\right\} =$$

$$= 2\cdot \frac{\left(1+\sqrt{x}\right)^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{4}{3}\sqrt{(1+\sqrt{x})^3}+k$$

Veamos como podemos realizar esta misma integral por el método de sustitución o cambio de variable.

Haciendo el cambio $1+\sqrt{x}=t$, calculamos dx: $\frac{1}{2\sqrt{x}}dx = dt \Rightarrow dx = 2\sqrt{x}dt$ y sustituimos en nuestra integral:

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = \int \frac{\sqrt{t}}{\sqrt{x}} \cdot 2\sqrt{x}dt = 2\int \sqrt{t}\,dt = 2\int t^{\frac{1}{2}}dt = 2\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + k = \frac{4}{3}t^{\frac{3}{2}} + k = \frac{4}{3}\sqrt{t^3} + k =$$

una vez realizada la integral hay que deshacer el cambio de variable y volver a la variable ***x*,** con lo que nos quedará:

$$= \frac{4}{3}\sqrt{(1+\sqrt{x})^3} + k$$

87. $\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}dx$ Sol: $4\sqrt{1+\sqrt{x}} + k$

$$\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}dx = \int \frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+\sqrt{x}}}dx = 2\int \frac{1}{2\sqrt{x}}\left(1+\sqrt{x}\right)^{-\frac{1}{2}}dx = \left\{\int f^{-\frac{1}{2}} \cdot f'\,dx\right\} =$$

$$= 2 \cdot \frac{\left(1+\sqrt{x}\right)^{\frac{1}{2}}}{\frac{1}{2}} + k = 4\sqrt{1+\sqrt{x}} + k$$

Veamos como podemos realizar esta misma integral por el método de sustitución o cambio de variable.

Haciendo el cambio $1+\sqrt{x} = t$, calculamos ***dx*:** $\frac{1}{2\sqrt{x}}dx = dt \Rightarrow dx = 2\sqrt{x}dt$ y sustituimos en nuestra integral:

$$\int \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}dx = \int \frac{1}{\sqrt{x}\sqrt{t}} \cdot 2\sqrt{x}dt = 2\int \frac{1}{\sqrt{t}}dt = 2\int t^{-\frac{1}{2}}dt = 2\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + k = 4t^{\frac{1}{2}} + k = 4\sqrt{t} + k =$$

una vez realizada la integral hay que deshacer el cambio de variable y volver a la variable ***x*,** con lo que nos quedará:

$$= 4\sqrt{1+\sqrt{x}} + k$$

88. $\int x\sqrt{x-1}\,dx$ Sol: $\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + k$

Hacemos la sustitución $x-1 = t^2 \quad \Rightarrow \quad x = t^2 + 1$

Calculamos la diferencial de *x*: $dx = 2t.dt$ y sustituimos en la integral que deseamos calcular. Tendremos:

$$\int x.\sqrt{x-1}\cdot dx=\int (t^2+1).\sqrt{t^2}\cdot 2tdt=2\int (t^2+1).t^2.dt=2\int (t^4+t^2).dt=2\left(\frac{t^5}{5}+\frac{t^3}{3}\right)+k=$$

$$=\frac{2}{5}\cdot t^5+\frac{2}{3}\cdot t^3+k=\frac{2}{5}\cdot (x-1)^{\frac{5}{2}}+\frac{2}{3}\cdot (x-1)^{\frac{3}{2}}+k$$

89. $\int x(5x^2-3)^7\,dx$ Sol: $\frac{1}{80}(5x^2-3)^8+k$

Directamente:

$$\int x(5x^2-3)^7\,dx=\frac{1}{10}\int 10x(5x^2-3)^7\,dx=\left\{\int f^7\cdot f'\,dx\right\}=\frac{1}{10}\cdot\frac{(5x^2-3)^8}{8}+k=$$

$$=\frac{1}{80}(5x^2-3)^8+k$$

Por sustitución:

Hacemos $5x^2-3=t\Rightarrow 10xdx=dt\Rightarrow dx=\frac{dt}{10x}$ y sustituimos en nuestra integral

$$\int x(5x^2-3)^7\,dx=\int xt^7\,\frac{dt}{10x}=\frac{1}{10}\int t^7dt=\frac{1}{10}\cdot\frac{t^8}{8}+k=\frac{1}{80}(5x^2-3)^8+k$$

90. $\int x(2x+5)^{10}dx$ Sol: $\frac{1}{4}\left[\frac{(2x+5)^{12}}{12}-\frac{5(2x+5)^{11}}{11}\right]+k$

Por sustitución:

Hacemos $2x+5=t\Rightarrow x=\frac{t-5}{2}\Rightarrow dx=\frac{1}{2}dt$ y sustituimos en nuestra integral

$$\int x(2x+5)^{10}dx=\int\frac{t-5}{2}\cdot t^{10}\cdot\frac{1}{2}dt=\frac{1}{4}\int (t-5)t^{10}dt=\frac{1}{4}\int (t^{11}-5t^{10})dt=$$

$$=\frac{1}{4}\left[\frac{t^{12}}{12}-5\cdot\frac{t^{11}}{11}\right]+k=\frac{1}{4}\left[\frac{(2x+5)^{12}}{12}-\frac{5(2x+5)^{11}}{11}\right]+k$$

91. $\int xe^x dx$ Sol: $e^x(x-1)+k$

Por el método de integración por partes:

$$\int xe^x dx=\left\{\begin{array}{l}u=x\Rightarrow du=dx\\ dv=e^x dx\Rightarrow v=e^x\end{array}\right\}=xe^x-\int e^x dx=xe^x-e^x+k=(x-1)e^x+k$$

92. $I = \int (x^2 - 3x + 5)e^x dx$ Sol: $e^x(x^2 - 5x + 10) + k$

Por el método de integración por partes:

$$I = \int (x^2 - 3x + 5)e^x dx = \left\{ \begin{array}{c} u = x^2 - 3x + 5 \Rightarrow du = (2x - 3)dx \\ dv = e^x dx \Rightarrow v = e^x \end{array} \right\} =$$

$$= (x^2 - 3x + 5) \ \ e^x - \int e^x (2x - 3)dx =$$

La integral que nos ha quedado es del mismo tipo que la que pretendemos calcular, por lo que nuevamente aplicaremos el método de integración de partes:

Hacemos $\left\{ \begin{array}{c} u = 2x - 3 \Rightarrow du = 2dx \\ dv = e^x dx \Rightarrow v = e^x \end{array} \right\}$ y sustituimos:

$$I = (x^2 - 3x + 5)e^x - \int e^x (2x - 3)dx = (x^2 - 3x + 5)e^x - \left[(2x - 3e^x - \int 2e^x dx \right] =$$

$$= (x^2 - 3x + 5)e^x - (2x - 3)e^x + 2\int e^x dx = (x^2 - 3x + 5)e^x - (2x - 3)e^x + 2e^x + k =$$

$$= \left[(x^2 - 3x + 5) - (2x - 3) + 2 \right] e^x + k = e^x (x^2 - 5x + 10) + k$$

93. $\int x \, \mathrm{Ln}(x) \, dx$ Sol: $\frac{1}{2} x^2 \left(\mathrm{Ln}(x) - \frac{1}{2} \right) + k$

Por el método de integración por partes:

$$\int x \, \mathrm{Ln}(x) \, dx = \left\{ \begin{array}{c} u = \mathrm{Ln}(x) \Rightarrow du = \frac{1}{x} dx \\ dv = x dx \Rightarrow v = \frac{1}{2} x^2 \end{array} \right\} = \frac{1}{2} x^2 \, \mathrm{Ln}(x) - \int \frac{1}{2} x^2 \cdot \frac{1}{x} dx =$$

$$= \frac{1}{2} x^2 \, \mathrm{Ln}(x) - \frac{1}{2} \int x \, dx = \frac{1}{2} x^2 \, \mathrm{Ln}(x) - \frac{1}{2} \cdot \frac{1}{2} x^2 + k = \frac{1}{2} x^2 \left(\mathrm{Ln}(x) - \frac{1}{2} \right) + k$$

94. $\int \mathrm{Ln}(x) \, dx$ Sol: $x(\mathrm{Ln}(x) - 1) + k$

Por el método de integración por partes:

$$\int \mathrm{Ln}(x) \, dx = \left\{ \begin{array}{c} u = \mathrm{Ln}(x) \Rightarrow du = \frac{1}{x} dx \\ dv = dx \Rightarrow v = x \end{array} \right\} = x \, \mathrm{Ln}(x) - \int x \cdot \frac{1}{x} dx =$$

$$= x\,\mathrm{Ln}(x) - \int dx = x\,\mathrm{Ln}(x) - x + k = x\big(\mathrm{Ln}(x) - 1\big) + k$$

95. $\int x \operatorname{sen} x\, dx$ Sol: $\operatorname{sen} x - x\cos x + k$

Por el método de integración por partes:

$$\int x \operatorname{sen} x\, dx = \left\{\begin{matrix} u = x \Rightarrow du = dx \\ dv = \operatorname{sen} x\, dx \Rightarrow v = -\cos x \end{matrix}\right\} = -x\cos x - \int -\cos x\, dx =$$

$$= -x\cos x + \int \cos x\, dx = -x\cos x + \operatorname{sen} x + k$$

96. $\int x\cos^2 x\, dx$ Sol: $\frac{x^2}{4} + \frac{1}{4} x \operatorname{sen} 2x + \frac{1}{8}\cos 2x + k$

Por el método de integración por partes, hacemos $u = x \Rightarrow du = dx$ y $dv = \cos^2 x dx$

Para calcular el valor de v recurrimos a las razones trigonométricas del ángulo mitad y tendremos que $\cos^2 x = \frac{1+\cos 2x}{2}$. Por tanto,

$$v = \int \cos^2 x dx = \int \frac{1+\cos 2x}{2} dx = \frac{1}{2}\int (1+\cos 2x) dx = \frac{1}{2}(x + \frac{1}{2}\operatorname{sen} 2x)$$

En consecuencia:

$$\int x\cos^2 x\, dx = x \cdot \frac{1}{2}(x + \frac{1}{2}\operatorname{sen} 2x) - \int \frac{1}{2}(x + \frac{1}{2}\operatorname{sen} 2x) dx =$$

$$= \frac{1}{2}(x^2 + \frac{1}{2} x\operatorname{sen} 2x) - \frac{1}{2}\int (x + \frac{1}{2}\operatorname{sen} 2x) dx = \frac{1}{2}(x^2 + \frac{1}{2} x\operatorname{sen} 2x) - \frac{1}{2}\cdot\left(\frac{x^2}{2} - \frac{1}{4}\cos 2x\right) + k =$$

$$= \frac{1}{2}x^2 + \frac{1}{4} x\operatorname{sen} 2x - \frac{x^2}{4} + \frac{1}{8}\cos 2x + k = \frac{x^2}{4} + \frac{1}{4} x\operatorname{sen} 2x + \frac{1}{8}\cos 2x + k$$

97. $\int e^{-x}\cos x dx$ Sol: $\frac{1}{2} e^{-x}(\operatorname{sen} x - \cos x) + k$

$$I = \int e^{-x}\cos x dx = \left\{\begin{matrix} u = e^{-x} \Rightarrow du = -e^{-x} dx \\ dv = \cos x dx \Rightarrow v = \operatorname{sen} x \end{matrix}\right\} = e^{-x}\operatorname{sen} x - \int -e^{-x}\operatorname{sen} x dx =$$

$$= e^{-x}\operatorname{sen} x + \int e^{-x}\operatorname{sen} x dx$$

Al aplicar el método de partes nos ha quedado una integral del mismo tipo que la que pretendemos calcular, por lo que volvemos a aplicar el mismo método. En ella hacemos:

$$u = e^{-x} \quad \Rightarrow \quad du = -e^{-x}dx$$
$$dv = \operatorname{sen} x\, dx \quad \Rightarrow \quad v = -\cos x$$

Sustituyendo en la expresión anterior nos queda:

$$\boldsymbol{I = e^{-x}\operatorname{sen} x + \int e^{-x} \operatorname{sen} x dx = e^{-x}\operatorname{sen} x + \left[-\cos x \cdot e^{-x} - \int -\cos x \cdot (-e^{-x}) dx \right] =}$$

$$\boldsymbol{= e^{-x}\operatorname{sen} x - \cos x \cdot e^{-x} - \int \cos x \cdot e^{-x} dx}$$

es decir, volvemos a la misma integral que pretendemos calcular. Entonces:

$$I = e^{-x}\operatorname{sen} x - \cos x \cdot e^{-x} - I \quad \Rightarrow \quad 2I = e^{-x}\operatorname{sen} x - \cos x \cdot e^{-x} \quad \Rightarrow \quad I = \frac{e^{-x}(\operatorname{sen} x - \cos x)}{2}$$

En consecuencia:

$$\boldsymbol{I = \int e^{-x} \cos x dx = \frac{e^{-x}(\operatorname{sen} x - \cos x)}{2} + k}$$

98. $\boldsymbol{\int \operatorname{Ln}(1-x)\, dx}$ Sol: $\boldsymbol{-x - (1-x)\operatorname{Ln}(1-x) + k}$

$$\boldsymbol{\int \operatorname{Ln}(1-x)\, dx = \left\{ \begin{matrix} u = \operatorname{Ln}(1-x) \Rightarrow du = \dfrac{-1}{1-x} dx \\ dv = dx \Rightarrow v = x \end{matrix} \right\} = x\operatorname{Ln}(1-x) - \int x \cdot \frac{-1}{1-x} dx =}$$

$$\boldsymbol{= x\operatorname{Ln}(1-x) - \int \frac{-x}{1-x} dx = x\operatorname{Ln}(1-x) - \int \frac{1-x-1}{1-x} dx = x\operatorname{Ln}(1-x) - \int \left(1 + \frac{-1}{1-x}\right) dx =}$$

$$\boldsymbol{= x\operatorname{Ln}(1-x) - \left(x + \operatorname{Ln}(1-x)\right) + k = x\operatorname{Ln}(1-x) - x - \operatorname{Ln}(1-x) + k =}$$

$$\boldsymbol{= -x - (1-x)\operatorname{Ln}(1-x) + k}$$

99. $\int x^n \,\mathrm{Ln}(x)\,dx$ Sol: $\dfrac{x^{n+1}}{n+1}\left(\mathrm{Ln}(x)-\dfrac{1}{n+1}\right)+k$

Por el método de integración por partes:

$$\int x^n \,\mathrm{Ln}(x)\,dx = \left\{\begin{matrix} u=\mathrm{Ln}(x)\Rightarrow du=\dfrac{1}{x}dx \\ dv = x^n dx \Rightarrow v=\dfrac{1}{n+1}x^{n+1}\end{matrix}\right\} = \frac{1}{n+1}x^{n+1}\,\mathrm{Ln}(x) - \int \frac{1}{n+1}x^{n+1}\cdot\frac{1}{x}dx =$$

$$= \frac{1}{n+1}x^{n+1}\,\mathrm{Ln}(x) - \frac{1}{n+1}\int x^n\,dx = \frac{1}{n+1}x^{n+1}\,\mathrm{Ln}(x) - \frac{1}{n+1}\cdot\frac{1}{n+1}x^{n+1} + k =$$

$$= \frac{x^{n+1}}{n+1}\left(\mathrm{Ln}(x)-\frac{1}{n+1}\right)+k$$

100. $\int \mathrm{arc\,sen}\,x\,dx$ Sol: $x\,\mathrm{arcsen}\,x + \sqrt{1-x^2}+k$

Hacemos el siguiente cambio: $\left\{\begin{matrix} u=\mathrm{arc\,sen}\,x \\ dv = dx\end{matrix}\right\} \Rightarrow \left\{\begin{matrix} du=\dfrac{1}{\sqrt{1-x^2}}\cdot dx \\ v = x\end{matrix}\right.$

Sustituyendo en la fórmula de integración por partes obtenemos:

$$\int \mathrm{arc\,sen}\,x.dx = x.\mathrm{arc\,sen}\,x - \int x\cdot\frac{1}{\sqrt{1-x^2}}\cdot dx = x.\mathrm{arc\,sen}\,x - \int x.(1-x^2)^{-\frac{1}{2}}.dx =$$

$$= x.\mathrm{arc\,sen}\,x + \frac{1}{2}\int -2x.(1-x^2)^{-\frac{1}{2}}.dx = x.\mathrm{arc\,sen}\,x + \frac{1}{2}\cdot\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}}+k =$$

$$= x.\mathrm{arc\,sen}\,x + \sqrt{1-x^2}+k$$

101. $\int \sqrt{1-x^2}\,dx$ Sol: $\dfrac{1}{2}\left(\mathrm{arcsen}\,x + x\sqrt{1-x^2}\right)+k$

$$\int \sqrt{1-x^2}\,dx = \int \frac{1-x^2}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{-x^2}{\sqrt{1-x^2}}dx =$$

$$= \mathrm{arc\,sen}\,x + \int \frac{-x^2}{\sqrt{1-x^2}}\cdot dx =$$

La integral que nos queda la realizaremos por partes:

$$\int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx = \int x \cdot \frac{-x}{\sqrt{1-x^2}} \cdot dx = \left\{ \begin{array}{c} u = x \Rightarrow dx \\ dv = \dfrac{-x}{\sqrt{1-x^2}} dx \Rightarrow v = \sqrt{1-x^2} \end{array} \right\} =$$

$$= x\sqrt{1-x^2} - \int \sqrt{1-x^2}\, dx$$

Sustituyendo nos queda:

$$\int \sqrt{1-x^2}\, dx = \operatorname{arc\,sen} x + \int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx = \operatorname{arc\,sen} x + x\sqrt{1-x^2} - \int \sqrt{1-x^2}\, dx$$

y se nos repite la misma integral. Entonces:

$$\int \sqrt{1-x^2}\, dx = \operatorname{arc\,sen} x + x\sqrt{1-x^2} - \int \sqrt{1-x^2}\, dx \Rightarrow$$

$$\Rightarrow 2\int \sqrt{1-x^2}\, dx = \operatorname{arc\,sen} x + x\sqrt{1-x^2} \Rightarrow \int \sqrt{1-x^2}\, dx = \frac{1}{2}\left(\operatorname{arc\,sen} x + x\sqrt{1-x^2}\right) + k$$

102. $\int x \operatorname{arc\,sen} x\, dx$ Sol: $\frac{1}{4}\left[(2x^2 - 1)\operatorname{arcsen} x + x\sqrt{1-x^2}\right] + k$

Hacemos el siguiente cambio: $\left\{ \begin{array}{l} u = \operatorname{arc\,sen} x \\ dv = x dx \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} du = \dfrac{1}{\sqrt{1-x^2}} \cdot dx \\ v = \dfrac{x^2}{2} \end{array} \right.$

Sustituyendo en la fórmula de integración por partes obtenemos:

$$\int x \operatorname{arc\,sen} x \cdot dx = \frac{x^2}{2} \cdot \operatorname{arc\,sen} x - \int \frac{x^2}{2} \cdot \frac{1}{\sqrt{1-x^2}} \cdot dx = \frac{x^2}{2} \cdot \operatorname{arc\,sen} x + \frac{1}{2}\int \frac{-x^2}{\sqrt{1-x^2}} dx =$$

Por el ejercicio anterior tenemos que :

$$\int \frac{-x^2}{\sqrt{1-x^2}} \cdot dx = \int x \cdot \frac{-x}{\sqrt{1-x^2}} \cdot dx = \left\{ \begin{array}{c} u = x \Rightarrow dx \\ dv = \dfrac{-x}{\sqrt{1-x^2}} dx \Rightarrow v = \sqrt{1-x^2} \end{array} \right\} =$$

$$= x\sqrt{1-x^2} - \int \sqrt{1-x^2}\, dx = x\sqrt{1-x^2} - \int \frac{1-x^2}{\sqrt{1-x^2}} dx \quad \Rightarrow$$

$$\int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}}dx - \int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \operatorname{arc\,sen} x - \int \frac{-x^2}{\sqrt{1-x^2}}dx$$

En consecuencia:

$$\int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \operatorname{arc\,sen} x - \int \frac{-x^2}{\sqrt{1-x^2}}dx$$

Por tanto:

$$2\int \frac{-x^2}{\sqrt{1-x^2}}dx = x\sqrt{1-x^2} - \operatorname{arc\,sen} x \Rightarrow \int \frac{-x^2}{\sqrt{1-x^2}}dx = \frac{1}{2}\left(x\sqrt{1-x^2} - \operatorname{arc\,sen} x\right)$$

Sustituyendo obtenemos:

$$\int x\operatorname{arc\,sen} x \cdot dx = \frac{x^2}{2}\cdot \operatorname{arc\,sen} x + \frac{1}{2}\int \frac{-x^2}{\sqrt{1-x^2}}dx =$$

$$= \frac{x^2}{2}\cdot \operatorname{arc\,sen} x + \frac{1}{2}\cdot\frac{1}{2}\left(x\sqrt{1-x^2} - \operatorname{arc\,sen} x\right) + k =$$

$$= \frac{x^2}{2}\cdot \operatorname{arc\,sen} x + \frac{1}{4}x\sqrt{1-x^2} - \frac{1}{4}\operatorname{arc\,sen} x + k =$$

$$= \frac{1}{4}\left[(2x^2-1)\operatorname{arcsen} x + x\sqrt{1-x^2}\right] + k$$

103. $\int \operatorname{arc\,tg} x dx$ Sol: $x \operatorname{arc\,tg} x - \frac{1}{2}\operatorname{Ln}(1+x^2) + k$

$$\int \operatorname{arc\,tg} x dx = \left\{\begin{matrix} u = \operatorname{arc\,tg} x \Rightarrow du = \frac{1}{1+x^2}dx \\ dv = dx \Rightarrow v = x \end{matrix}\right\} = x\cdot \operatorname{arc\,tg} x - \int x \cdot \frac{1}{1+x^2}dx =$$

$$= x\cdot \operatorname{arc\,tg} x - \int \frac{x}{1+x^2}dx = x\cdot \operatorname{arc\,tg} x - \frac{1}{2}\int \frac{2x}{1+x^2}dx = x\cdot \operatorname{arc\,tg} x - \frac{1}{2}\operatorname{Ln}(1+x^2) + k$$

104. $\int \operatorname{arc\,tg}\sqrt{x}\, dx$ Sol: $(x+1)\operatorname{arc\,tg}\sqrt{x} - \sqrt{x} + k$

$$\int \operatorname{arc\,tg}\sqrt{x}\, dx = \left\{\begin{matrix} u = \operatorname{arc\,tg}\sqrt{x} \Rightarrow du = \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}dx \\ dv = dx \Rightarrow v = x \end{matrix}\right\} = x\operatorname{arc\,tg}\sqrt{x} - \int x\frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}dx =$$

$$= x\operatorname{arc\,tg}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{1+x}\cdot dx = \left\{\begin{matrix} x = t^2 \Rightarrow \\ \Rightarrow dx = 2tdt \end{matrix}\right\} = x\operatorname{arc\,tg}\sqrt{x} - \frac{1}{2}\int \frac{t}{1+t^2}\cdot 2tdt =$$

$$= x\,\text{arc tg}\sqrt{x} - \int \frac{t^2}{1+t^2}\,dt = x\,\text{arc tg}\sqrt{x} - \int \frac{1+t^2-1}{1+t^2}\,dt =$$

$$= x\,\text{arc tg}\sqrt{x} - \int \frac{1+t^2-1}{1+t^2}\,dt = x\,\text{arc tg}\sqrt{x} - \int \left(1 - \frac{1}{1+t^2}\right)dt =$$

$$= x\,\text{arc tg}\sqrt{x} - \int dt + \int \frac{1}{1+t^2}\,dt = x\,\text{arc tg}\sqrt{x} - t + \text{arc tg}\, t + k =$$

$$= x\,\text{arc tg}\sqrt{x} - \sqrt{x} + \text{arc tg}\sqrt{x} + k = (x+1)\,\text{arc tg}\sqrt{x} - \sqrt{x} + k$$

105. $\int \text{Ln}\,(x + \sqrt{1+x^2})\,dx$ Sol: $x\,\text{Ln}\,(x + \sqrt{1+x^2}) - \sqrt{1+x^2} + k$

Hacemos: $u = \text{Ln}\,(x + \sqrt{1+x^2})$ y $dv = dx$ con lo cual

$$du = \frac{1}{x+\sqrt{1+x^2}} \cdot \left(1 + \frac{2x}{2\sqrt{1+x^2}}\right)dx = \frac{1}{x+\sqrt{1+x^2}} \cdot \left(1 + \frac{x}{\sqrt{1+x^2}}\right)dx \Rightarrow$$

$$\Rightarrow du = \frac{1}{x+\sqrt{1+x^2}} \cdot \left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right)dx = \frac{1}{\sqrt{1+x^2}}\,dx \quad \text{y} \quad v = x$$

Sustituyendo en la fórmula de integración por partes, obtenemos:

$$\int \text{Ln}\,(x+\sqrt{1+x^2})\,dx = x \cdot \text{Ln}\,(x+\sqrt{1+x^2}) - \int x \cdot \frac{1}{\sqrt{1+x^2}}\,dx =$$

$$= x \cdot \text{Ln}\,(x+\sqrt{1+x^2}) - \int \frac{2x}{2\sqrt{1+x^2}}\,dx = x \cdot \text{Ln}\,(x+\sqrt{1+x^2}) - \sqrt{1+x^2} + k$$

106. $\int \frac{x\,\text{arc sen}\,x}{\sqrt{1-x^2}}\,dx$ Sol: $x - \sqrt{1-x^2}\,\text{arc sen}\,x + k$

$$\int \frac{x\,\text{arc sen}\,x}{\sqrt{1-x^2}}\,dx = \left\{ \begin{array}{c} u = \text{arc sen}\,x \Rightarrow du = \frac{1}{\sqrt{1-x^2}}\,dx \\ dv = \frac{x}{\sqrt{1-x^2}}\,dx \Rightarrow v = -\int \frac{-2x}{2\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} \end{array} \right\} =$$

$$= -\sqrt{1-x^2} \cdot \text{arc sen}\,x - \int -\sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} \cdot \text{arc sen}\,x + \int dx =$$

$$= -\sqrt{1-x^2} \cdot \text{arc sen}\,x + x + k$$

107. $\displaystyle\int \frac{2x-1}{(x-1)(x-2)}dx$ Sol: $\text{Ln}\left|\frac{(x-2)^3}{x-1}\right|+k$

Tenemos una integral de tipo racional donde el grado del numerador es menor que el grado del denominador. Vamos a descomponer el integrando en fracciones simples:

$$(x-1)(x-2)=0 \Rightarrow \begin{cases} x=1 \\ x=2 \end{cases} \text{ (raíces reales simples)}$$

Entonces:

$$\frac{2x-1}{(x-1)(x-2)}=\frac{A}{x-1}+\frac{B}{x-2}=\frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$

Vamos a calcular los coeficientes indeterminados. Al ser los denominadores iguales, los numeradores también lo serán. Por tanto:

$$2x-1=A(x-2)+B(x-1) \Rightarrow \begin{cases} x=1 \rightarrow 1=-A \Rightarrow A=-1 \\ x=2 \rightarrow 3=B \quad \Rightarrow B=3 \end{cases}$$

Por tanto,

$$\int \frac{2x-1}{(x-1)(x-2)}dx=\int\left(\frac{-1}{x-1}+\frac{3}{x-2}\right)dx=-\int \frac{1}{x-1}dx+3\int \frac{1}{x-2}dx=$$

$$=-\text{Ln }(x-1)+3\ \ \text{Ln }(x-2)+k=\text{Ln}\left|\frac{(x-2)^3}{x-1}\right|+k$$

108. $\displaystyle\int \frac{xdx}{(x+1)(x+3)(x+5)}$ Sol: $\frac{1}{8}\text{ Ln}\left|\frac{(x+3)^6}{(x+1)(x+5)^5}\right|+k$

Tenemos una integral de tipo racional donde el grado del numerador es menor que el grado del denominador. Vamos a descomponer el integrando en fracciones simples:

$$(x+1)(x+3)(x+5)=0 \Rightarrow \begin{cases} x=-1 \\ x=-3 \\ x=-5 \end{cases} \text{ (raíces reales simples)}$$

Entonces:

$$\frac{x}{(x+1)(x+3)(x+5}=\frac{A}{x+1}+\frac{B}{x+3}+\frac{C}{x+5}=$$

$$=\frac{A(x+3)(x+5)+B(x+1)(x+5)+C(x+1)(x+3)}{(x+1)(x+3)(x+5)}$$

Para calcular los coeficientes indeterminados, al ser los denominadores iguales, los numeradores también lo serán. Por tanto:

$$x = A(x+3)(x+5) + B(x+1)(x+5) + C(x+1)(x+3) \Rightarrow \begin{cases} x=-1 \to -1 = 8A \Rightarrow A = -\frac{1}{8} \\ x=-3 \to -3 = -4B \Rightarrow B = \frac{3}{4} \\ x=-5 \to -5 = 8C \Rightarrow C = -\frac{5}{8} \end{cases}$$

Por tanto,

$$\int \frac{xdx}{(x+1)(x+3)(x+5)} = \int \left(\frac{-\frac{1}{8}}{x+1} + \frac{\frac{3}{4}}{x+3} + \frac{-\frac{5}{8}}{x+5} \right) dx =$$

$$= -\frac{1}{8}\int \frac{1}{x+1}dx + \frac{3}{4}\int \frac{1}{x+3}dx - \frac{5}{8}\int \frac{1}{x+5}dx = -\frac{1}{8}\mathrm{Ln}(x+1) + \frac{3}{4}\mathrm{Ln}(x+3) - \frac{5}{8}\mathrm{Ln}(x+5) + k =$$

$$= \frac{1}{8}\left(\mathrm{Ln}\,(x+1) + 6\,\mathrm{Ln}\,(x+3) - 5\,\mathrm{Ln}\,(x+5)\right) + k = \frac{1}{8}\mathrm{Ln}\left|\frac{(x+3)^6}{(x+1)(x+5)^5}\right| + k$$

109. $\displaystyle\int \frac{x^5 + x^4 - 8}{x^3 - 4x} dx$ Sol: $\displaystyle\frac{x^3}{3} + \frac{x^2}{2} + 4x + \mathrm{Ln}\left|\frac{x^2(x-2)^5}{(x+2)^3}\right| + k$

Al ser el grado del numerador mayor que el grado del denominador, antes de aplicar el método de descomposición en fracciones simples tendremos que dividir. De esta forma obtenemos:

$$\frac{x^5 + x^4 - 8}{x^3 - 4x} = x^2 + x + 4 + \frac{4x^2 + 16x - 8}{x^3 - 4x}$$

En consecuencia:

$$\int \frac{x^5 + x^4 - 8}{x^3 - 4x} \cdot dx = \int (x^2 + x + 4)dx + \int \frac{4x^2 + 16x - 8}{x^3 - 4x} \cdot dx =$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \int \frac{4x^2 + 16x - 8}{x^3 - 4x} \cdot dx$$

A la integral que nos queda le aplicamos el método de descomposición en fracciones simples. Calculamos las raíces del denominador:

$$x^3 - 4x = 0 \quad \to \quad x \cdot (x^2 - 4) = 0 \quad \to \quad \begin{cases} x = 0 \\ x = \pm 2 \end{cases}$$

Entonces:

$$\frac{4x^2 + 16x - 8}{x^3 - 4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} = \frac{A(x-2)(x+2) + Bx(x+2) + Cx(x-2)}{x(x-2)(x+2)}$$

Como los denominadores son iguales, los numeradores también lo serán; por tanto:

$$4x^2 + 16x - 8 = A(x-2)(x+2) + Bx(x+2) + Cx(x-2)$$

Calculamos los coeficientes indeterminados: le vamos asignando los valores de las raíces

$$\begin{array}{lllll} x = 0 & \to & -8 = -4A & \to & A = 2 \\ x = 2 & \to & 40 = \ 8B & \to & B = 5 \\ x = -2 & \to & -24 = 8C & \to & C = -3 \end{array}$$

Por tanto, la fracción descompuesta en fracciones simples nos queda:

$$\frac{4x^2 + 16x - 8}{x^3 - 4x} = \frac{2}{x} + \frac{5}{x-2} - \frac{3}{x+2}$$

La integral de la función pedida será:

$$\int \frac{x^5 + x^4 - 8}{x^3 - 4x} \cdot dx = \frac{x^3}{3} + \frac{x^2}{2} + 4x + \int \frac{4x^2 + 16x - 8}{x^3 - 4x} \cdot dx =$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \int \left(\frac{2}{x} + \frac{5}{x-2} - \frac{3}{x+2} \right) \cdot dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \int \frac{2}{x} \cdot dx + \int \frac{5}{x-2} \cdot dx - \int \frac{3}{x+2} \cdot dx =$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 4x + 2\int \frac{1}{x} \cdot dx + 5\int \frac{1}{x-2} \cdot dx - 3\int \frac{1}{x+2} \cdot dx =$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 4x + 2 \cdot \text{Ln}\,|x| + 5 \cdot \text{Ln}\,|x-2| - 3 \cdot \text{Ln}\,|x+2| + k =$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \text{Ln}\left| \frac{x^2 (x-2)^5}{(x+2)^3} \right| + k$$

110. $\displaystyle\int \frac{x^4 dx}{(x^2-1)(x+2)}$ Sol: $\displaystyle\frac{x^2}{2}-2x+\frac{1}{6}\ \text{Ln}\left|\frac{(x-1)}{(x+1)^3}\right|+\frac{16}{3}\cdot\text{Ln}\,|\,x+2\,|+k$

Como el grado del numerador es mayor que el del denominador, tenemos que dividir, obteniendo:

$$\frac{x^4}{(x^2-1)(x+2)}=x-2+\frac{5x^2-4}{(x^2-1)(x+2)}$$

Con lo que

$$\int\frac{x^4\cdot dx}{(x^2-1)(x+2)}=\int(x-2)\,dx+\int\frac{5x^2-4}{(x^2-1)(x+2)}dx=\frac{x^2}{2}-2x+\int\frac{5x^2-4}{(x^2-1)(x+2)}dx$$

y tendremos que integrar la función racional que nos queda, donde el grado del numerador es menor que el grado del denominador.

Descomponemos en fracciones simples:

$$\frac{5x^2-4}{(x^2-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+2}=\frac{A(x+1)(x+2)+B(x-1)(x+2)+C(x-1)(x+1)}{(x^2-1)(x+2)}$$

Como los denominadores son iguales, también lo serán los numeradores. Entonces:

$$5x^2-4=A(x+1)(x+2)+B(x-1)(x+2)+C(x-1)(x+1)$$

Calculamos los coeficientes indeterminados:

$$x=1 \quad\to\quad 1=6A \quad\to\quad A=\frac{1}{6}$$

$$x=-1 \quad\to\quad 1=-2B \quad\to\quad B=-\frac{1}{2}$$

$$x=-2 \quad\to\quad 16=3C \quad\to\quad C=\frac{16}{3}$$

Entonces: $$\frac{5x^2-4}{(x^2-1)(x+2)}=\frac{\frac{1}{6}}{x-1}+\frac{-\frac{1}{2}}{x+1}+\frac{\frac{16}{3}}{x+2}$$

Y, por tanto:

$$\int \frac{x^4 \cdot dx}{(x^2-1)(x+2)} = \frac{x^2}{2} - 2x + \int \frac{5x^2-4}{(x^2-1)(x+2)} dx = \frac{x^2}{2} - 2x + \int \left(\frac{\frac{1}{6}}{x-1} + \frac{-\frac{1}{2}}{x+1} + \frac{\frac{16}{3}}{x+2} \right) \cdot dx =$$

$$= \frac{x^2}{2} - 2x + \int \frac{\frac{1}{6}}{x-1} \cdot dx + \int \frac{-\frac{1}{2}}{x+1} \cdot dx + \int \frac{\frac{16}{3}}{x+2} \cdot dx =$$

$$= \frac{x^2}{2} - 2x + \frac{1}{6} \int \frac{1}{x-1} \cdot dx - \frac{1}{2} \int \frac{1}{x+1} \cdot dx + \frac{16}{3} \int \frac{1}{x+2} \cdot dx =$$

$$= \frac{x^2}{2} - 2x + \frac{1}{6} \cdot \text{Ln}\,|x-1| - \frac{1}{2} \cdot \text{Ln}\,|x+1| + \frac{16}{3} \cdot \text{Ln}\,|x+2| + k =$$

$$= \frac{x^2}{2} - 2x + \frac{1}{6} \cdot \left(\text{Ln}\,|x-1| - 3 \cdot \text{Ln}\,|x+1| \right) + \frac{16}{3} \cdot \text{Ln}\,|x+2| + k =$$

$$= \frac{x^2}{2} - 2x + \frac{1}{6} \cdot \text{Ln} \left| \frac{x-1}{(x+1)^3} \right| + \frac{16}{3} \cdot \text{Ln}\,|x+2| + k$$

111. $\displaystyle\int \frac{dx}{(x-1)^2(x-2)}$ Sol: $\displaystyle\frac{1}{x-1} + \text{Ln} \left| \frac{x-2}{x-1} \right| + k$

Como el grado del numerador es menor que el grado del denominador aplicamos la descomposición en fracciones simples directamente:

$$\frac{1}{(x-1)^2(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-2)} = \frac{A(x-1)(x-2) + B(x-2) + C(x-1)^2}{(x-1)^2(x-2)} \quad \rightarrow$$

$$\rightarrow \quad 1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

Calculamos los coeficientes:

$x = 1: \quad 1 = -B \quad \rightarrow \quad B = -1$

$x = 2: \quad 1 = C$

$x = 0: \quad 1 = 2A - 2B + C \quad \rightarrow \quad 1 = 2A + 2 + 1 \quad \rightarrow \quad A = -1$

Entonces:

$$\int \frac{1}{(x-1)^2(x-2)} \cdot dx = \int \frac{-1}{x-1} \cdot dx + \int \frac{-1}{(x-1)^2} \cdot dx + \int \frac{1}{(x-2)} \cdot dx =$$

$$= -\int \frac{1}{x-1}\cdot dx - \int (x-1)^{-2}\cdot dx + \int \frac{1}{x-2}\cdot dx =$$

$$= -\text{Ln}\,|x-1| - \frac{(x-1)^{-1}}{-1} + \text{Ln}\,|x-2| + k = \frac{1}{x-1} + \text{Ln}\left|\frac{x-2}{x-1}\right| + k$$

112. $\int \frac{x-8}{x^3-4x^2+4x}dx$ Sol: $\frac{3}{x-2} + \text{Ln}\,\frac{(x-2)^2}{x^2} + k$

Igual que en el anterior, aplicamos la descomposición en fracciones simples:

Calculamos las raíces del denominador:

$$x^3-4x^2+4x=0 \;\rightarrow\; x\cdot(x^2-4x+4)=0 \;\rightarrow\; x\cdot(x-2)^2=0 \;\rightarrow\; \begin{cases} x=0 \\ x=2 \quad \text{(doble)} \end{cases}$$

Entonces:

$$\frac{x-8}{x^3-4x^2+4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2} = \frac{A(x-2)^2 + Bx(x-2) + Cx}{x(x-2)^2} \quad \Rightarrow$$

$$\Rightarrow \quad x-8 = A(x-2)^2 + Bx(x-2) + Cx \quad \Rightarrow$$

Calculamos los coeficientes:

$$x=0 \;\rightarrow\; -8=4A \;\rightarrow\; A=-2$$
$$x=2 \;\rightarrow\; -6=2C \;\rightarrow\; C=-3$$
$$x=1 \;\rightarrow\; -7=A-B+C \;\rightarrow\; B=7+A+C=7-2-3=2 \;\rightarrow\; B=2$$

Entonces:

$$\int \frac{x-8}{x^3-4x^2+4x}\,dx = \int\left(\frac{-2}{x} + \frac{2}{x-2} + \frac{-3}{(x-2)^2}\right)\cdot dx =$$

$$= -2\int \frac{1}{x}\cdot dx + 2\int \frac{1}{x-2}\cdot dx - 3\int \frac{1}{(x-2)^2}\cdot dx = -2\,\text{Ln}\,|x| + 2\,\text{Ln}\,|x-2| - 3\int (x-2)^{-2}dx =$$

$$= -2\,\text{Ln}\,|x| + 2\,\text{Ln}\,|x-2| - 3\cdot\frac{(x-2)^{-1}}{-1} + k = \frac{3}{x-2} + \text{Ln}\,\frac{(x-2)^2}{x^2} + k$$

113. $\int \frac{3x+2}{x(x+1)^3}\,dx$ Sol: $\frac{4x+3}{2(x+1)^2} + \text{Ln}\,\frac{x^2}{(x+1)^2} + k$

114. $\int \frac{3x-2}{x(x+1)^3}dx$ Sol: $\text{Ln}\,\frac{(x+1)^2}{x^2} - \frac{4x+9}{2(x+1)^2} + k$

Descomponemos el integrando en fracciones simples:

$$\frac{3x-2}{x(x+1)^3}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}+\frac{D}{(x+1)^3}=\frac{A(x+1)^3+Bx(x+1)^2+Cx(x+1)+Dx}{x\cdot(x+1)^3}\quad\rightarrow$$

$$\rightarrow\quad 3x-2=A(x+1)^3+Bx(x+1)^2+Cx(x+1)+Dx\quad\rightarrow$$

Calculamos los coeficientes:

$x=0 \quad\rightarrow\quad -2=A$

$x=-1\rightarrow\quad -5=-D\quad\rightarrow\quad D=5$

$x=1\quad\rightarrow\quad 1=8A+4B+2C+D\quad\rightarrow\quad 1=-16+4B+2C+5\quad\rightarrow\quad 2B+C=6$

$x=-2\rightarrow\quad -8=-A-2B+2C-2D\quad\rightarrow\quad -8=2-2B+2C-10\quad\rightarrow\quad -B+C=0$

Resolviendo el sistema resultante, obtenemos:

$$\left.\begin{array}{r}2B+C=6\\-B+C=0\end{array}\right\}\quad\rightarrow\quad\left\{\begin{array}{l}2B+B=6\quad\rightarrow\quad B=2\\C=B=2\end{array}\right.$$

Entonces:

$$\int\frac{3x-2}{x(x+1)^3}\cdot dx=\int\frac{-2}{x}\cdot dx+\int\frac{2}{x+1}\cdot dx+\int\frac{2}{(x+1)^2}\cdot dx+\int\frac{5}{(x+1)^3}\cdot dx=$$

$$=-2\int\frac{1}{x}\cdot dx+2\int\frac{1}{x+1}\cdot dx+2\int\frac{1}{(x+1)^2}\cdot dx+5\int\frac{1}{(x+1)^3}\cdot dx=$$

$$=-2\cdot\text{Ln}\,|x|+2\cdot\text{Ln}\,|x+1|+2\int(x+1)^{-2}\cdot dx+5\int(x+1)^{-3}\cdot dx=$$

$$=-2\cdot\text{Ln}\,|x|+2\cdot\text{Ln}\,|x+1|+2\,\frac{(x+1)^{-1}}{-1}+5\,\frac{(x+1)^{-2}}{-2}+k=$$

$$=\text{Ln}\,\frac{(x+1)^2}{x^2}-\frac{2}{x+1}-\frac{5}{2(x+1)^2}+k=\text{Ln}\,\frac{(x+1)^2}{x^2}-\frac{4(x+1)+5}{2(x+1)^2}+k=$$

$$=\text{Ln}\,\frac{(x+1)^2}{x^2}-\frac{4x+9}{2(x+1)^2}+k$$

115. $\int \frac{x^2 dx}{(x+2)^2 (x+4)^2}$ Sol: $-\frac{5x+12}{x^2+6x+8} + \text{Ln}\left(\frac{x+4}{x+2}\right)^2 + k$

116. $\int \frac{\sqrt{x}}{\sqrt[4]{x^3}+1} dx$ Sol: $\frac{4}{3}\left(\sqrt[4]{x^3} - \text{Ln}\left(\sqrt[4]{x^3}+1\right)\right) + k$

117. $\int \frac{\sqrt{x^3} - \sqrt[3]{x}}{6\sqrt[4]{x}} dx$ Sol: $\frac{2}{27}\sqrt[4]{x^9} - \frac{2}{13}\sqrt[12]{x^{13}} + k$

118. $\int \frac{\sqrt[6]{x}+1}{\sqrt[6]{x^7}+\sqrt[4]{x^5}} dx$ Sol: $-\frac{6}{\sqrt[6]{x}} + \frac{12}{\sqrt[12]{x}} + 2\,\text{Ln}\,x - 24\,\text{Ln}\,(\sqrt[12]{x}+1) + k$

119. $\int \frac{\sqrt[7]{x}+\sqrt{x}}{\sqrt[7]{x^8}+\sqrt[14]{x^{15}}} dx$ Sol: $4\left[\sqrt[14]{x} - \frac{1}{2}\sqrt[7]{x} + \frac{1}{3}\sqrt[14]{x^3} - \frac{1}{4}\sqrt[7]{x^2} + \frac{1}{5}\sqrt[14]{x^5}\right] + k$

120. $\int \frac{dx}{\sqrt{1-x}+\sqrt[3]{1-x}}$ Sol: $6\left[\frac{\sqrt{1-x}}{3} - \frac{\sqrt[3]{1-x}}{2} + \sqrt[6]{1-x} - Ln\left(1+\sqrt[6]{1-x}\right)\right] + k$

121. $\int \frac{e^x}{e^{2x}+e^x-2} dx$ Sol: $\frac{1}{3}\ \text{Ln}\left|\frac{e^x-1}{e^x+2}\right| + k$

122. $\int \frac{dx}{e^x+1}$ Sol: $x - \text{Ln}\,(e^x+1) + k$

123. $\int \frac{e^x}{e^{2x}+3e^x+2} dx$ Sol: $\text{Ln}\left|\frac{e^x+1}{e^x+2}\right| + k$

124. $\int \text{sen}^3 x\, dx$ Sol: $\frac{1}{3}\cos^3 x - \cos x + k$

125.