- Calcule las siguientes integrales

$$\int \frac{\operatorname{sen} 3x}{e^{x}}\, dx$$

$$\int \frac{\operatorname{sen} x}{5-2\cos x}\, dx$$

$$\int \frac{\ln x}{\sqrt{x}}\, dx$$

$$\int \frac{x}{\cos^{2} x}\, dx$$

$$\int \frac{x^{2}}{4+x^{6}}\, dx$$

$$\int \sqrt{x^{4}-5x^{2}}\, dx$$

$$\int \frac{2x^{3}-9x^{2}+9x+6}{x^{2}-5x+6}\, dx$$

- Calcula los siguientes integrales:

$$\int \frac{\text{sen } 3x}{e^{x}}\,dx = \int e^{-x} \cdot \text{sen } 3x\,dx \rightarrow \text{por partes}$$

$$u = \text{sen } 3x \rightarrow du = 3\cos 3x\,dx$$
$$dv = e^{-x} \cdot dx \rightarrow v = \int e^{-x}\,dx = -e^{-x}$$

$$\int e^{-x} \text{sen } 3x\,dx = -e^{-x} \cdot \text{sen } 3x - \int -e^{-x}\, 3\cos 3x\,dx =$$

$$\Rightarrow u = \cos 3x \Rightarrow du = -3\,\text{sen } 3x\,dx$$
$$dv = e^{-x}dx \Rightarrow v = \int e^{-x}dx = -e^{-x}$$

$$= -e^{-x} \cdot \text{sen } 3x + 3\left[-e^{-x} \cdot \cos 3x - \int + e^{-x} \cdot 3\,\text{sen } 3x\,dx = \right.$$

$$\Rightarrow \underbrace{\int e^{-x} \cdot \text{sen } 3x\,dx}_{I} = -e^{-x} \cdot \text{sen } 3x - 3e^{-x} \cdot \cos 3x - 9 \cdot \underbrace{\int e^{-x} \text{sen } 3x\,dx}_{I} \Rightarrow$$

$$\boxed{I = \int e^{-x} \cdot \text{sen } 3x\,dx}$$

$$I = -e^{-x} \cdot \text{sen} 3x - 3e^{-x} \cdot \cos 3x - 9 \cdot I$$

$$9I + I = -e^{-x} \cdot \text{sen } 3x - 3e^{-x} \cdot \cos 3x$$

$$I = \frac{-e^{-x} \cdot (\text{sen} 3x + 3\cos 3x)}{10} \Rightarrow$$

$$\int e^{-x} \cdot \text{sen } 3x\,dx = \frac{-e^{-x}(\text{sen } 3x + 3\cos 3x)}{10} + C$$

$$\cdot \int \frac{\operatorname{sen} x}{5-2\cos x}\,dx = \frac{1}{2}\int \frac{2\operatorname{sen} x}{5-2\cos x}\,dx =$$

$$= \frac{1}{2}\ln|5-2\cos x| + C$$

$$\cdot \int \frac{3x-8}{x^3+4x}\,dx = \int \frac{3x-8}{x(x^2+4)} = \int \frac{-2}{x}\,dx + \int \frac{2x+3}{x^2+4}\,dx =$$

$$\frac{3x-8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$3x-8 = A(x^2+4) + Bx(x-2) + Cx(x+2)$$

SP $x=2 \rightarrow \quad 6-8 = 8A + 8C$

SP $x=-2 \rightarrow \quad -6-8 = 8A + 8B$

$x=0 \rightarrow \quad -8 = 4A \rightarrow \boxed{A=-2}$

$$\rightarrow B = \frac{-14+16}{8} = \frac{2}{8} = \boxed{\frac{1}{4}}$$

$$C = \frac{-2+16}{8} = \frac{14}{8} = \boxed{\frac{7}{4}}$$

$$= \int \frac{-2}{x} + \int \frac{1}{4x+8}\,dx + \int \frac{7}{4x-2}\,dx =$$

$$= -2\ln|x| +$$

- $$\int \frac{\ln x}{\sqrt{x}}\,dx = \left[\begin{array}{l} \sqrt{x}=t \Rightarrow x=t^2 \\ \frac{1}{2\sqrt{x}}\,dx = dt \Rightarrow dx = 2\sqrt{x}\,dt \\ \qquad dx = 2t\,dt \end{array}\right.$$

$$\Rightarrow \int \frac{\ln t^2}{t}\,2t\,dt = \int \frac{2\ln t}{t}\cdot 2t\,dt = 4\int \ln t\,dt =$$

$$= 4(t\ln t - t) + C = 4(\sqrt{x}\ln\sqrt{x} - \sqrt{x}) + C$$

$$= 4\left(\frac{\sqrt{x}}{2}\ln x - \sqrt{x}\right) + C = \boxed{2\sqrt{x}\ln x - 4\sqrt{x} + C}$$

- $$\int \frac{x}{\cos^2 x}\,dx = \left[\begin{array}{l} u = x \Rightarrow du = dx \\ dv = \frac{1}{\cos^2 x} \Rightarrow v = \operatorname{tg} x \end{array}\right.$$

$$\Rightarrow x\cdot \operatorname{tg} x - \int \operatorname{tg} x \cdot dx = x\operatorname{tg} x - \int \frac{\operatorname{sen} x}{\cos x}\,dx =$$

$$= x\operatorname{tg} x - (-\ln(\cos x)) + C = x\operatorname{tg} x + \ln(\cos x) + C$$

- $$\int \frac{x^2}{4+x^6}\,dx = \int \frac{x^2}{4+(x^3)^2}\,dx = \int \frac{x^2/4}{\frac{4}{4}+\frac{(x^3)^2}{4}}\,dx = \int \frac{x^2/4}{1+\left(\frac{x^3}{2}\right)^2} =$$

$$= \frac{1}{4}\int \frac{x^2}{1+\left(\frac{x^3}{2}\right)^2}\,dx = \frac{1}{4}\cdot\frac{2}{3}\int \frac{3x^2/2}{1+\left(\frac{x^3}{2}\right)^2}\,dx = \frac{1}{6}\operatorname{arctg}\frac{x^3}{2} + C$$

$$\int \sqrt{x^4-5x^2}\,dx = \int \sqrt{x^2(x^2-5)}\,dx = \int x\sqrt{x^2-5}\,dx =$$

$$= \frac{1}{2}\int 2x(x^2-5)^{1/2}dx = \frac{1}{2}\,\frac{(x^2-5)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$$

$$= \frac{1}{2}\,\frac{(x^2-5)^{3/2}}{3/2} + C = \frac{\sqrt{(x^2-5)^3}}{3} + C = \frac{(x^2-5)\sqrt{x^2-5}}{3} + C$$

• $$\int \frac{2x^3-9x^2+9x+6}{x^2-5x+6}\,dx \Rightarrow$$

$$2x^3-9x^2+9x+6 = (2x+1)(x^2-5x+6)+2x$$

$$\Rightarrow \int (2x+1)dx + \int \frac{2x}{x^2-5x+6}$$

$$\frac{2x}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3} = \frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$$

$$\Rightarrow \begin{cases} A+B=2 \\ -3A-2B=0 \end{cases} \quad \begin{cases} A=-4 \\ B=6 \end{cases}$$

$$\Rightarrow \int (2x+1)dx - 4\int \frac{1}{x-2} + 6\int \frac{1}{x-3}\,dx =$$

$$x^2+x-4\ln|x-2) + 6\ln(x-3) + C$$