Discuta, según los valores del parámetro m, el sistema: $\begin{cases} x + (m-3)y + mz = 1 \\ (m-3)y + (m^2-m)z = 1 \\ x + m^2 z = 0 \end{cases}$

Discuta, según los valores del parámetro m, el sistema: $\begin{cases} x + 2y = m \\ my + 3z = 1 \\ x + (m+2)y + (m+1)z = m+1 \end{cases}$

Discuta, según los valores del parámetro m, el sistema: $\begin{cases} mx + y + z = 2m \\ mx + (m+1)y + z = 1 \\ mx + (m+1)y + 2z = m+1 \end{cases}$

Resuélvelos para todos los casos en los que sean compatibles

Galicia Ord 2022

(1) $A^* = \left(\begin{array}{ccc|c} 1 & m-3 & m & 1 \\ 0 & m-3 & m^2-m & 1 \\ 1 & 0 & m^2 & 0 \end{array}\right)$ $\qquad A = \begin{pmatrix} 1 & m-3 & m \\ 0 & m-3 & m^2-m \\ 1 & 0 & m^2 \end{pmatrix}$

[Nota: Como $C_2 = (m-3)\cdot C_4 \Rightarrow rg\,A = rg\,A^*$]

(1º) $|A| = m^2\cdot(m-3) + (m^2-m)\cdot(m-3) - m\cdot(m-3) =$
$= 2m^2(m-3) - 2m(m-3) = 2m(m-3)(m-1)$

(2º) $|A| = 0 \Leftrightarrow \begin{cases} m=0 \\ m=3 \\ m=1 \end{cases}$

polo tto, se $m \neq 0,1,3$, $|A| \neq 0 \Rightarrow rg\,A = 3 = rg\,A^* \Rightarrow$ SCD Única solución

(3º) Estudamos os casos nos que se anula $|A|$ e calculamos en cada caso, $rg\,A$ e $rg\,A^*$.

$\underline{m=0}$

$A^* = \left(\begin{array}{ccc|c} 1 & -3 & 0 & 1 \\ 0 & -3 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array}\right)$

$A = \begin{pmatrix} 1 & -3 & 0 \\ 0 & -3 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

A? $\exists \begin{vmatrix} 1 & -3 \\ 0 & -3 \end{vmatrix} = -3 \neq 0$

menor orde 2 $\neq 0$ de A

$\Downarrow$

$rg\,A = 2$

A^*? $\begin{vmatrix} 1 & -3 & 1 \\ 0 & -3 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0$ ... todos os menores de orde 3 son cero de A^*

($C_4 = C_2 : (-3)$) e $C_3 = 0$)

$\Rightarrow rg\,A^* = 2$

$\Longrightarrow rg\,A = 2 = rg\,A^* < 3 =$ nº incógnitas $\Rightarrow$ SCI Inf. solucións

$\underline{m=3}$ $A^* = \left(\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 0 & 6 & 1 \\ 1 & 0 & 9 & 0 \end{array}\right)$ $A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 0 & 6 \\ 1 & 0 & 9 \end{pmatrix}$

A?

$\exists$ menor orde 2 de A: $\begin{vmatrix} 1 & 3 \\ 0 & 6 \end{vmatrix} = 6 \neq 0 \Rightarrow rg A = 2$

A^*?

$\exists \begin{vmatrix} 1 & 3 & 1 \\ 0 & 6 & 1 \\ 1 & 9 & 0 \end{vmatrix} = 3-6-9 \neq 0 \Rightarrow rg A^* = 3$

menor orde 3 de A^*

$\Rightarrow rg A = 2 \neq 3 = rg A^* \Rightarrow$ S.I Non hai solución

$\underline{m=1}$

$A^* = \left(\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{array}\right)$ $A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$\begin{pmatrix} C_2 = -2C_4 \\ rg A = rg A^* \end{pmatrix}$

A?

$\exists$ menor orde 2 de A $\begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} = -2 \neq 0 \Rightarrow rg A = 2$

A^*? todos os menores de orde 3 de A^* son iguais a cero

pq $C_1 = C_3$
$C_2 = -2C_4$

$\Rightarrow rg A^* = 2$

$\Rightarrow rg A = 2 = rg A^* < 3 = n^o$ incógnitas $\Rightarrow$ SCI

Infinitas solucións

Conclusión n^o incóg $= 3$

valor de m	rg A	rg A^*	clasificación
$m=0$	2	2	SCI
$m=1$	2	2	SCI
$m=3$	2	3	SI
$m\neq 0,1,3$	3	3	SCD

Resolvemos nos casos posibles:

$m=0$ SCI

$$\left\{\begin{array}{l} \cancel{x-3y=1} \\ -3y=1 \Rightarrow y=-1/3 \\ x \qquad =0 \Rightarrow x=0 \end{array}\right. \Rightarrow \left\{\left(0,-\frac{1}{3},\alpha\right);\ \alpha\in\mathbb{R}\right\}$$

z cq valor

$m=1$ SCI

$$\left\{\begin{array}{l} \cancel{x-2y+z=1} \\ -2y \quad =1 \Rightarrow y=-1/2 \\ x \quad +z=0 \Rightarrow z=-x \end{array}\right. \Rightarrow \left\{\left(+\alpha,-\frac{1}{2},-\alpha\right),\ \alpha\in\mathbb{R}\right\}$$

x cq vr.

$m\neq 0,1,3$ SCD

Regra de Cramer

$$x=\frac{\begin{vmatrix} 1 & m-3 & m \\ 1 & m-3 & m^2-m \\ 0 & 0 & m^2 \end{vmatrix}}{|A|}=\frac{m^2(m-3)-m^2(m-3)}{|A|}=0$$

$$y=\frac{\begin{vmatrix} 1 & 1 & m \\ 0 & 1 & m^2-m \\ 1 & 0 & m^2 \end{vmatrix}}{|A|}=\frac{m^2+m^2-m-m}{|A|}=\frac{2m^2-2m}{|A|}=\frac{2m(m-1)}{2m(m-3)(m-1)}=\frac{1}{m-3}$$

$$z=\frac{\begin{vmatrix} 1 & m-3 & 1 \\ 0 & m-3 & 1 \\ 1 & 0 & 0 \end{vmatrix}}{|A|}=0 \qquad \text{Sol } \left\{\left(0,\frac{1}{m-3},0\right)\right\}$$

Galicia Ord 2021

2

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & m & 3 \\ 1 & m+2 & m+1 \end{pmatrix} \qquad A^* = \left(\begin{array}{ccc|c} 1 & 2 & 0 & m \\ 0 & m & 3 & 1 \\ 1 & m+2 & m+1 & m+1 \end{array}\right)$$

$|A| = m(m+1) + 6 - 3(m+2) = m^2 - 2m$

$|A| = 0 \iff \begin{cases} m = 0 \\ m = 2 \end{cases}$

Polo tto, se $m \neq 0, 2$, $|A| \neq 0 \Rightarrow rg\, A = 3 = rg\, A^* \Rightarrow$ SCD Sol. única

$\underset{\text{nº incógnitas}}{\overset{\shortparallel}{3}}$

Estudamos os casos nos que $|A| = 0$.

$\underline{m = 0}$

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \\ 1 & 2 & 1 \end{pmatrix} \qquad A^* = \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 1 \\ 1 & 2 & 1 & 1 \end{array}\right)$$

A? $\exists$ menor orde 2 en A $\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6 \neq 0 \Rightarrow rg\, A = 2$

A^*? $\exists$ menor orde 3 en A^* $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 3 - 1 = 2 \neq 0 \Rightarrow rg\, A^* = 3$

$\Rightarrow rg\, A = 2 \neq 3 = rg\, A^* \Rightarrow$ S.I. Non hai solución

$\underline{m = 2}$

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 4 & 3 \end{pmatrix} \qquad A^* = \left(\begin{array}{ccc|c} 1 & 2 & 0 & 2 \\ 0 & 2 & 3 & 1 \\ 1 & 4 & 3 & 3 \end{array}\right)$$

($F_3 = F_1 + F_2$
$\Rightarrow rg\, A^* \leq 2$)

A? $\exists$ menor orde 2 en A: $\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 \neq 0 \Rightarrow rg\, A = 2$

A^*? todos os menores de orde 3 de A^* son iguais a 0
$\Rightarrow rg\, A^* = 2$

Polo tto, $rg\, A = 2 = rg\, A^* < 3 =$ nº incógnitas $\Rightarrow$ SCI
Inf. solucións

Resumo $\quad$ nº incógnitas = 3

Valor de m	rg A	rg A^*	Clasificación
$m=0$	2	3	SI - non hai sol
$m=2$	2	2	SCI - inf. sol.
$m\neq 0,2$	3	3	SCD - Única sol.

Resolvemos os casos posibles:

$m=2$

$$\begin{cases} x+2y=2 \\ \quad 2y+3z=1 \\ x+4y+3z=3 \end{cases} \rightarrow \quad \begin{array}{l} x = 2-2y = 2-(1-3z)=1+3z \\ y=\dfrac{1-3z}{2} \\ z \text{ cq val.} \end{array}$$

$$\Rightarrow \left\{\left(1+3\alpha, \frac{1-3\alpha}{2}, \alpha\right);\ \alpha\in\mathbb{R}\right\}$$

$m\neq 0,2$ $\quad$ Regra de Cramer

$$x=\frac{\begin{vmatrix} m & 2 & 0 \\ 1 & m & 3 \\ m+1 & m+2 & m+1\end{vmatrix}}{|A|}=\frac{m^2(m+1)+6(m+1)-3m(m+2)-2(m+1)}{|A|}=$$

$$=\frac{m^3+m^2+\cancel{6m}+6-3m^2-\cancel{6m}-2m-2}{|A|}=\frac{m^3-2m^2-2m+4}{|A|}=$$

$$=\frac{(m-2)(m^2-2)}{m(m-2)}=\frac{m^2-2}{m}$$

$$\begin{array}{r|rrrr} & 1 & -2 & -2 & 4 \\ 2 & & 2 & 0 & -4 \\ \hline & 1 & 0 & -2 & \boxed{0} \end{array}$$

$$y=\frac{\begin{vmatrix} 1 & m & 0 \\ 0 & 1 & 3 \\ 1 & m+1 & m+1\end{vmatrix}}{|A|}=\frac{m+1+3m-3(m+1)}{|A|}=\frac{m-2}{m(m-2)}=\frac{1}{m}$$

$$z=\frac{\begin{vmatrix} 1 & 2 & m \\ 0 & m & 1 \\ 1 & m+2 & m+1\end{vmatrix}}{|A|}=\frac{m(m+1)+2-m^2-(m+2)}{|A|}=0$$

$$\text{Sol } \left\{\left(\frac{m^2-2}{m}, \frac{1}{m}, 0\right)\right\}$$

Galiza Extraord. 2021

3

$$A = \begin{pmatrix} m & 1 & 1 \\ m & m+1 & 1 \\ m & m+1 & 2 \end{pmatrix} \qquad A^* = \left(\begin{array}{ccc|c} m & 1 & 1 & 2m \\ m & m+1 & 1 & 1 \\ m & m+1 & 2 & m+1 \end{array}\right)$$

$$|A| = 2m(m+1) + m(m+1) + m - m(m+1) - m(m+1) - 2m =$$
$$= m(m+1) - m = m^2$$

$|A| = 0 \iff m = 0$

$\longrightarrow$ Polo tanto, se $m \neq 0$, $|A| \neq 0 \Rightarrow rg A = 3 = rg A^* \Rightarrow$ SCD sol. única

Se $m = 0$

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & \boxed{1} & \boxed{1} \\ 0 & \boxed{1} & \boxed{2} \end{pmatrix} \qquad A^* = \left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{array}\right)$$

A? $\exists$ menor de orde 2 : $\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1 \neq 0 \Rightarrow rg A = 2$
de $A \neq 0$

A^*? $\exists$ menor de orde 3 : $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = 1+1-2-1 = -1 \neq 0 \Rightarrow$
de $A^* \neq 0$

$\Rightarrow rg A^* = 3$

$\Rightarrow rg A = 2 \neq 3 = rg A^* \Rightarrow$ S.I
Non hai solución

Resumo

- $m \neq 0$ $\quad rg A = 3 = rg A^* \Rightarrow$ SCD
- $m = 0$ $\quad rg A = 2 \neq 3 = rg A^* \Rightarrow$ SI

Resolvemos no caso $m \neq 0$

$$x = \frac{1}{|A|} \cdot \begin{vmatrix} 2m & 1 & 1 \\ 1 & m+1 & 1 \\ m+1 & m+1 & 2 \end{vmatrix} = \frac{4m(m+1) + (m+1) + m+1 - (m+1)^2 - 2m(m+1) - 2}{|A|} =$$

$$= \frac{4m^2 + 4m + 2m + 2 - m^2 - 2m - 1 - 2m^2 - 2m - 2}{m^2} =$$

$$= \frac{m^2 + 2m - 1}{m^2}$$

$$y = \frac{1}{|A|} \cdot \begin{vmatrix} m & 2m & 1 \\ m & 1 & 1 \\ m & m+1 & 2 \end{vmatrix} = \frac{2m + m(m+1) + 2m^2 - m - m(m+1) - 2m \cdot 2m}{|A|}$$

$$= \frac{2m + \cancel{m^2} + \cancel{m} + \boxed{2m^2} - \cancel{m} - \cancel{m^2} - m - 4m^2}{m^2} = \frac{-2m^2 + m}{m^2} = \frac{m-2}{m}$$

$$z = \frac{1}{|A|} \cdot \begin{vmatrix} m & 1 & 2m \\ m & m+1 & 1 \\ m & m+1 & m+1 \end{vmatrix} = \frac{m(m+1)^2 + 2m^2(m+1) + m - 2m^2(m+1) - 2m(m+1)}{|A|}$$

$$= \frac{m(m^2 + 2m + 1) + \cancel{2m^3} + \cancel{2m^2} + m - \cancel{2m^3} - \cancel{2m^2} - 2m^2 - 2m}{|A|} =$$

$$= \frac{m^3 + \cancel{2m^2} + \cancel{m} + \cancel{m} - \cancel{2m^2} - \cancel{2m}}{m^2} = m$$

Sol. $\left\{ \left(\frac{m^2 + 2m - 1}{m^2}, \frac{m-2}{m}, m \right) \right\}$

$$\left| \frac{2m - \frac{1-2m}{m} - m}{m} = \frac{2m^2 - 1 + 2m - m^2}{m^2} = \frac{m^2 + 2m - 1}{m^2} \right|$$