

TRIGONOMETRÍA 1º BACHILLERATO

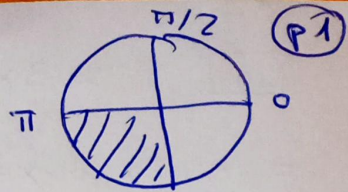
1. Siendo $\tan x = \frac{3}{5}$, con $\pi < x < \frac{3\pi}{2}$, calcular $\sin x$ y $\cos x$.

$$(\cos x = \pm \frac{5}{\sqrt{34}}, \sin x = -\frac{3\sqrt{34}}{34})$$

Mal, las + no valen

2. Resolver el triángulo $A = 32^\circ$ $B = 48^\circ$ $a = 10$ ($b = 14,02$ $c = 18,58$)
3. Resolver el triángulo $A = 75^\circ$ $a = 28$ $b = 12$ ($\hat{B} = 24^\circ 27' 16''$ $c = 28,59$)
4. Cuando una persona que mide 170 cm arroja una sombra de 84 cm, la de un edificio es de 32 metros. ¿Qué altura tiene el edificio? (64,76 m)
5. Un globo está sujeto a un puente de 84 m de largo. Los ángulos de elevación del globo desde cada uno de los extremos del puente son 53° y 74° . ¿Cuál es la altura del globo? (80,73 m)
6. El ángulo de elevación con el que se ve la parte superior de un edificio es de 50° . Avanzando 20 m hacia él, el ángulo es de 65° . ¿Cuál es la altura del edificio? (53,66 m)
7. Desde dos torres de observación separadas entre sí 2485 m se ve un punto (alineado con ellas) bajo ángulos respectivos de 84° y 72° . ¿A qué distancia de cada una de las torres se encuentra dicho punto? (1877,63 m)
8. Calcular el área de un hexágono regular de 5 cm de lado. ($\frac{75\sqrt{3}}{2}$)
9. Desde un punto del suelo se observa un repetidor de televisión situado encima de un monte de 548 m. Los ángulos de elevación de la base del repetidor y de su punto más alto son, respectivamente, 53° y $54^\circ 30'$. ¿Cuál es la altura del repetidor? (30,93 m)
10. Hallar, sin hacer uso de la calculadora, las razones de 105° . Lo mismo, con 345° .
11. Escribir como suma o diferencia el producto $\sin 3x \cdot \cos x$
12. Simplificar la expresión $\frac{1 + \sec x}{\tan x + \sin x}$.
13. Demostrar que $\frac{\sec^4 x - \tan^4 x}{\sec^2 x} = 1 + \sin^2 x$
14. Resolver las ecuaciones
- $2 \tan x \cdot \sec x - \tan x = 0$ ($x = 0^\circ + k \cdot 180^\circ$, $\forall k \in \mathbb{Z}$)
 - $\sin^2 x + 3 \sin x - 2 = 0$ ($x = 34^\circ 9' 48'' + k \cdot 360^\circ$, $x = 145^\circ 50' 12'' + k \cdot 360^\circ$)
 - $\cos x + 2 \sin x \tan x = 1$ ($x = 0^\circ + k \cdot 360^\circ$)
 - $\tan^2 x + 2 \sec^2 x = 1$ (sin solución)
15. Demostrar que $\frac{1 - \tan x}{1 + \tan x} = \frac{1 - \sin 2x}{\cos 2x}$ solo es cierto si $x \neq \frac{\pi}{4} + \frac{k\pi}{2}, \frac{\pi}{2} + k\pi$
16. Simplificar $\cos 2x \cdot \cos 3x + \sin 2x \cdot \sin 3x$
17. Resuelve la ecuación $\cos 2x + \sin x = 4 \sin^2 x$. (Soluciones:)
 $x = 30^\circ + k \cdot 360^\circ$, $x = 150^\circ + k \cdot 360^\circ$, $x = 199^\circ 28' 16'' + k \cdot 360^\circ$, $x = 340^\circ 31' 44'' + k \cdot 360^\circ$
18. Resuelve la ecuación $\cos x + \sqrt{3} \sin x = 2$. ($x = 60^\circ + k \cdot 360^\circ$)
19. Resolver el sistema $\begin{cases} \sin^2 x + y = 1 \\ \cos^2 x + y = 2 \end{cases}$ ($x = k \cdot 180^\circ$, $y = 1$)
20. Resolver el sistema $\begin{cases} \sin x \cdot \sin y = -\frac{1}{2} \\ \cos x \cdot \cos y = -\frac{1}{2} \end{cases}$
- ~~($x = 135^\circ$, $y = 45^\circ$)~~ Mal
 $(x = 135^\circ, y = 315^\circ)$
 $(x = 225^\circ, y = 45^\circ)$

$$[1] \quad \operatorname{tg} x = \frac{3}{5} \quad \text{com } \pi < x < \frac{3\pi}{2} \quad (3^\circ \text{mad})$$



$$\frac{\operatorname{sen} x}{\cos x} = \frac{3}{5} \quad \left\{ \begin{array}{l} \operatorname{sen} x = \frac{3}{5} \cos x \\ \operatorname{sen}^2 x + \cos^2 x = 1 \end{array} \right. \quad \left(\frac{3}{5} \right)^2 \cos^2 x + \cos^2 x = 1;$$

$$\frac{\operatorname{sen} x}{\cos x} = \frac{3\pi}{2} \quad \text{em } 3^\circ \text{mad} \quad \operatorname{sen} y \cos \neq 0$$

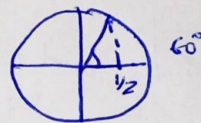
$$\frac{9}{25} \cos^2 x + \cos^2 x = 1 \quad ; \quad \left(\frac{9}{25} + 1 \right) \cos^2 x = 1 \quad ; \quad \frac{34}{25} \cos^2 x = 1 \quad ; \quad \cos^2 x = \frac{25}{34}$$

$$\cos x = -\frac{5}{\sqrt{34}} = \boxed{-\frac{5\sqrt{34}}{34}} \quad ; \quad \operatorname{sen} x = \frac{3}{5} \left(-\frac{5\sqrt{34}}{34} \right) = \boxed{-\frac{3\sqrt{34}}{34}}$$

$3^\circ \text{mad. cos -}$

$$[10] \quad a) \quad 105^\circ = 60^\circ + 45^\circ$$

$$\left. \begin{array}{l} \operatorname{sen}(\alpha + \beta) = \operatorname{sen} \alpha \cos \beta + \cos \alpha \operatorname{sen} \beta \\ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \operatorname{sen} \alpha \operatorname{sen} \beta \end{array} \right\} \Rightarrow$$



$$\operatorname{sen}(105^\circ) = \operatorname{sen} 60^\circ \cos 45^\circ + \cos 60^\circ \operatorname{sen} 45^\circ = \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + \frac{1}{2} \frac{\sqrt{2}}{2} =$$

$$= \boxed{\frac{\sqrt{2}(1 + \sqrt{3})}{4}}$$

$$\cos 105^\circ = \cos 60^\circ \cos 45^\circ - \operatorname{sen} 60^\circ \operatorname{sen} 45^\circ = \frac{1}{2} \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} =$$

$$= \boxed{\frac{\sqrt{2}(1 - \sqrt{3})}{4}}$$

$$b) \quad 345^\circ = (360^\circ + 45^\circ) = ((360^\circ - 60^\circ) + 45^\circ) = ((-60^\circ) + 45^\circ)$$

$$\left. \begin{array}{l} \operatorname{sen} 345^\circ = \operatorname{sen}(-60^\circ) \cos 45^\circ + \cos(-60^\circ) \operatorname{sen} 45^\circ \\ \cos 345^\circ = \cos(-60^\circ) \cos 45^\circ - \operatorname{sen}(-60^\circ) \operatorname{sen} 45^\circ \end{array} \right\} \quad \left\{ \begin{array}{l} \operatorname{sen}(-60^\circ) = -\operatorname{sen} 60^\circ \\ \cos(-60^\circ) = \cos 60^\circ \end{array} \right.$$

$$\operatorname{sen} 345^\circ = -\frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + \frac{1}{2} \frac{\sqrt{2}}{2} = \frac{\sqrt{2}(-\sqrt{3} + 1)}{4} = \boxed{\frac{\sqrt{2}(1 - \sqrt{3})}{4}}$$

$$\cos 345^\circ = \cos(-60^\circ) \cos 45^\circ - \operatorname{sen}(-60^\circ) \operatorname{sen} 45^\circ = \frac{1}{2} \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2} \right) \frac{\sqrt{2}}{2} =$$

$$= \boxed{\frac{\sqrt{2}(1 + \sqrt{3})}{4}}$$

[11]

$$\operatorname{sen} 3x \cdot \cos x = \operatorname{sen}(2x+x) \cdot \cos x =$$

$$= (\operatorname{sen} 2x \cos x + \cos 2x \operatorname{sen} x) \cos x =$$

$$= (2 \operatorname{sen} x \cos^2 x + (\cos^2 x - \operatorname{sen}^2 x) \operatorname{sen} x) \cos x =$$

$$= (2 \operatorname{sen} x \cos^2 x + \cos^2 x \operatorname{sen} x - \operatorname{sen}^3 x) \cos x =$$

$$= (3 \operatorname{sen} x \cos^2 x - \operatorname{sen}^3 x) \cos x$$

$$= \operatorname{sen} x \cdot \cos x (3 \cos^2 x - \operatorname{sen}^2 x) =$$

$$= \left[\operatorname{sen} x \cdot \cos x (\sqrt{3} \cos x + \operatorname{sen} x)(\sqrt{3} \cos x - \operatorname{sen} x) \right]$$

[12]

$$\frac{1 + \sec x}{\operatorname{tg} x + \operatorname{sen} x} = \frac{1 + \frac{1}{\cos x}}{\frac{\operatorname{sen} x}{\cos x} + \operatorname{sen} x} = \frac{\frac{\cos x + 1}{\cos x}}{\frac{\operatorname{sen} x + \operatorname{sen} x \cos x}{\cos x}} =$$

para que $\exists$
 $\operatorname{tg} x + \operatorname{sen} x \neq 0$

para que $\exists$
 $\cos \neq 0$

para que $\exists$
 $\operatorname{sen} x (1 + \cos x) \neq 0$

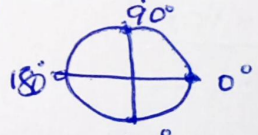
$$= \frac{\frac{\cos x + 1}{\cos x}}{\frac{\operatorname{sen} x (1 + \cos x)}{\cos x}} = \frac{1}{\operatorname{sen} x} = \operatorname{cosec} x$$

si $\cos x \neq 0$
y $1 + \cos x \neq 0$

Entonces siempre q. $\cos x \cdot \operatorname{sen} x \neq 0$; $\cos x \neq -1$

$$\frac{1 + \sec x}{\operatorname{tg} x + \operatorname{sen} x} = \operatorname{cosec} x$$

$x \neq 0, 90, 180, 270$ $+ n^{\circ}$ entero de vueltas



[13]

$$\frac{\sec^4 x - \operatorname{tg}^4 x}{\sec^2 x} = 1 + \operatorname{sen}^2 x$$

para que $\exists \Rightarrow \sec \neq 0$ i.e. $\frac{1}{\cos x} \neq 0$ cierto siempre $\forall x \in \mathbb{R}$

$$\frac{\frac{1}{\cos^4 x} - \frac{\operatorname{sen}^4 x}{\cos^4 x}}{\frac{1}{\cos^2 x}} = \frac{\frac{1 - \operatorname{sen}^4 x}{\cos^4 x}}{\frac{1}{\cos^2 x}} = \frac{(1 - \operatorname{sen}^4 x) \cos^2 x}{\cos^4 x} = \frac{(1 - \operatorname{sen}^4 x)}{\cos^2 x} =$$

$$= \frac{(1 + \operatorname{sen}^2 x)(1 - \operatorname{sen}^2 x)}{(1 + \operatorname{sen}^2 x) \cos^2 x} \quad \text{si } \cos x \neq 0$$

13) continuación, entonces para q. se cumpla p③
la igualdad $\cos x \neq 0$ i.e. $x \in \mathbb{R} - \{90^\circ + k \cdot 180^\circ / k \in \mathbb{Z}\}$

14) Ecuaciones

a) $2 \operatorname{tg} x \cdot \sec x - \operatorname{tg} x = 0$; $\operatorname{tg} x (2 \cdot \sec x - 1) = 0 \Leftrightarrow$

$$\begin{cases} \operatorname{tg} x = 0 & \Leftrightarrow \sec x = 0 ; x \in \{0^\circ + 180^\circ k / k \in \mathbb{Z}\} \\ \text{o} \\ 2 \sec x - 1 = 0 & \Leftrightarrow \sec x = \frac{1}{2} \Leftrightarrow \frac{1}{\cos x} = \frac{1}{2} \Leftrightarrow \cos x = 2 \text{ ¡IMPOSIBLE!} \\ & -1 \leq \cos x \leq 1 \end{cases}$$

Soluciones $x \in \{180^\circ + k / k \in \mathbb{Z}\}$

b) $\sin^2 x + 3 \sin x - 2 = 0$ $\sin x = a$ (cambio variable)

$$a^2 + 3a - 2 = 0 ; a = \frac{-3 \pm \sqrt{9 - 4(-2)}}{2} = \frac{-3 \pm \sqrt{17}}{2}$$

$$\sin x = \frac{-3 + \sqrt{17}}{2} \Rightarrow \text{log. de la calculadora}$$

$$\sin x = \frac{-3 - \sqrt{17}}{2} \rightarrow \text{idem.}$$

c) $\cos x + 2 \sin x \tan x = 1$; $\cos x + \frac{2 \sin^2 x}{\cos x} = 1$

para que $\exists \cos x \neq 0$ (i.e. $x \neq 90^\circ + k \cdot 180^\circ / k \in \mathbb{Z}$)

en ese caso $\frac{\cos^2 x + 2 \sin^2 x}{\cos x} = 1 \Rightarrow (\cos^2 x + 2 \sin^2 x = \underbrace{\cos^2 x + \sin^2 x}_1 + \sin^2 x)$

$$\Rightarrow \left[\frac{1 + \sin^2 x}{\cos x} = 1 \right] \Leftrightarrow 1 + \sin^2 x = \cos x ; 1 + 1 - \cos^2 x = \cos x$$

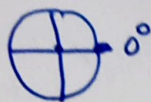
$$\sin^2 x = 1 - \cos^2 x$$

$$2 - \cos^2 x - \cos x = 0 ; \cos^2 x + \cos x - 2 = 0 ; a = \cos x$$

$$a^2 + a - 2 = 0 ; a = \frac{-1 \pm \sqrt{1 - 4(-2)}}{2} = \frac{-1 \pm 3}{2} = \begin{cases} 1 \\ 0 \\ -2 \end{cases}$$

~~IMPOSIBLE~~
 $-1 < \cos x < 1$

Si $\cos x = 1 \Leftrightarrow$



$x = 0^\circ + k \cdot 360^\circ / k \in \mathbb{Z}$ (P.4)

Soluciones:

$\{ k \cdot 360^\circ / k \in \mathbb{Z} \}$

d) $\tan^2 x + 2 \sec^2 = 1$; $\frac{\sec^2 x}{\cos^2 x} + 2 \frac{1}{\cos^2 x} = 1$

para que $\exists \cos^2 x \neq 0$ i.e. $\cos x \neq 0$ portanto $x \neq 90^\circ + k \cdot 180^\circ / k \in \mathbb{Z}$

en ese caso:

$\frac{\sec^2 x + 2}{\cos^2 x} = 1$; $\sec^2 x + 2 = \cos^2 x$; $\sec^2 x - \cos^2 x = -2$

$\Leftrightarrow \underbrace{\cos^2 x - \sec^2 x}_{\cos 2x} = 2$ i $\cos 2x = 2$ impossible!
 $-1 \leq \cos \alpha \leq 1$

$\Rightarrow$ No tiene solución

[15]

Para qué ángulos se cumple la igualdad:

pág 5

$$\frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} = \frac{1 - \operatorname{sen} 2x}{\cos 2x}$$

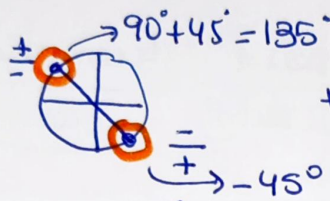
Solución

1º) Para q. exista la fracción $\frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} = \frac{1 - \operatorname{tg} x}{1 + \frac{\operatorname{sen} x}{\cos x}} =$

$$= \frac{1 + \operatorname{tg} x}{\frac{\cos x + \operatorname{sen} x}{\cos x}}$$

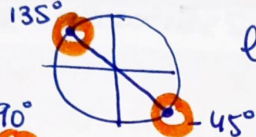
tiene q. ocurrir $\left\{ \begin{array}{l} 1 + \operatorname{tg} x \neq 0 \quad a) \\ \cos x + \operatorname{sen} x \neq 0 \quad b) \\ \cos x \neq 0 \quad c) \end{array} \right.$

a.) $\operatorname{tg} x \neq -1?$



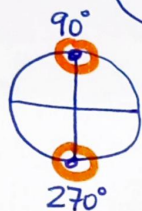
+ n° entero de vueltas

b.) $\cos x + \operatorname{sen} x \neq 0$
 $\operatorname{sen} x \neq -\cos x$



las mismas de antes

c.) $\cos x \neq 0$



+ n° entero de vueltas



Todas juntas: $x \neq 90^\circ, 135^\circ, 270^\circ, -45^\circ, \dots$

2º) Para q. $\exists \frac{1 - \operatorname{sen} x}{\cos 2x} \Rightarrow \cos 2x \neq 0$

$$\cos 2x = 0 \Leftrightarrow 2x = 90^\circ, 270^\circ, \dots$$

Es lo mismo q. $x = 45^\circ, 135^\circ, \dots$

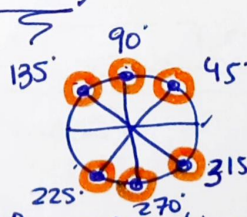
$$x \neq \frac{90^\circ}{2}, \frac{270^\circ}{2}, \dots$$

$$45^\circ, 135^\circ$$

$$225^\circ, 315^\circ = -45^\circ$$



+ media vuelta entera (180°)



+ media vuelta entera 180°

Si unimos todo nos queda

$$x \notin \{ 45^\circ + 90^\circ k \mid k \in \mathbb{Z} \} \cup \{ 90^\circ + 180^\circ k \mid k \in \mathbb{Z} \}$$

En radianes $x \notin \{ \frac{\pi}{4} + k\frac{\pi}{2} \mid k \in \mathbb{Z} \} \cup \{ \frac{\pi}{2} + k\pi \mid k \in \mathbb{Z} \}$

Continuación del 15

Ya sabemos que para que $\exists$ tiene q. ocurrir

que x no sea ni $\frac{\pi}{4} + k\frac{\pi}{2}$, ni $\frac{\pi}{2} + k\pi$, en ese caso:

demostramos la igualdad $A = B$

$$\textcircled{A} = \frac{1 - \operatorname{tg} x}{1 + \operatorname{tg} x} = \frac{1 - \frac{\operatorname{sen} x}{\cos x}}{1 + \frac{\operatorname{sen} x}{\cos x}} = \frac{\frac{\cos x - \operatorname{sen} x}{\cos x}}{\frac{\cos x + \operatorname{sen} x}{\cos x}} = \frac{\cos x - \operatorname{sen} x}{\cos x + \operatorname{sen} x}$$

$$\textcircled{B} \quad \frac{1 - \operatorname{sen}^2 x}{\cos 2x} = \frac{1 - 2 \operatorname{sen} x \cos x}{\cos^2 x - \operatorname{sen}^2 x} = \frac{1 - 2 \operatorname{sen} x \cos x}{(\cos x + \operatorname{sen} x)(\cos x - \operatorname{sen} x)}$$

$$A = B \Leftrightarrow A - B = 0 \Rightarrow$$

$$\frac{\cos x - \operatorname{sen} x}{(\cos x + \operatorname{sen} x)} - \frac{1 - 2 \operatorname{sen} x \cos x}{(\cos x + \operatorname{sen} x)(\cos x - \operatorname{sen} x)} =$$

$$= \frac{(\cos x - \operatorname{sen} x)(\cos x - \operatorname{sen} x) - 1 + 2 \operatorname{sen} x \cos x}{(\cos x + \operatorname{sen} x)(\cos x - \operatorname{sen} x)} =$$

$$= \frac{\cos^2 x - 2 \operatorname{sen} x \cos x + \operatorname{sen}^2 x - 1 + 2 \operatorname{sen} x \cos x}{(\cos^2 x - \operatorname{sen}^2 x)} = \frac{\cos^2 x + \operatorname{sen}^2 x - 1}{(\cos^2 x - \operatorname{sen}^2 x)} =$$

$$= \frac{1 - 1}{\cos^2 x - \operatorname{sen}^2 x} = \frac{0}{\cos^2 x - \operatorname{sen}^2 x} \overset{\uparrow}{=} 0$$

por $\cos^2 x - \operatorname{sen}^2 x \neq 0$ según pedimos al principio.

16 Simplificar $\cos 2x \cos 3x + \sin 2x \cdot \sin 3x$ (p. 7)

$$(\cos^2 x - \sin^2 x) \cos(2x+x) + 2 \sin x \cos x (\sin(2x+x)) = (*)$$

$$\left. \begin{aligned} \sin(\alpha+\beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta & (\alpha=2x, \beta=x) \\ \cos(\alpha+\beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta & (\alpha=2x, \beta=x) \end{aligned} \right\}$$

$$(*) = (\cos^2 x - \sin^2 x) (\cos 2x \cdot \cos x - \sin 2x \sin x) + 2 \sin x \cos x (\sin(2x+x))$$

$$= \underbrace{(\cos^2 x - \sin^2 x)(\cos 2x \cos x - \sin 2x \sin x)}_{(1)} + \underbrace{2 \sin x \cos x (\sin 2x \cos x + \cos 2x \sin x)}_{(2)} =$$

$\Rightarrow$ separarmos:

$$(1) = (\cos^2 x - \sin^2 x) [(\cos^2 x - \sin^2 x) \cos x - 2 \sin x \cos x \sin x] =$$

$$(\cos^2 x - \sin^2 x) [\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x] =$$

$$(\cos^2 x - \sin^2 x) [\cos^3 x - 3 \sin^2 x \cos x] =$$

$$= \cos^5 x - \underbrace{3 \sin^2 x \cos^3 x - \sin^2 x \cos^3 x + 3 \sin^4 x \cos x}_{(1)} =$$

$$= \boxed{\cos^5 x - 4 \sin^2 x \cos^3 x + 3 \sin^4 x \cos x} = (1)$$

$$(2) = 2 \sin x \cos x [2 \sin x \cos x \cos x + (\cos^2 x - \sin^2 x) \sin x] =$$

$$= 2 \sin x \cos x [2 \sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x] =$$

$$= 2 \sin x \cos x [3 \sin x \cos^2 x - \sin^3 x] =$$

$$= \boxed{6 \sin^2 x \cos^3 x - 2 \sin^4 x \cos x} = (2)$$

$$(1) + (2) = \cos^5 x - \underbrace{4 \sin^2 x \cos^3 x + 3 \sin^4 x \cos x}_{(1)} + \underbrace{6 \sin^2 x \cos^3 x - 2 \sin^4 x \cos x}_{(2)} =$$

$$= \cos^5 x + 2 \sin^2 x \cos^3 x + \sin^4 x \cos x =$$

$$= \cos x [\cos^4 x + 2 \sin^2 x \cos^2 x + \sin^4 x] =$$

$$= \cos x [(\cos^2 x + \sin^2 x)^2] =$$

$$= \cos x (\cos^2 x + 1 - \cos^2 x)^2 = \cos x (1)^2 = \boxed{\cos x}$$

Suponiendo q. se cumplen las condiciones antes indicadas enjerto que $\frac{1-\operatorname{tg} x}{1+\operatorname{tg} x} = \frac{1-\operatorname{sen} 2x}{\cos 2x}$? (p.8)

$$[1] = \frac{1-\operatorname{tg} x}{1+\operatorname{tg} x} = \frac{1-\frac{\operatorname{sen} x}{\cos x}}{1+\frac{\operatorname{sen} x}{\cos x}} = \frac{\frac{\cos x - \operatorname{sen} x}{\cos x}}{\frac{\cos x + \operatorname{sen} x}{\cos x}} \stackrel{\cos x \neq 0}{=} \frac{\cos x - \operatorname{sen} x}{\cos x + \operatorname{sen} x}$$

$$[2] = \frac{1-\operatorname{sen} 2x}{\cos 2x} = \frac{1-2\operatorname{sen} x \cos x}{\cos^2 x - \operatorname{sen}^2 x} = \frac{1-2\operatorname{sen} x \cos x}{1-\operatorname{sen}^2 x - \operatorname{sen}^2 x} =$$

$$= \frac{1-2\operatorname{sen} x \cos x}{1-2\operatorname{sen}^2 x}$$

Para q. $[1] = [2]$ tendr a q. ocurrir que :

$$\frac{\cos x - \operatorname{sen} x}{\cos x + \operatorname{sen} x} = \frac{1-2\operatorname{sen} x \cos x}{1-2\operatorname{sen}^2 x} \text{ esto ser a cierto}$$

además por condiciones $\cos x + \operatorname{sen} x \neq 0$ y $1-2\operatorname{sen}^2 x \neq 0$

$$\Leftrightarrow (\cos x - \operatorname{sen} x)(1-2\operatorname{sen}^2 x) = (\cos x + \operatorname{sen} x)(1-2\operatorname{sen} x \cos x)$$

para q. sea cierto $(\Rightarrow)$

$$(\cos x - \operatorname{sen} x)(1-2\operatorname{sen}^2 x) - (\cos x + \operatorname{sen} x)(1-2\operatorname{sen} x \cos x) = 0$$

Veamos si es verdad:

$$\underbrace{\cos x}_{\cancel{\cos x}} - \underbrace{2\operatorname{sen}^2 x \cos x}_{\cancel{2\operatorname{sen}^2 x \cos x}} - \operatorname{sen} x + 2\operatorname{sen}^3 x - \underbrace{\cos x}_{\cancel{\cos x}} - \operatorname{sen} x + 2\operatorname{sen} x \cos^2 x + \underbrace{2\operatorname{sen}^2 x \cos x}_{\cancel{2\operatorname{sen}^2 x \cos x}}$$

$$= -\operatorname{sen} x + 2\operatorname{sen}^3 x - \operatorname{sen} x + 2\operatorname{sen} x \cos^2 x =$$

$$= -2\operatorname{sen} x + 2\operatorname{sen}^3 x + 2\operatorname{sen} x \cos^2 x =$$

$$= 2\operatorname{sen} x \underbrace{(-1 + \operatorname{sen}^2 x + \cos^2 x)}_{\substack{\text{" } \forall x \in \mathbb{R} \\ 1}} = 2\operatorname{sen} x (-1 + 1) = \boxed{0}$$

Si, es cierto

Resumen la igualdad es cierta siempre q.

$$x \in \mathbb{R} - \left\{ \frac{\pi}{4} + k\frac{\pi}{2} \mid k \in \mathbb{Z} \right\} \cup \left\{ \frac{\pi}{2} + k\pi \mid k \in \mathbb{Z} \right\}$$

17

$$\cos 2x + \operatorname{sen} x = 4 \operatorname{sen}^2 x$$

$$\cos^2 x - \operatorname{sen}^2 x + \operatorname{sen} x = 4 \operatorname{sen}^2 x$$

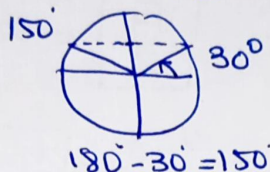
$$1 - \operatorname{sen}^2 x - \operatorname{sen}^2 x + \operatorname{sen} x - 4 \operatorname{sen}^2 x = 0$$

$$1 - 6 \operatorname{sen}^2 x + \operatorname{sen} x = 0$$

$$6 \operatorname{sen}^2 x - \operatorname{sen} x - 1 = 0 ; b = \operatorname{sen} x ; 6b^2 - b - 1 = 0$$

$$\operatorname{sen} x = b = \frac{1 \pm \sqrt{1 - 4 \cdot 6 \cdot (-1)}}{12} = \frac{1 \pm 5}{12} \begin{cases} 1/2 \\ -1/3 \end{cases}$$

$$1^a) \text{ si } \operatorname{sen} x = \frac{1}{2}$$



Comprobación

$$a) x = 30^\circ$$

$$\cos 60^\circ + \operatorname{sen} 30^\circ - 4 \operatorname{sen}^2 30^\circ =$$

$$= \frac{1}{2} + \frac{1}{2} - 4 \left(\frac{1}{2}\right)^2 = 0$$

Si, VALE

$$b) \text{ si } x = 150^\circ$$

$$\cos 300^\circ + \operatorname{sen} 150^\circ = 4 \operatorname{sen}^2 150^\circ ?$$

$$\frac{1}{2} + \frac{1}{2} = 4 \left(\frac{1}{2}\right)^2 \quad \text{Si, VALE}$$

$$S_1 = \{ 30^\circ + k \cdot 360^\circ / k \in \mathbb{Z} \}$$

$$S_2 = \{ 150^\circ + k \cdot 360^\circ / k \in \mathbb{Z} \}$$

$$2^a) \text{ si } \operatorname{sen} x = -\frac{1}{3}$$



$$-19,47122^\circ \quad 4^\circ \text{ cuad.}$$

$$180^\circ + 19,47122^\circ = 199,47122^\circ \quad 3^\circ \text{ cuad.}$$

Comprobación

$$a) \cos(2 \cdot (-19,47122^\circ)) + \operatorname{sen}(-19,47122^\circ) = 4 \cdot \operatorname{sen}^2(-19,47122^\circ)$$

$$\frac{7}{9} + \left(-\frac{1}{3}\right) = 4 \cdot \left(-\frac{1}{3}\right)^2 ?$$

$$S_3 = \{ -19,47122^\circ + k \cdot 360^\circ \}$$

$$-19,47122^\circ = 340,52878^\circ$$

$$\frac{7-3}{9} = 4 \cdot \frac{1}{9} ?$$

$$\frac{4}{9} = \frac{4}{9} \quad \text{Si, VALE //}$$

$$b) \cos(2 \cdot (199,47122^\circ)) + \operatorname{sen}(199,47122^\circ) = 4 \operatorname{sen}^2(199,47122^\circ)$$

$$\frac{7}{9} + \left(-\frac{1}{3}\right) = 4 \cdot \left(-\frac{1}{3}\right)^2 ? \quad S_4 = \{ 199,47122^\circ + k \cdot 360^\circ \}$$

$$\frac{7-3}{9} = \frac{4}{9} ?$$

$$\frac{4}{9} = \frac{4}{9} \quad \text{Si, VALE //}$$

$$\text{Soluciones} = \{ S_1 \cup S_2 \cup S_3 \cup S_4 \}$$

18

$$\begin{cases} \cos x + \sqrt{3} \operatorname{sen} x = 2 \\ \operatorname{sen}^2 x + \cos^2 x = 1 \end{cases} \quad \begin{cases} a = \cos x, b = \operatorname{sen} x \\ a + \sqrt{3} b = 2 \\ a^2 + b^2 = 1 \end{cases}$$

p. 10

$$a = 2 - \sqrt{3}b \Rightarrow (2 - \sqrt{3}b)^2 + b^2 = 1; 4 - 4\sqrt{3}b + 3b^2 + b^2 = 1$$

$$4b^2 - 4\sqrt{3}b + 3 = 0; b = \frac{4\sqrt{3} \pm \sqrt{16 \cdot 3 - 4 \cdot 4 \cdot 3}}{8} =$$

$$= \frac{4\sqrt{3} \pm \sqrt{48 - 48}}{8} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2} = \operatorname{sen} x$$

$$a = 2 - \sqrt{3} \cdot \frac{\sqrt{3}}{2} = 2 - \frac{3}{2} = \frac{4-3}{2} = \frac{1}{2} = \cos x$$

1^{er} cuadr.

Soluciones $\{ 60^\circ + k360^\circ / k \in \mathbb{Z} \}$



19

$$\begin{cases} \operatorname{sen}^2 x + y = 1 \\ \cos^2 x + y = 2 \end{cases} = \begin{cases} \operatorname{sen}^2 x + y = 1 \\ 1 - \operatorname{sen}^2 x + y = 2 \end{cases} = \text{lamo} \\ a = \operatorname{sen} x$$

$$\begin{cases} a^2 + y = 1 \\ 1 - a^2 + y = 2 \end{cases} \Rightarrow \begin{cases} a^2 + y = 1 \\ -a^2 + y = 1 \end{cases}$$

$$2y = 2 \Rightarrow y = 1 \Rightarrow a^2 + 1 = 1 \Rightarrow a = 0$$

$$\Rightarrow \operatorname{sen} x = 0$$



Soluciones

$$x = \{ k \cdot 180^\circ / k \in \mathbb{Z} \}$$

$$y = 1$$

20

$$\operatorname{sen} x \cdot \operatorname{sen} y = -\frac{1}{2}$$

$$\cos x \cdot \cos y = -\frac{1}{2}$$

$$1^a \text{ Ec} + 2^a \text{ Ec} \Rightarrow$$

$$\operatorname{sen} x \operatorname{sen} y + \cos x \cos y = -1$$

$$\cos(x-y) = -1$$

$$\text{hago } 1^a \text{ Ec} - 2^a \text{ Ec} \Rightarrow \operatorname{sen} x \operatorname{sen} y - \cos x \cos y = 0$$

$$-\cos(x+y) = 0$$

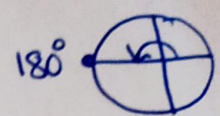
$$\cos(x-y) = -1$$

Resumiendo:

$$\begin{cases} \cos(x-y) = -1 \\ \cos(x+y) = 0 \end{cases}$$

$$\left. \begin{aligned} \cos(x-y) &= -1 \\ \cos(x+y) &= 0 \end{aligned} \right\} \Rightarrow \boxed{x-y=180^\circ}$$

$$\cos(x+y)=0 \Rightarrow \left. \begin{aligned} x+y &= 90^\circ \\ x+y &= 270^\circ \end{aligned} \right\}$$



Salen 2 posibilidades

1ª) $\left. \begin{aligned} x-y &= 180^\circ \\ x+y &= 90^\circ \end{aligned} \right\}$

$$\begin{aligned} 2x &= 270 \Rightarrow x = \boxed{135^\circ} \\ y &= x - 180^\circ = -45^\circ = \boxed{315^\circ} \end{aligned}$$

$$S_1 = \left\{ \begin{aligned} x &= 135^\circ + k \cdot 360^\circ \\ y &= 315^\circ + k \cdot 360^\circ \end{aligned} / k \in \mathbb{Z} \right\}$$

Comprobación

$$\cos(135-315) = -1 \quad \text{Si, VALE}$$

$$\cos(135+315) = 0 \quad \text{Si, VALE}$$

2ª) $\left. \begin{aligned} x-y &= 180^\circ \\ x+y &= 270^\circ \end{aligned} \right\}$

$$\begin{aligned} 2x &= 450 \Rightarrow x = \frac{450}{2} = 225^\circ \\ y &= 225^\circ - 180^\circ = 45^\circ \end{aligned}$$

$$S_2 = \left\{ \begin{aligned} x &= 225^\circ + k \cdot 360^\circ \\ y &= 45^\circ + k \cdot 360^\circ \end{aligned} / k \in \mathbb{Z} \right\}$$

Comprobación

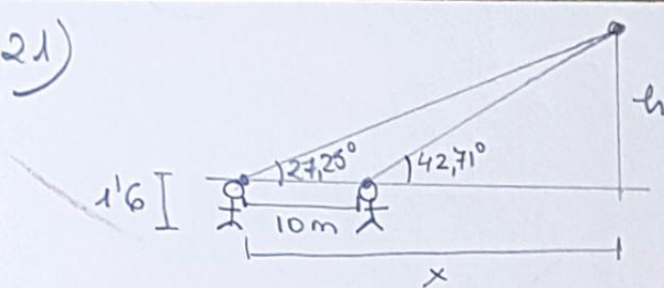
$$\cos(225-45) = -1$$

$$\cos(225+45) = 0 \quad \text{Si, VALE}$$

$$\text{Soluciones} = \boxed{S_1 \cup S_2}$$



21)



$$\left. \begin{aligned} \operatorname{tg} 27,25^\circ &= \frac{h}{x} \\ \operatorname{tg} 42,71^\circ &= \frac{h}{x-10} \end{aligned} \right\}$$

(p.13)

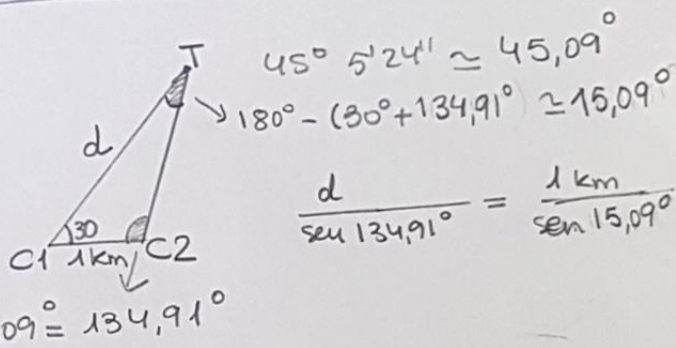
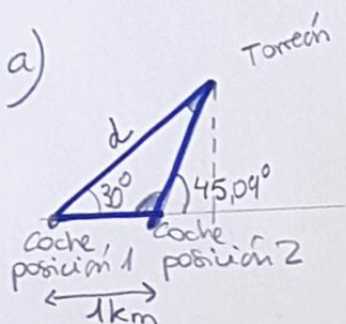
$$x = \frac{h}{\operatorname{tg} 27,25^\circ} ; \quad \operatorname{tg} 42,71^\circ (x-10) = h$$

$$\operatorname{tg} 42,71^\circ \left(\frac{h}{\operatorname{tg} 27,25^\circ} - 10 \right) = h ; \quad \frac{\operatorname{tg} 42,71^\circ}{\operatorname{tg} 27,25^\circ} h - 10 \operatorname{tg} 42,71^\circ = h$$

$$\left(\frac{\operatorname{tg} 42,71^\circ}{\operatorname{tg} 27,25^\circ} - 1 \right) h = 10 \operatorname{tg} 42,71^\circ ; \quad h = \frac{10 \operatorname{tg} 42,71^\circ}{\left(\frac{\operatorname{tg} 42,71^\circ}{\operatorname{tg} 27,25^\circ} - 1 \right)} \approx \boxed{11,65 \text{ m}}$$

$$\Rightarrow H \approx 11,65 + 1,6 \approx \boxed{13,25 \text{ m}}$$

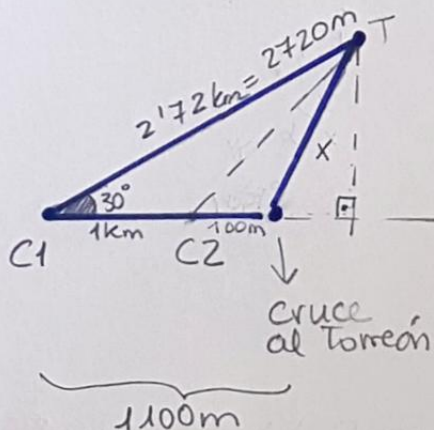
22) a)



$$\frac{d}{\operatorname{sen} 134,91^\circ} = \frac{1 \text{ km}}{\operatorname{sen} 15,09^\circ}$$

$$d = \frac{\operatorname{sen} 134,91^\circ}{\operatorname{sen} 15,09^\circ} = \boxed{2,72 \text{ km}}$$

b)



$$v = 50 \text{ km/h} = \frac{50000}{60} \frac{\text{m}}{\text{min}}$$

th. coseno

$$x^2 = 2720^2 + 1100^2 - 2 \cdot 2720 \cdot 1100 \cos 30^\circ$$

$$x \approx 1850,97 \text{ m}$$

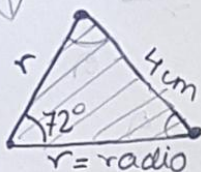
$$\frac{50000 \text{ m}}{1850,97 \text{ m}} = \frac{60'}{t}$$

$$\text{ tarda: } t = 2,22' \text{ minutos}$$



$$\alpha = \frac{360^\circ}{5} = 72^\circ$$

$$(\text{En general } \alpha = \frac{360^\circ}{n^\circ \text{ lados}})$$



$$\text{Área triángulo} = \frac{r \cdot r \cdot \sin \frac{360}{5}}{2}$$

$$\text{Área polígono} = 5 \cdot \text{área del triángulo} = 5 \cdot \frac{r^2 \sin 72^\circ}{2}$$

* Calculamos el radio por el th. coseno.

$$4^2 = r^2 + r^2 - 2r \cdot r \cos 72^\circ$$

$$4^2 = 2r^2 - 2r^2 \cos 72^\circ$$

$$4^2 = 2r^2 (1 - \cos 72^\circ)$$

$$r^2 = \frac{4^2}{2(1 - \cos 72^\circ)} ; \quad r \approx 3.4 \text{ cm}$$

$$\Rightarrow \text{Área polígono} = \frac{5 \cdot 3.4^2 \sin 72^\circ}{2} \approx 27.49 \text{ cm}^2$$

* En general, el área de un polígono regular

$$A = n^\circ \text{ lados} \cdot \frac{r^2 \sin \left(\frac{360^\circ}{n^\circ \text{ lados}} \right)}{2}$$

$$\text{siendo } r = \text{radio} = \sqrt{\frac{l^2}{2(1 - \cos \frac{360^\circ}{n^\circ \text{ lados}})}}$$

$l = \text{lado}$

$n = n^\circ \text{ lados}$

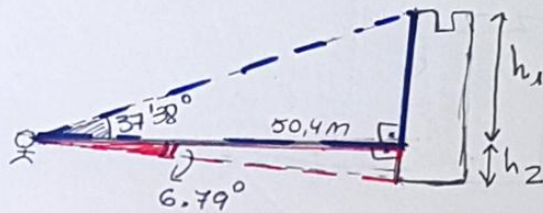
$r = \text{radio}$

$l = \text{lado}$

$$A = \frac{n \cdot r^2 \sin \left(\frac{360}{n} \right)}{2}$$

$$r = \sqrt{\frac{l^2}{2(1 - \cos(\frac{360}{n}))}}$$

(24)



p. 15
Tenemos 2 triángulos rectángulos

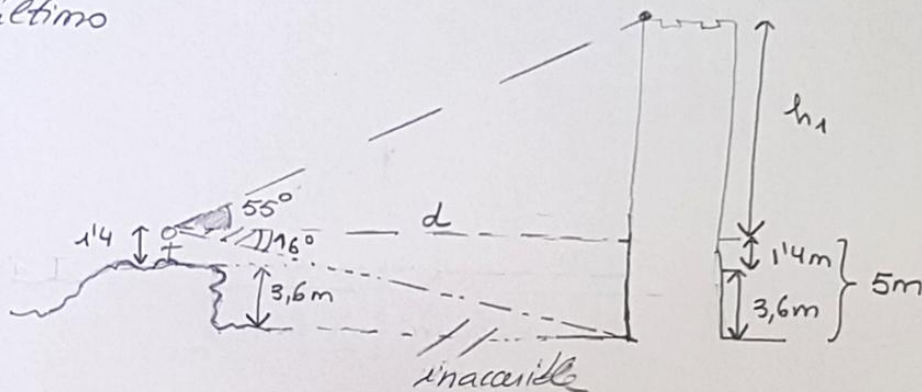
$$\operatorname{tg} 37,38^\circ = \frac{h_1}{50,4}$$

$$\operatorname{tg} 6,79^\circ = \frac{h_2}{50,4}$$

la altura del torreón $h = h_1 + h_2 = 50,4 \cdot \operatorname{tg} 37,38^\circ + 50,4 \cdot \operatorname{tg} 6,79^\circ$

$$\boxed{h \approx 44,51 \text{ m}}$$

(25) último



$$\operatorname{tg} 55^\circ = \frac{h_1}{d}$$

$$\operatorname{tg} 16^\circ = \frac{5}{d} \Rightarrow d = \frac{5}{\operatorname{tg} 16^\circ} \approx 17,44 \text{ m} \Rightarrow h_1 = d \operatorname{tg} 55^\circ$$

$$\boxed{h_1 \approx 24,9 \text{ m}}$$

$$\Rightarrow H = 24,9 + 5 = \boxed{29,9 \text{ m}}$$

altura
de la torre