Números complejos

1. Dados los siguientes números complejos:

$$z_1 = 4-5i \quad z_2 = 2+3i \quad z_3 = -3+5i \quad z_4 = 6+2i$$
$$z_5 = (7,8) \quad z_6 = (-4,-9) \quad z_7 = (-12,2) \quad z_8 = (4,5)$$

efectúa las siguientes operaciones algebraicas:

1) $z_1 + z_2$ 2) $z_4 - z_3$ 3) $\dfrac{z_5 \cdot z_3}{z_6}$ 4) $\left(z_8 + z_7\right) \cdot z_4$

2. Escribe en forma polar el resultado del cociente: $\dfrac{i^5 - i^{-8}}{i\sqrt{2}}$

3. Dados los números complejos $z_1 = 5_{\pi/4}$, $z_2 = 2_{15^\circ}$ y $z_3 = 4i$, calcula

a) $z_3 \cdot z_2$ b) $\dfrac{z_1}{(z_2)^2}$ c) $\dfrac{z_1 \cdot z_2^3}{z_3}$ d) $\dfrac{(z_1)^3}{z_2 \cdot (z_3)^2}$

4. Sea $z = \sqrt{3} - i$. Calcular: a) $\overline{z}$ b) $\dfrac{1}{z}$ c) z^4 d) $\sqrt[4]{z}$

5. Expresa en forma polar:
a) $4-3i$ b) $5+12i$ c) $-3+3i$ d) $-2-4i$

6. Expresa en forma trigonométrica los complejos:
a) $-3+3\sqrt{3}i$ b) $1-i$ c) $6-5i$ d) $-9-8i$

7. Expresa en forma binómica los siguientes complejos:
a) 7_{120° b) $2_{\pi/6}$ c) $3_{3\pi/4}$ d) 5_{135°

8. Realiza las operaciones en forma polar y después pasa a forma binómica:
a) $3_{45^\circ} \cdot 2_{15^\circ}$ b) $6_{-21^\circ} : 2_{24^\circ}$ c) $(2_{25^\circ})^3 \cdot 3_{15^\circ}$ d) $\left(\sqrt{2} - i\right)^6$

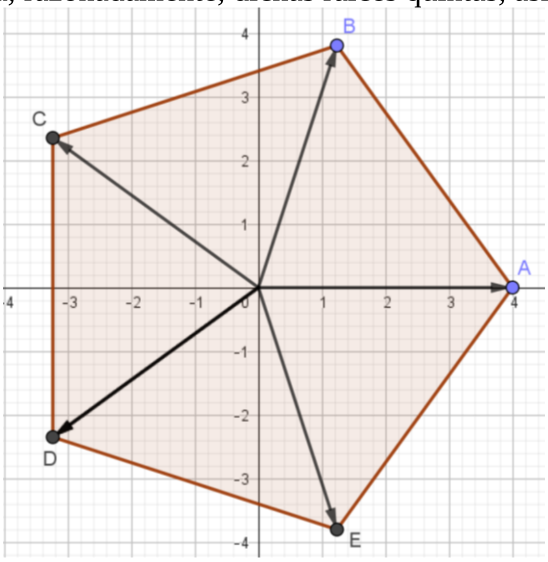
9. Halla las siguientes raíces:
a) $\sqrt[3]{1+i}$ b) $\sqrt[3]{-i}$ c) $\sqrt[6]{-64}$ d) $\sqrt[3]{-27}$

10. Calcula:
a) $\sqrt[5]{\dfrac{32}{-i}}$ b) $\left(\dfrac{i^5 - i^{-8}}{\sqrt{2}i}\right)^5$ c) $\left(\dfrac{1+i}{1-i}\right)^5$ d) $\sqrt[4]{-8+8\sqrt{3}i}$

11. *Resuelve las siguientes ecuaciones polinómicas:*

1) $x^2+1=0$	6) $z^2+2iz+2=0$
2) $x^2-2x+5=0$	7) $z^3+2iz^2+2z=0$
3) $x^2+x+1=0$	8) $z^2+3z+7=0$
4) $z^4+1=0$	9) $\dfrac{z-3}{2z-i}=1-i$
5) $z^2+2=0$	10) $\dfrac{z}{2i}+\dfrac{z+1}{4-2i}=3$

12. Los vértices del pentágono regular de la siguiente figura son las raíces quintas de un número complejo. Determina, razonadamente, dichas raíces quintas, así como el número complejo.



1

1) $z_1 + z_2 = (4-5i) + (2+3i) = \boxed{6-2i}$

2) $z_4 - z_3 = (6+2i) - (-3+5i) = \boxed{9-3i}$

3) $\dfrac{z_5 \cdot z_3}{z_6} = \dfrac{(7,8)\cdot(-3+5i)}{(-4,-9)} = \dfrac{(7+8i)\cdot(-3+5i)}{-4-9i} = \dfrac{-21+35i-24i-40}{-4-9i} =$

$= \dfrac{-61+11i}{-4-9i} = \dfrac{(-61+11i)(-4+9i)}{(-4-9i)(-4+9i)} = \dfrac{244-549i-44i-99}{16-36i+36i+81} = \dfrac{145-593i}{97} =$

$= \boxed{\dfrac{145}{97} - \dfrac{593}{97}i}$

4) $(z_8+z_7)\cdot z_4 = [(4,5)+(-12,2)]\cdot(6+2i) = (-8,7)\cdot(6+2i) = (-8+7i)\cdot(6+2i) =$

$= -48-16i+42i-14 = \boxed{-62+26i}$

2

$z = \dfrac{i^5 - i^{-8}}{i\sqrt{2}} = \dfrac{i - \frac{1}{i^8}}{i\sqrt{2}} = \dfrac{i - \frac{1}{i^0}}{i\sqrt{2}} = \dfrac{i - \frac{1}{1}}{i\sqrt{2}} = \dfrac{i-1}{i\sqrt{2}} = \dfrac{(i-1)(-i\sqrt{2})}{(i\sqrt{2})(-i\sqrt{2})} =$

$= \dfrac{\sqrt{2}+i\sqrt{2}}{2} = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i$

Módulo: $|z| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4}+\frac{2}{4}} = \sqrt{\frac{4}{4}} = 1$

Argumento: $\alpha = \text{arctg}\, \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \text{arctg}\, 1 = 45°$

Por tanto, $\boxed{z = 1_{45°}}$

3 Como las operaciones en forma polar son muy sencillas, escribimos todos los nºs complejos en dicha forma:

$z_1 = 5_{\frac{\pi}{4}} = 5_{45°}$, $z_2 = 2_{15°}$, $z_3 = 4i = 4_{90°}$

a) $z_3 \cdot z_2 = 4_{90°} \cdot 2_{15°} = (4\cdot 2)_{90°+15°} = \boxed{8_{105°}}$

b) $\dfrac{z_1}{z_2^2} = \dfrac{5_{45°}}{(2_{15°})^2} = \dfrac{5_{45°}}{4_{15°\cdot 2}} = \dfrac{5_{45°}}{4_{30°}} = \left(\dfrac{5}{4}\right)_{45°-30°} = \boxed{\left(\dfrac{5}{4}\right)_{15°}}$

c) $\dfrac{z_1 \cdot z_2^3}{z_3} = \dfrac{5_{45°}\cdot(2_{15°})^3}{4_{90°}} = \dfrac{5_{45°}\cdot 8_{45°}}{4_{90°}} = \dfrac{40_{90°}}{4_{90°}} = \left(\dfrac{40}{4}\right)_{90°-90°} = \boxed{10}$

d) $\dfrac{z_1^3}{z_2 \cdot z_3^2} = \dfrac{(5_{45°})^3}{2_{15°}\cdot(4_{90°})^2} = \dfrac{125_{135°}}{2_{15°}\cdot 16_{180°}} = \dfrac{125_{135°}}{32_{195°}} = \left(\dfrac{125}{32}\right)_{-60°} = \boxed{\left(\dfrac{125}{32}\right)_{300°}}$

④ En los apartados a) y b) trabajaremos en forma binómica y en los apartados c) y d) en forma polar:

a) $\boxed{\bar{z} = \sqrt{3}+i}$

b) $\frac{1}{z} = \frac{1}{\sqrt{3}-i} = \frac{\sqrt{3}+i}{(\sqrt{3}-i)(\sqrt{3}+i)} = \frac{\sqrt{3}+i}{3+\sqrt{3}i-\sqrt{3}i+1} = \frac{\sqrt{3}+i}{4} = \boxed{\frac{\sqrt{3}}{4}+\frac{1}{4}i}$

c) $z = \sqrt{3}-i$

$$\left.\begin{array}{l}\text{Módulo : } |z| = \sqrt{(\sqrt{3})^2+(-1)^2} = 2 \\ \text{Argumento: } \alpha = 360^\circ + \operatorname{arctg}\frac{-1}{\sqrt{3}} = 360^\circ - 30^\circ = 330^\circ\end{array}\right\} z = 2_{330^\circ}$$

$$z^4 = (2_{330^\circ})^4 = (2^4)_{4\cdot 330^\circ} = 16_{1320} = \boxed{16_{240^\circ}}$$

d)

$$\sqrt[4]{z} = \sqrt[4]{2_{330^\circ}} = \boxed{\begin{cases}\sqrt[4]{2}_{\beta_0} = \sqrt[4]{2}_{82,5^\circ} \\ \sqrt[4]{2}_{\beta_1} = \sqrt[4]{2}_{172,5^\circ} \\ \sqrt[4]{2}_{\beta_2} = \sqrt[4]{2}_{262,5^\circ} \\ \sqrt[4]{2}_{\beta_3} = \sqrt[4]{2}_{352,5^\circ}\end{cases}}$$

$$\beta_0 = \frac{330^\circ+0\cdot 360^\circ}{4} = 82,5^\circ \quad , \quad \beta_1 = \frac{330^\circ+1\cdot 360^\circ}{4} = 262,5^\circ$$

$$\beta_2 = \frac{330^\circ+2\cdot 360^\circ}{4} = 172,5^\circ \quad , \quad \beta_3 = \frac{330^\circ+3\cdot 360^\circ}{4} = 352,5^\circ$$

⑤ a) $z_1 = 4-3i$

$$\left.\begin{array}{l}\text{Módulo : } |z_1| = \sqrt{4^2+(-3)^2} = 5 \\ \text{Argumento : } \alpha_1 = 360^\circ + \operatorname{arctg}\frac{-3}{4} = 360^\circ - 37^\circ = 323^\circ\end{array}\right\} \boxed{z_1 = 5_{323^\circ}}$$

b) $z_2 = 5+12i$

$$\left.\begin{array}{l}\text{Módulo : } |z_2| = \sqrt{5^2+12^2} = 13 \\ \text{Argumento : } \alpha_2 = \operatorname{arctg}\frac{12}{5} \simeq 67^\circ\end{array}\right\} \boxed{z_2 = 13_{67^\circ}}$$

c) $z_3 = -3+3i$

$$\left.\begin{array}{l}\text{Módulo : } |z_3| = \sqrt{(-3)^2+3^2} = \sqrt{18} \\ \text{Argumento : } \alpha_3 = 180^\circ + \operatorname{arctg}\left(\frac{3}{-3}\right) = 180^\circ - 45^\circ = 135^\circ\end{array}\right\} \boxed{z_3 = \sqrt{18}_{135^\circ}}$$

d) $z_4 = -2-4i$

Módulo : $|z_4| = \sqrt{(-2)^2+(-4)^2} = \sqrt{20}$

Argumento : $\alpha_4 = 180° + \text{arctg}\,\frac{-4}{-2} = 180° + 63° \simeq 243°$

$\boxed{z_4 = \sqrt{20}_{\,243°}}$

(6) a) $z_1 = -3+3\sqrt{3}\,i$

Módulo : $|z_1| = \sqrt{(-3)^2+(3\sqrt{3})^2} = 6$

Argumento : $\alpha_1 = 180° + \text{arctg}\,\frac{3\sqrt{3}}{-3} = 180° - 60° = 120°$

$z_1 = 6_{120°} = \boxed{6\,(\cos 120° + i\,\text{sen}\,120°)}$

b) $z_2 = 1-i$

Módulo : $|z_2| = \sqrt{1^2+(-1)^2} = \sqrt{2}$

Argumento : $\alpha_2 = 360° + \text{arctg}\,\frac{-1}{1} = 360° - 45° = 315°$

$z_2 = \sqrt{2}_{\,315°} = \boxed{\sqrt{2}\,(\cos 315° + i\,\text{sen}\,315°)}$

c) $z_3 = 6-5i$

Módulo : $|z_3| = \sqrt{6^2+(-5)^2} = \sqrt{61}$

Argumento : $\alpha_3 = 360° + \text{arctg}\,\frac{-5}{6} \simeq 360° - 40° = 320°$

$z_3 = \sqrt{61}_{\,320°} = \boxed{\sqrt{61}\,(\cos 320° + i\,\text{sen}\,320°)}$

d) $z_4 = -9-8i$

Módulo : $|z_4| = \sqrt{(-9)^2+(-8)^2} = \sqrt{145}$

Argumento : $\alpha_4 = 180° + \text{arctg}\left(\frac{-8}{-9}\right) \simeq 180° + 42° = 222°$

$z_4 = \sqrt{145}_{\,222°} = \boxed{\sqrt{145}\,(\cos 222° + i\,\text{sen}\,222°)}$

(7) a) $z_1 = 7_{120°} = 7\,(\cos 120° + i\,\text{sen}\,120°) = 7\cos 120° + i\,(7\,\text{sen}\,120°) =$

$= \boxed{-3{,}5 + 6{,}1\,i}$

b) $z_2 = 2_{\pi/6} = 2_{30°} = 2\,(\cos 30° + i\,\text{sen}\,30°) = 2\cos 30° + i\,(2\,\text{sen}\,30°) =$

$= \boxed{1{,}7 + i}$

c) $z_3 = 3_{\frac{3\pi}{4}} = 3_{135^\circ} = 3(\cos 135^\circ + i \operatorname{sen} 135^\circ) = 3\cos 135^\circ + i(3 \operatorname{sen} 135^\circ) =$

$= \boxed{-2{,}1 + 2{,}1i}$

d) $z_4 = 5_{135^\circ} = 5(\cos 135^\circ + i \operatorname{sen} 135^\circ) = 5\cos 135^\circ + i(5 \operatorname{sen} 135^\circ) =$

$= \boxed{-3{,}5 + 3{,}5i}$

8 a) $3_{45^\circ} \cdot 2_{15^\circ} = (3 \cdot 2)_{45^\circ + 15^\circ} = \boxed{6_{60^\circ}}$

$= 6\cos 60^\circ + i(6 \operatorname{sen} 60^\circ) = \boxed{3 + 5{,}2i}$

b) $6_{-21^\circ} : 2_{24^\circ} = (6:2)_{-21^\circ - 24^\circ} = 3_{-45^\circ} = \boxed{3_{315^\circ}}$

$= 3\cos 315^\circ + i(3 \operatorname{sen} 315^\circ) = \boxed{2{,}1 - 2{,}1i}$

c) $(2_{25^\circ})^3 \cdot 3_{15^\circ} = 8_{75^\circ} \cdot 3_{15^\circ} = \boxed{24_{90^\circ}}$

$= 24\cos 90^\circ + i(24 \operatorname{sen} 90^\circ) = \boxed{24i}$

d) $(\sqrt{2} - i)^6$

$z = \sqrt{2} - i$

$\left.\begin{array}{l} \text{Módulo: } \sqrt{(\sqrt{2})^2 + (-1)^2} = \sqrt{3} \\ \text{Argumento: } \alpha = 360^\circ + \operatorname{arctg} \frac{-1}{\sqrt{2}} \simeq 360^\circ - 35^\circ = 325^\circ \end{array}\right\} z = \sqrt{3}_{325^\circ}$

$z^6 = \left(\sqrt{3}_{325^\circ}\right)^6 = 27_{1950^\circ} = \boxed{27_{150^\circ}}$

$= 27\cos 150^\circ + i(27 \operatorname{sen} 150^\circ) = \boxed{-23{,}4 + 13{,}5i}$

9 a) $\sqrt[3]{1+i}$

$z = 1 + i$

$\left.\begin{array}{l} \text{Módulo: } |z| = \sqrt{1^2 + 1^2} = \sqrt{2} \\ \text{Argumento: } \alpha = \operatorname{arctg} \frac{1}{1} = 45^\circ \end{array}\right\} z = \sqrt{2}_{45^\circ}$

$$\sqrt[3]{\sqrt{2}_{45^\circ}} = \boxed{\left\{\begin{array}{l} \left(\sqrt[3]{\sqrt{2}}\right)_{\beta_0} = \left(\sqrt[6]{2}\right)_{15^\circ} \\ \left(\sqrt[3]{\sqrt{2}}\right)_{\beta_1} = \left(\sqrt[6]{2}\right)_{135^\circ} \\ \left(\sqrt[3]{\sqrt{2}}\right)_{\beta_2} = \left(\sqrt[6]{2}\right)_{255^\circ} \end{array}\right.}$$

$\beta_0 = \dfrac{45^\circ + 0 \cdot 360^\circ}{3} = 15^\circ$

$\beta_1 = \dfrac{45^\circ + 1 \cdot 360^\circ}{3} = 135^\circ$

$\beta_2 = \dfrac{45^\circ + 2 \cdot 360^\circ}{3} = 255^\circ$

b) $\sqrt[3]{-i} = \sqrt[3]{1_{270^\circ}} = \begin{cases} 1_{\beta_0} = 1_{90^\circ} \\ 1_{\beta_1} = 1_{210^\circ} \\ 1_{\beta_2} = 1_{330^\circ} \end{cases}$

$$\beta_0 = \frac{270^\circ + 0\cdot 360^\circ}{3} = 90^\circ$$
$$\beta_1 = \frac{270^\circ + 1\cdot 360^\circ}{3} = 210^\circ$$
$$\beta_2 = \frac{270^\circ + 2\cdot 360^\circ}{3} = 330^\circ$$

c) $\sqrt[6]{-64} = \sqrt[6]{64_{180^\circ}} = \begin{cases} 2_{\beta_0} = 2_{30^\circ} \\ 2_{\beta_1} = 2_{90^\circ} \\ 2_{\beta_2} = 2_{150^\circ} \\ 2_{\beta_3} = 2_{210^\circ} \\ 2_{\beta_4} = 2_{270^\circ} \\ 2_{\beta_5} = 2_{330^\circ} \end{cases}$

$$\beta_0 = \frac{180^\circ + 0\cdot 360^\circ}{6} = 30^\circ$$
$$\beta_1 = \frac{180^\circ + 1\cdot 360^\circ}{6} = 90^\circ$$
$$\beta_2 = \frac{180^\circ + 2\cdot 360^\circ}{6} = 150^\circ$$
$$\beta_3 = \frac{180^\circ + 3\cdot 360^\circ}{6} = 210^\circ$$
$$\beta_4 = \frac{180^\circ + 4\cdot 360^\circ}{6} = 270^\circ$$
$$\beta_5 = \frac{180^\circ + 5\cdot 360^\circ}{6} = 330^\circ$$

d) $\sqrt[3]{-27} = \sqrt[3]{27_{180^\circ}} = \begin{cases} 3_{\beta_0} = 3_{60^\circ} \\ 3_{\beta_1} = 3_{180^\circ} \\ 3_{\beta_2} = 3_{300^\circ} \end{cases}$

$$\beta_0 = \frac{180^\circ + 0\cdot 360^\circ}{3} = 60^\circ$$
$$\beta_1 = \frac{180^\circ + 1\cdot 360^\circ}{3} = 180^\circ$$
$$\beta_2 = \frac{180^\circ + 2\cdot 360^\circ}{3} = 300^\circ$$

10 a) $\sqrt[5]{\frac{32}{-i}} = \sqrt[5]{32i} = \sqrt[5]{32_{90^\circ}}$

$$z = \frac{32}{-i} = \frac{32i}{-i\cdot i} = 32i = 32_{90^\circ}$$

$\sqrt[5]{32_{90^\circ}} = \begin{cases} 2_{\beta_0} = 2_{18^\circ} \\ 2_{\beta_1} = 2_{90^\circ} \\ 2_{\beta_2} = 2_{162^\circ} \\ 2_{\beta_3} = 2_{234^\circ} \\ 2_{\beta_4} = 2_{306^\circ} \end{cases}$

$$\beta_0 = \frac{90^\circ + 0\cdot 360^\circ}{5} = 18^\circ$$
$$\beta_1 = \frac{90^\circ + 1\cdot 360^\circ}{5} = 90^\circ$$
$$\beta_2 = \frac{90^\circ + 2\cdot 360^\circ}{5} = 162^\circ$$
$$\beta_3 = \frac{90^\circ + 3\cdot 360^\circ}{5} = 234^\circ$$
$$\beta_4 = \frac{90^\circ + 4\cdot 360^\circ}{5} = 306^\circ$$

b) Por el ejercicio 2 sabemos que $z = \dfrac{i^5 - i^{-8}}{\sqrt{2}\, i} = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} i = 1_{45°}$

$$z^5 = \left(1_{45°}\right)^5 = 1_{5 \cdot 45°} = \boxed{1_{225°}}$$

c) $\left(\dfrac{1+i}{1-i}\right)^5 = i^5 = i^1 = \boxed{i}$

$$\frac{1+i}{1-i} = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+i+i-1}{1-i+i+1} = \frac{2i}{2} = i$$

d) $\sqrt[4]{-8+8\sqrt{3}i}$

$z = -8+8\sqrt{3}i$

$$\left.\begin{array}{l} \text{Módulo: } |z| = \sqrt{(-8)^2+(8\sqrt{3})^2} = 16 \\ \text{Argumento: } \alpha = 180° + \operatorname{arctg} \dfrac{8\sqrt{3}}{-8} = 180° - 60° = 120° \end{array}\right\} z = 16_{120°}$$

$$\sqrt[4]{16_{120°}} = \boxed{\begin{cases} z_{\beta_0} = 2_{30°} \\ z_{\beta_1} = 2_{120°} \\ z_{\beta_2} = 2_{210°} \\ z_{\beta_3} = 2_{300°} \end{cases}}$$

$$\beta_0 = \frac{120° + 0 \cdot 360°}{4} = 30°$$

$$\beta_1 = \frac{120° + 1 \cdot 360°}{4} = 120°$$

$$\beta_2 = \frac{120° + 2 \cdot 360°}{4} = 210°$$

$$\beta_3 = \frac{120° + 3 \cdot 360°}{4} = 300°$$

11 Resuelve las siguientes ecuaciones:

① $x^2+1=0 \Rightarrow x^2=-1 \Rightarrow x=\pm\sqrt{-1}=\pm i$ (dos soluciones)

② $x^2-2x+5=0$

$$x=\frac{2\pm\sqrt{(-2)^2-4\cdot 1\cdot 5}}{2\cdot 1}=\frac{2\pm\sqrt{-16}}{2}=\frac{2\pm\sqrt{16}\,i}{2}=\frac{2\pm 4i}{2}=\begin{cases}\frac{2+4i}{2}=\\ \frac{2-4i}{2}=\end{cases}$$

$$=\begin{cases}1+2i\\ 1-2i\end{cases}$$ (dos soluciones)

③ $x^2+x+1=0$

$$x=\frac{-1\pm\sqrt{1^2-4\cdot 1\cdot 1}}{2\cdot 1}=\frac{-1\pm\sqrt{-3}}{2}=\frac{-1\pm\sqrt{3}\,i}{2}=\begin{cases}\frac{-1+\sqrt{3}\,i}{2}\\ \frac{-1-\sqrt{3}\,i}{2}\end{cases}$$

④ $z^4+1=0 \Rightarrow z^4=-1 \Rightarrow z=\sqrt[4]{-1}$

Calculamos $\sqrt[4]{-1}$:

Expresión en forma polar de -1: $-1=1_{180^\circ}$

<u>Módulo</u> (de las raíces cuartas)

$\sqrt[4]{1}=1$

<u>Argumentos</u> (de las raíces cuartas)

$$\beta_0=\frac{180^\circ+360^\circ\cdot 0}{4}=45^\circ \qquad \beta_2=\frac{180^\circ+360^\circ\cdot 2}{4}=225^\circ$$

$$\beta_1=\frac{180^\circ+360^\circ\cdot 1}{4}=135^\circ \qquad \beta_3=\frac{180^\circ+360^\circ\cdot 3}{4}=315^\circ$$

Las soluciones de la ecuación dada son:

$$z_1=1_{45^\circ}=1\cdot\cos 45^\circ+(1\cdot \operatorname{sen} 45^\circ)i=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$$

$$z_2=1_{135^\circ}=1\cdot\cos 135^\circ+(1\cdot \operatorname{sen} 135^\circ)i=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$$

$$z_3=1_{225^\circ}=1\cdot\cos 225^\circ+(1\cdot \operatorname{sen} 225^\circ)i=-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$$

$$z_4=1_{315^\circ}=1\cdot\cos 315^\circ+(1\cdot \operatorname{sen} 315^\circ)i=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$$

⑤ $z^2+2=0 \Rightarrow z^2=-2 \Rightarrow z=\pm\sqrt{-2}=\pm\sqrt{2}\,i$

⑥ $z^2+2iz+2=0$

$$z=\frac{-2i\pm\sqrt{(2i)^2-4\cdot1\cdot2}}{2\cdot1}=\frac{-2i\pm\sqrt{-4-8}}{2}=\frac{-2i\pm\sqrt{-12}}{2}=\frac{-2i\pm\sqrt{12}\,i}{2}=$$

$$=\begin{cases}\dfrac{-2i+2\sqrt{3}\,i}{2}=-i+\sqrt{3}\,i=(\sqrt{3}-1)\,i\\[2mm]\dfrac{-2i-2\sqrt{3}\,i}{2}=-i-\sqrt{3}\,i=(-\sqrt{3}-1)\,i\end{cases}$$

(7) $z^3+2iz^2+2z=0 \Rightarrow$ (sacando factor común) $z(z^2+2iz+2)=0 \Rightarrow$

$\Rightarrow$ (un producto es cero, cuando al menos uno de los factores es cero)

$$\begin{cases}z=0\\ z^2+2iz+2=0 \Rightarrow z=\begin{cases}(\sqrt{3}-1)\,i\\(-\sqrt{3}-1)\,i\end{cases}\end{cases}$$ (tres soluciones)

$\uparrow$

es la ecuación anterior

(8) $z^2+3z+7=0$

$$z=\frac{-3\pm\sqrt{3^2-4\cdot1\cdot7}}{2\cdot1}=\frac{-3\pm\sqrt{-19}}{2}=\frac{-3\pm\sqrt{19}\,i}{2}=\begin{cases}\dfrac{-3+\sqrt{19}\,i}{2}\\[2mm]\dfrac{-3-\sqrt{19}\,i}{2}\end{cases}$$

(9) $\dfrac{z-3}{2z-1}=1-i \Rightarrow z-3=(1-i)(2z-1) \Rightarrow z-3=2z-1+(1-2z)i \Rightarrow$

$\Rightarrow z-3-2z+1-i+2zi=0 \Rightarrow (2i-1)z-2-i=0 \Rightarrow z=\dfrac{2+i}{2i-1}=$

$=\dfrac{(2+i)(-1-2i)}{(-1+2i)(-1-2i)}=\dfrac{-5i}{5}=-i$

(10) $\dfrac{z}{2i}+\dfrac{z+1}{4-2i}=3 \Rightarrow$ (reduciendo a común denominador)

$\dfrac{(4-2i)z+(z+1)2i}{2i(4-2i)}=\dfrac{3(2i(4-2i))}{2i(4-2i)}=\dfrac{24i+12}{2i(4-2i)} \Rightarrow$ (igualando numeradores)

$(4-2i)z+(z+1)2i=24i+12 \Rightarrow 4z+2i=24i+12 \Rightarrow$

$\Rightarrow 4z=12+24i-2i \Rightarrow 4z=12+22i \Rightarrow z=\dfrac{12+22i}{4}=3+\dfrac{11}{2}i$

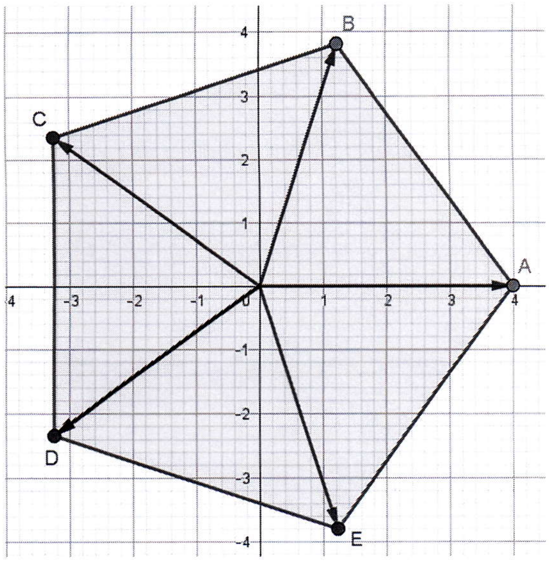
(11) $\dfrac{x+1}{3}+\dfrac{5}{x+1}+2=0 \Rightarrow \dfrac{(x+1)^2+5\cdot3+2\cdot3\cdot(x+1)}{3(x+1)}=0 \Rightarrow$

$\Rightarrow (x+1)^2+15+6(x+1)=0 \Rightarrow x^2+1+2x+15+6x+6=0 \Rightarrow$

$\Rightarrow x^2+8x+22=0 \Rightarrow x=\dfrac{-8\pm\sqrt{8^2-4\cdot2\cdot22}}{2\cdot1}=\dfrac{-8\pm\sqrt{-24}}{2}=\dfrac{-8\pm\sqrt{24}\,i}{2}=$

$$=\begin{cases}-4+\sqrt{6}\,i\\-4-\sqrt{6}\,i\end{cases}$$

12 Los vértices del pentágono regular de la siguiente figura son las raíces quintas de un número complejo. Determina, razonadamente, dichas raíces así como el número complejo.



Una de las raíces es $z_1 = (4,0) = 4_{0°}$

Módulo del nº complejo: $z = r_\alpha$

$$\sqrt[5]{r} = 4 \Rightarrow (\sqrt[5]{r})^5 = 4^5 \Rightarrow r = 1024$$

Argumento del número complejo

$$\frac{\alpha + 360°\cdot 0}{5} = 0° \Rightarrow \alpha = 0°$$

Así, $z = 1024_{0°} = 1024$

Calculamos ahora las otras raíces quintas:

$$\beta_0 = \frac{0° + 360°\cdot 0}{5} = 0°$$

$$\beta_1 = \frac{0° + 360°\cdot 1}{5} = 72°$$

$$\beta_2 = \frac{0° + 360°\cdot 2}{5} = 144°$$

$$\beta_3 = \frac{0° + 360°\cdot 3}{5} = 216°$$

$$\beta_4 = \frac{0° + 360°\cdot 4}{5} = 288°$$

Por tanto, el nº complejo pedido es $z = 1024$ y sus raíces quintas son:

$z_1 = 4_{0°}$

$z_2 = 4_{72°}$

$z_3 = 4_{144°}$

$z_4 = 4_{216°}$

$z_5 = 4_{288°}$