

Trigonometría y Complejos

- 1) Demostrar que: $\frac{1 + \operatorname{tg}^2 x}{\operatorname{tg}^2 x} = \operatorname{cosec}^2 x$
- 2) Resolver: $\cos 2x + \cos x = 0$
- 3) Resolver un triángulo del que conocemos $a = 6$, $b = 8$ y $C = 60^\circ$.
- 4) Calcular $(-\sqrt{3} + i)^{11}$, dando el resultado final en forma polar, trigonométrica, binómica y cartesiana.
- 5) Calcular las raíces sextas de $-i$.

$$\textcircled{1} \quad \frac{1 + \operatorname{tg}^2 x}{\operatorname{tg}^2 x} = \frac{1/\cos^2 x}{\frac{\sin^2 x}{\cos^2 x}} = \frac{\cos^2 x}{\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x} = \operatorname{cosec}^2 x$$

$$\textcircled{2} \quad \cos 2x + \cos x = 0 \Rightarrow \cos^2 x - \sin^2 x + \cos x = 0 \Rightarrow \cos^2 x - (1 - \cos^2 x) + \cos x = 0$$

$$\Rightarrow \cos^2 x - 1 + \cos^2 x + \cos x = 0 \Rightarrow 2\cos^2 x + \cos x - 1 = 0 \Rightarrow \cos x = \frac{-1 \pm \sqrt{1+8}}{4} =$$

$$= \frac{-1 \pm 3}{4} \Rightarrow \begin{cases} = -1 \Rightarrow x = 180^\circ + 360^\circ k, k \in \mathbb{Z} \\ = \frac{2}{4} = \frac{1}{2} \Rightarrow x = \pm 60^\circ + 360^\circ k, k \in \mathbb{Z} \end{cases}$$

$$\textcircled{3} \quad \text{T. Coseno: } c = \sqrt{a^2 + b^2 - 2ab \cos C} = \sqrt{36 + 64 - 2 \cdot 6 \cdot 8 \cdot \frac{1}{2}} = \sqrt{100 - 48} = \sqrt{52} \approx 7,21$$

En el caso de 2 lados y el ángulo comprendido hay sol. única. Buscamos la solución por el T. del seno: $b^2 = a^2 + c^2 - 2ac \cos B \Rightarrow 2ac \cos B = a^2 + c^2 - b^2 \Rightarrow$

$$\Rightarrow \cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{36 + 52 - 64}{2 \cdot 6 \cdot \sqrt{52}} \Rightarrow B = 73,90^\circ = 73^\circ 53' 52,4''$$

$$\Rightarrow A = 180^\circ - B - C = 46,10^\circ = 46^\circ 6' 36,1''$$

$$\textcircled{4} \quad \text{Llamamos } z = -\sqrt{3} + i \Rightarrow |z| = \sqrt{3+1} = 2; \quad \operatorname{arg} z = 2^\circ \text{ wad.} \Rightarrow \alpha = 150^\circ$$

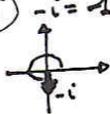
$$\Rightarrow z = 2_{150} \Rightarrow z^{11} = (2_{150})^{11} = (2^{11})_{1650} = 2048_{1650} = 2048_{210} \quad \text{Polar}$$

Trigonométrica: $z^{11} = 2048(\cos 210 + i \operatorname{sen} 210)$

Binómica: $z^{11} = 2048\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) = -1024\sqrt{3} - 1024i$

Cartesiana: $z^{11} = (-1024\sqrt{3}, -1024)$

$$\textcircled{5} \quad -i = 1_{270} \Rightarrow \text{Módulo de las raíces: } \sqrt[6]{1} = 1. \text{ Argumentos:}$$



$$\begin{aligned} \beta_1 &= \frac{270}{6} = 45 \Rightarrow 1_{45} \\ \beta_2 &= \frac{270}{6} + \frac{360}{6} = 105 \Rightarrow 1_{105} \\ \beta_3 &= \frac{270}{6} + 2 \cdot \frac{360}{6} = 165 \Rightarrow 1_{165} \\ \beta_4 &= \frac{270}{6} + 3 \cdot \frac{360}{6} = 225 \Rightarrow 1_{225} \\ \beta_5 &= \frac{270}{6} + 4 \cdot \frac{360}{6} = 285 \Rightarrow 1_{285} \\ \beta_6 &= \frac{270}{6} + 5 \cdot \frac{360}{6} = 345 \Rightarrow 1_{345} \end{aligned}$$