

1. Resuelve: $3^{2(x+1)} - 28 \cdot 3^x + 3 = 0$

2. Resuelve: $2 \log x - 1 = \log (x^2 - 6)$

3. Resuelve: $\begin{cases} x - y = 3 \\ 2^x - 2^y = \frac{7}{4} \end{cases}$

4. Si $\log 2 = 0,3010$, $\log 3 = 0,4771$, $\log 5 = 0,6990$. Calcula:

a. $\log \sqrt[6]{\frac{1}{0,128}}$ b. $\log \frac{540}{256}$

5. Escribe la expresión algebraica de: $\log A = 2 + 5 \log x - \frac{3}{5} \log(y) - 4 \log z - 3 \log \frac{x}{y}$

6. Resuelve: $\begin{cases} \log x - \log y = 1 \\ 3x + 2y = 64 \end{cases}$

7. Resuelve $\frac{\log 2 + \log(11-x^2)}{\log(5-x)} = 2$

8. Resuelve $4^{1+x} + 2^{x+3} = 320$

$$(1) 3^{2(x+1)} - 28 \cdot 3^x + 3 = 0$$

$$3^{2x+2} - 28 \cdot 3^x + 3 = 0 \rightarrow (3^x)^2 \cdot 3^2 - 28 \cdot 3^x + 3 = 0 \xrightarrow{3^x = t}$$

$$9t^2 - 28t + 3 = 0 \rightarrow t = \frac{28 \pm \sqrt{28^2 - 4 \cdot 9 \cdot 3}}{18} = \begin{matrix} 3 \\ 1/9 \end{matrix}$$

$$t_1 = 3 \rightarrow 3^x = 3 \rightarrow x = 1$$

$$t_2 = \frac{1}{9} \rightarrow 3^x = 3^{-2} \rightarrow x = -2$$

$$(2) 2 \log x - 1 = \log (x^2 - 6)$$

$$\log x^2 - \log 10 = \log (x^2 - 6) \rightarrow \log \frac{x^2}{10} = \log (x^2 - 6) \rightarrow \frac{x^2}{10} = x^2 - 6$$

$$x^2 = 10x^2 - 60 \rightarrow 60 = 9x^2 \rightarrow x = \pm \sqrt{\frac{60}{9}} \text{ so both } x = +\sqrt{\frac{60}{9}} = \frac{2\sqrt{15}}{3}$$

(3)

$$\begin{cases} x - y = 3 \\ 2^x - 2^y = \frac{7}{4} \end{cases} \rightarrow \begin{cases} x = 3 + y \\ 2^{3+y} - 2^y = \frac{7}{4} \end{cases} \rightarrow 2^3 \cdot 2^y - 2^y = \frac{7}{4} \xrightarrow{2^y = t}$$

$$8t - t = \frac{7}{4} \rightarrow 7t = \frac{7}{4} \rightarrow t = \frac{1}{4} \rightarrow 2^y = \frac{1}{4} = 2^{-2} \rightarrow y = -2$$

$$x = 3 - 2 = 1 \rightarrow \boxed{\begin{matrix} x = 1 \\ y = -2 \end{matrix}}$$

$$(4) \log 2 = 0,3010, \log 3 = 0,4771, \log 5 = 0,6990$$

$$\begin{aligned} a) \log \sqrt[6]{\frac{1}{0,128}} &= \log \sqrt[6]{\frac{125}{16}} = \frac{1}{6} \log \frac{125}{16} = \frac{1}{6} [\log 125 - \log 16] = \\ &= \frac{1}{6} [\log 5^3 - \log 2^4] = \frac{1}{6} [3 \log 5 - 4 \log 2] = \\ &= \frac{1}{6} [3 \cdot 0,6990 - 4 \cdot 0,3010] = 0,1488 \end{aligned}$$

$$\begin{aligned} b) \log \frac{540}{256} &= \log \frac{2^2 \cdot 3^3 \cdot 5}{2^8} = \log 2^2 \cdot 3^3 \cdot 5 - \log 2^8 = \\ &= 2 \log 2 + 3 \log 3 + \log 5 - 8 \log 2 = 0,3243 \end{aligned}$$

$$(5) \log A = 2 + 5 \log x + \frac{3}{5} \log y - 4 \log z - 3 \log \frac{x}{y}$$

$$\log A = \log 100 + \log x^5 + \log \sqrt[5]{y^3} - \log z^4 - \log \left(\frac{x}{y}\right)^3$$

$$A = \frac{100 \cdot x^5 \cdot \sqrt[5]{y^3}}{z^4 \cdot \frac{x^3}{y^3}} = \frac{100 x^2 \cdot y^3 \sqrt[5]{y^3}}{z^4}$$

$$\begin{aligned}
 \textcircled{6} \quad \log x - \log y = 1 & \quad \log \frac{x}{y} = \log 10 & \quad \begin{cases} x = 10y \\ 3x + 2y = 64 \end{cases} \\
 3x + 2y = 64 & \quad 3x + 2y = 64 & \quad \begin{cases} 3 \cdot 10y + 2y = 64 \\ 32y = 64 \end{cases} \\
 & & \quad \begin{cases} y = 2 \\ x = 20 \end{cases}
 \end{aligned}$$

$$\textcircled{7} \quad \frac{\log 2 + \log (11 - x^2)}{\log (5 - x)} = 2$$

$$\log 2 + \log (11 - x^2) = 2 \log (5 - x)$$

$$\log 2 (11 - x^2) = \log (5 - x)^2$$

$$22 - 2x^2 = 25 - 10x + x^2$$

$$-3x^2 + 10x - 3 = 0 \quad \begin{cases} x_1 = 3 \\ x_2 = \frac{1}{3} \end{cases}$$

$$\textcircled{8} \quad 4^{1+x} + 2^{x+3} = 320$$

$$(2^2)^{1+x} + 2^{x+3} = 320 \rightarrow 2^2 \cdot (2^x)^2 + 2^x \cdot 2^3 = 320 \rightarrow \boxed{2^x = t}$$

$$4t^2 + 8t - 320 = 0 \quad \begin{cases} t_1 = 8 \rightarrow 2^x = 8 \rightarrow x = 3 \\ t_2 = -10 \rightarrow 2^x = -10 \quad \text{not possible} \end{cases}$$