

EJERCICIOS RESUELTOS DE FRACCIONES, POTENCIAS Y LOGARITMOS

1. Operaciones con fracciones:

$$\text{a) } \frac{-3 - \left[-\frac{8}{2} - 50 \cdot \left(1 - \frac{24}{25} \right) \right]}{-4 - \left(\frac{1}{2} - 1 \right)} = \frac{-3 - \left[-4 - 50 \cdot \frac{1}{25} \right]}{-4 - \left(-\frac{1}{2} \right)} = \frac{-3 - [-6]}{-4 + \frac{1}{2}} = \frac{3}{-\frac{7}{2}} = 3 : \left(-\frac{7}{2} \right) = -\frac{6}{7}$$

$$\text{b) } \frac{3 - \left[-\frac{1}{2} - 5 \cdot \left(1 - \frac{1}{2} \right) \right]}{-4 + \frac{1}{2}} = \frac{3 - \left[-\frac{1}{2} - \frac{5}{2} \right]}{-\frac{7}{2}} = \frac{3 - \left[-\frac{6}{2} \right]}{-\frac{7}{2}} = \frac{3 + 3}{-\frac{7}{2}} = \frac{6}{-\frac{7}{2}} = -\frac{12}{7}$$

$$\begin{aligned} \text{c) } 1 - \frac{1 - \frac{1}{10}}{1 - \frac{1}{1 - \frac{1}{10}}} &= 1 - \frac{\frac{10-1}{10}}{1 - \frac{1}{\frac{10-1}{10}}} = 1 - \frac{\frac{9}{10}}{1 - \frac{1}{\frac{9}{10}}} = 1 - \frac{\frac{9}{10}}{1 - \frac{10}{9}} = 1 - \frac{\frac{9}{10}}{\frac{9-10}{9}} = \\ &= 1 - \frac{\frac{9}{10}}{\frac{-1}{9}} = 1 + \frac{9}{10} : \frac{1}{9} = 1 + \frac{81}{10} = \frac{10+81}{10} = \frac{91}{10} \end{aligned}$$

$$\text{d) } \frac{\frac{\frac{2}{3} + \frac{1}{4}}{\frac{3}{4} - \frac{1}{2}} \cdot \frac{3 - \frac{3}{2}}{2 + \frac{1}{5}}}{5 \cdot \frac{1}{2}} = \frac{\frac{\frac{11}{12} \cdot \frac{3}{2}}{\frac{1}{4} \cdot \frac{11}{5}} \cdot \frac{3 \cdot 5}{2 \cdot 11}}{\frac{5}{2}} = \frac{\frac{11 \cdot 4 \cdot 3 \cdot 5}{12 \cdot 2 \cdot 11}}{\frac{5}{2}} = \frac{\cancel{11} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{5}}{\cancel{12} \cdot \cancel{2} \cdot \cancel{11} \cdot \cancel{5}} = 1$$

2. Operaciones con potencias:

$$\text{a) } 16^{-2} \cdot 4^3 = (2^4)^{-2} \cdot (2^2)^3 = 2^{-8} \cdot 2^6 = 2^{-8+6} = 2^{-2} = \frac{1}{4}$$

$$\text{b) } \frac{4^2 \cdot 12^3 \cdot 15^2}{9^3 \cdot 8^2 \cdot 3^3} = \frac{(2^2)^2 \cdot (2^2 \cdot 3)^3 \cdot (3 \cdot 5)^2}{(3^2)^3 \cdot (2^3)^2 \cdot 3^3} = \frac{2^4 \cdot 2^6 \cdot 3^3 \cdot 3^2 \cdot 5^2}{3^6 \cdot 2^6 \cdot 3^3} = \frac{2^{10} \cdot 3^5 \cdot 5^2}{2^6 \cdot 3^9} = 2^4 \cdot 3^{-4} \cdot 5^2$$

$$\text{c) } \frac{8^4 \cdot 15^3 \cdot 18^2 \cdot 12^{-3}}{20^3 \cdot 27^2 \cdot 3^{-3}} = \frac{(2^3)^4 \cdot (3 \cdot 5)^3 \cdot (2 \cdot 3^2)^2 \cdot (2^2 \cdot 3)^{-3}}{(2^2 \cdot 5)^3 \cdot (3^3)^2 \cdot 3^{-3}} =$$

$$= \frac{2^{12} \cdot 3^3 \cdot 5^3 \cdot 2^2 \cdot 3^4 \cdot 2^{-6} \cdot 3^{-3}}{2^6 \cdot 5^3 \cdot 3^6 \cdot 3^{-3}} = \frac{2^8 \cdot 3^4 \cdot 5^3}{2^6 \cdot 3^3 \cdot 5^3} = 2^2 \cdot 3 = 12$$

$$\text{d)} \frac{27^{-1} \cdot 81 \cdot 3^4 \cdot \left(\frac{2^3}{3}\right)^{-1} \cdot 2^3}{36 \cdot \left(\frac{1}{3}\right)^{-2} \cdot \frac{4}{3} \cdot \frac{27}{16} \cdot (2^0)^{-2}} = \frac{(3^3)^{-1} \cdot 3^4 \cdot 3^4 \cdot \frac{3}{2^3} \cdot 2^3}{3^2 \cdot 2^2 \cdot 3^2 \cdot \frac{2^2}{3} \cdot \frac{3^3}{2^4} \cdot 1} = \frac{3^6}{3^6} = 1$$

3. Haz las siguientes operaciones con radicales:

$$\text{a)} \frac{\sqrt[4]{27}}{\sqrt[3]{18}} = \frac{\sqrt[4]{3^3}}{\sqrt[3]{2 \cdot 3^2}} = \sqrt[12]{\frac{(3^3)^3}{(2 \cdot 3^2)^4}} = \sqrt[12]{\frac{3^9}{2^4 \cdot 3^8}} = \sqrt[12]{\frac{3}{2^4}} = \sqrt[12]{\frac{3}{16}}$$

$$\begin{aligned} \text{b)} \quad \sqrt[4]{-80} : \sqrt[3]{18} &= \frac{-\sqrt[4]{2^4 \cdot 5}}{\sqrt[3]{2 \cdot 3^2}} = -\frac{2\sqrt[4]{5}}{\sqrt[3]{2 \cdot 3^2}} = -\frac{2\sqrt[4]{5^3}}{\sqrt[4]{(2 \cdot 3^2)^4}} = 2 \cdot \sqrt[4]{\frac{5^3}{2^4 \cdot 3^8}} = \\ &= \frac{\cancel{2}}{\cancel{2} \cdot 3^2} \cdot \sqrt[4]{5^3} = \frac{\sqrt[4]{75}}{9} \end{aligned}$$

$$\text{c)} \left(\sqrt[15]{-\frac{1}{243}}\right)^3 = \left(-\sqrt[15]{\frac{1}{3^5}}\right)^3 = -\sqrt[15]{\left(\frac{1}{3^5}\right)^3} = -\sqrt[5]{\frac{1}{3^{15}}} = -\frac{1}{3^3} = -\frac{1}{27}$$

$$\text{d)} \sqrt[3]{\sqrt{2}} \cdot \sqrt[3]{16} = \sqrt[6]{2} \cdot \sqrt[3]{16} = \sqrt[6]{2 \cdot 16^2} = \sqrt[6]{2 \cdot (2^4)^2} = \sqrt[6]{2^9} = \sqrt[6]{2^6 \cdot 2^3} = 2 \cdot \sqrt[6]{2^3} = 2 \cdot \sqrt{2}$$

$$\text{e)} \sqrt[3]{\sqrt{2}} \cdot \sqrt[3]{16} = \sqrt[6]{2} \cdot \sqrt[3]{16} = \sqrt[6]{2 \cdot 16^2} = \sqrt[6]{2 \cdot (2^4)^2} = \sqrt[6]{2^9} = \sqrt[6]{2^6 \cdot 2^3} = 2 \cdot \sqrt[6]{2^3} = 2 \cdot \sqrt{2}$$

$$\text{f)} \sqrt{\frac{3\sqrt{2}}{8}} = \sqrt{\frac{\sqrt{3^2 \cdot 2}}{2 \cdot 2^2}} = \frac{1}{2} \sqrt{\frac{3^2 \cdot 2}{2^2}} = \frac{1}{2} \sqrt[4]{\frac{3^2 \cdot 2}{2^2}} = \frac{1}{2} \sqrt[4]{\frac{9}{2}}$$

$$\text{g)} \frac{(\sqrt[4]{3^2})^2 \cdot (\sqrt[3]{3})^6}{(\sqrt[12]{3^4})^6} = \frac{\sqrt[4]{3^4} \cdot \sqrt[3]{3^6}}{\sqrt[12]{3^{24}}} = \frac{3 \cdot 3^2}{3^2} = 3$$

$$\text{h)} \frac{(\sqrt[5]{3})^4 \cdot (\sqrt[3]{3})^2}{(\sqrt{3^4})^3} = \frac{\sqrt[5]{3^4} \cdot \sqrt[3]{3^2}}{\sqrt{3^{12}}} = \frac{\sqrt[15]{(3^4)^3 \cdot (3^2)^5}}{3^2} = \frac{\sqrt[15]{3^{12} \cdot 3^{10}}}{3^2} = \frac{\sqrt[15]{3^{22}}}{3^2} = \sqrt[15]{\frac{3^{22}}{3^{30}}} = \sqrt[15]{\frac{1}{3^8}}$$

$$\text{i)} \sqrt{\sqrt[3]{\sqrt[4]{64^4}}} = \sqrt{\sqrt[3]{4 \cdot (2^6)^4}} = {}^{2 \cdot 3 \cdot 4}\sqrt{2^{24}} = {}^{24}\sqrt{2^{24}} = 2$$

$$\text{j)} \sqrt{8} - \sqrt{50} - \frac{1}{2}\sqrt{98} = \sqrt{2^2 \cdot 2} - \sqrt{2 \cdot 5^2} - \frac{1}{2}\sqrt{7^2 \cdot 2} = 2\sqrt{2} - 5\sqrt{2} - 7\sqrt{2} = -10\sqrt{2}$$

$$\begin{aligned} \text{k)} \quad \frac{1}{2}\sqrt{3} - \sqrt{12} - \frac{3}{4}\sqrt{75} &= \frac{1}{2}\sqrt{3} - \sqrt{2^2 \cdot 3} - \frac{3}{4}\sqrt{5^2 \cdot 3} = \frac{1}{2}\sqrt{3} - 2\sqrt{3} - \frac{3 \cdot 5}{4}\sqrt{3} = \\ &= \frac{1}{2}\sqrt{3} - 2\sqrt{3} - \frac{15}{4}\sqrt{3} = -\frac{21}{4}\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{l)} \quad \sqrt{9xy} + \frac{xy}{\sqrt{4xy}} + \frac{\sqrt[6]{(xy)^{21}}}{x^3y^3} &= 3\sqrt{xy} - \frac{xy}{2\sqrt{xy}} - \sqrt[6]{\frac{(xy)^{21}}{(x^3y^3)^6}} = 3\sqrt{xy} - \frac{1}{2}\sqrt{\frac{(xy)^2}{xy}} - \sqrt[6]{(xy)^3} = \\ &= 3\sqrt{xy} - \frac{1}{2}\sqrt{xy} - \sqrt{xy} = \frac{3}{2}\sqrt{xy} \end{aligned}$$

4. Racionaliza:

$$\text{a)} \quad \frac{1}{2 \cdot \sqrt[3]{5}} = \frac{1}{2 \cdot \sqrt[3]{5}} \frac{\sqrt[3]{5} \sqrt[3]{5}}{\sqrt[3]{5} \sqrt[3]{5}} = \frac{\sqrt[3]{25}}{2 \cdot 5} = \frac{\sqrt[3]{25}}{10}$$

$$\text{b)} \quad \frac{1}{\sqrt[5]{x^4}} = \frac{1}{\sqrt[5]{x^4}} \left(\frac{\sqrt[5]{x^4}}{\sqrt[5]{x^4}} \right)^4 = \frac{\left(\sqrt[5]{x^4} \right)^4}{\left(\sqrt[5]{x^4} \right)^5} = \frac{\left(x^4 \right)^{\frac{1}{5} \cdot 4}}{x^4} = \frac{x^{\frac{16}{5}}}{x^4} = \frac{\sqrt[5]{x^{16}}}{x^4} = \frac{\sqrt[5]{x^{15} \cdot x}}{x^4} = \frac{x^3 \cdot \sqrt[5]{x}}{x^4} = \frac{\sqrt[5]{x}}{x}$$

$$\text{c)} \quad \frac{\sqrt[3]{x}}{\sqrt[5]{x^5}} = \frac{\sqrt[3]{x}}{\sqrt[5]{x^5}} \left(\frac{\sqrt[6]{x^5}}{\sqrt[6]{x^5}} \right)^5 = \frac{\sqrt[3]{x} \left(\sqrt[6]{x^5} \right)^5}{\left(\sqrt[6]{x^5} \right)^6} = \frac{x^{\frac{1}{3}} \left(x^5 \right)^{\frac{1}{6} \cdot 5}}{x^5} = \frac{x^{\frac{1}{3}} \cdot x^{\frac{25}{6}}}{x^5} = \frac{x^{\frac{2+25}{6}}}{x^5} = \frac{x^{\frac{27}{6}}}{x^5} = \frac{\sqrt[6]{x^{27}}}{x^5} = \frac{\sqrt[6]{x^3}}{x}$$

$$\text{d)} \quad \frac{\sqrt{2}}{\sqrt{3}+1} = \frac{\sqrt{2} \cdot (\sqrt{3}-1)}{(\sqrt{3}+1) \cdot (\sqrt{3}-1)} = \frac{\sqrt{2} \cdot (\sqrt{3}-1)}{(\sqrt{3})^2 - 1^2} = \frac{\sqrt{2} \cdot (\sqrt{3}-1)}{3-1} = \frac{\sqrt{2} \cdot (\sqrt{3}-1)}{2}$$

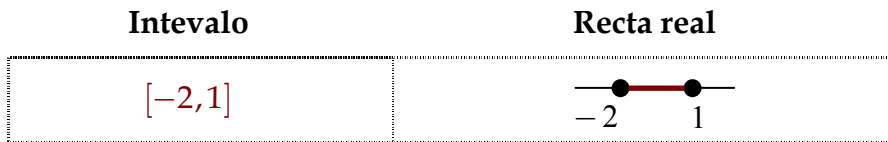
$$\text{e)} \quad \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \cdot \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})^2}{(\sqrt{2})^2 - (\sqrt{3})^2} = \frac{(\sqrt{2}+\sqrt{3})^2}{2-3} = -(\sqrt{2}+\sqrt{3})^2$$

$$\begin{aligned} \text{f)} \quad \frac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}} &= \frac{(2\sqrt{3}+\sqrt{2}) \cdot (2\sqrt{3}+\sqrt{2})}{(2\sqrt{3}-\sqrt{2}) \cdot (2\sqrt{3}+\sqrt{2})} = \frac{(2\sqrt{3})^2 + 2 \cdot 2\sqrt{3} + (\sqrt{2})^2}{(2\sqrt{3})^2 - (\sqrt{2})^2} = \frac{4 \cdot 3 + 4\sqrt{3} + 2}{4 \cdot 3 - 2} = \\ &= \frac{7+2\sqrt{3}}{5} \end{aligned}$$

5. Expresa mediante intervalos los siguientes conjuntos y represéntalos en la recta real.

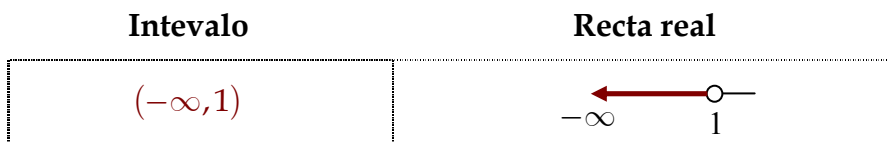
a) $A = \{x / x \in \mathbb{R}, -2 \leq x \leq 1\}$

Solución:



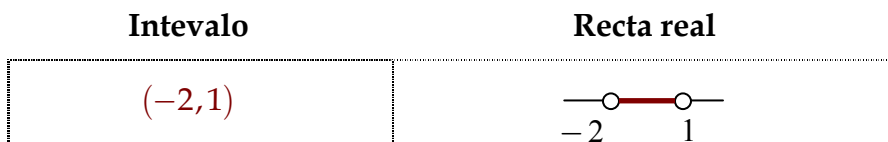
b) $B = \{x / x \in \mathbb{R}, -\infty < x < 1\}$

Solución:



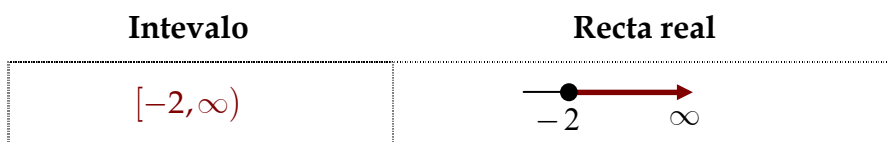
c) $C = \{x / x \in \mathbb{R}, -2 < x < 1\}$

Solución:



d) $D = \{x / x \in \mathbb{R}, -2 \leq x < \infty\}$

Solución:



e) $E = \left\{x / x \in \mathbb{R}, \frac{1}{2} \leq x \leq 2\right\}$

Solución:

Intervalo	Recta real
$\left[\frac{1}{2}, 2\right]$	

f) $F = \{x / x \in \mathbb{R}, 2 \leq x < 12\}$

Solución:

Intervalo	Recta real
$[2, 12)$	

6. Teniendo en cuenta los conjuntos anteriores, escribe como un solo intervalo:

a) $A \cup B$ y $A \cap B$

Solución:

La unión de A y B son el conjunto de valores que pertenecen a A, a B o a ambos, mientras que la intersección son el conjunto de valores comunes a A y B. Entonces:

- $A \cup B = (-\infty, 1]$
- $A \cap B = [-2, 1)$

b) $C \cup D$ y $C \cap D$

Solución:

- $C \cup D = [-2, \infty)$
- $C \cap D = (-2, 1)$

c) $E \cup F$ y $E \cap F$

Solución:

- $E \cup F = \left[\frac{1}{2}, 12\right)$
- $C \cap D = 2$

7. Halla el valor de los siguientes logaritmos

a) $\log 10000 = \log 10^4 = 4 \log 10 = 4$

b) $\log_2 64 = \log_2 2^6 = 6 \log_2 2 = 6$

c) $\log_3 27 = \log_3 3^3 = 3 \log_3 3 = 3$

d) $\log_{\frac{1}{3}} 27 = \log_{\frac{1}{3}} 3^3 = \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-3} = -3 \log_{\frac{1}{3}} \left(\frac{1}{3}\right) = -3$

e) $\log_5 \sqrt{\frac{1}{125}} = \log_5 \left(\frac{1}{5^3}\right)^{\frac{1}{2}} = \log_5 (5^{-3})^{\frac{1}{2}} = \log_5 5^{-\frac{3}{2}} = -\frac{3}{2} \log_5 5 = -\frac{3}{2}$

f)
$$\frac{\log_2 \sqrt[3]{2} - \log_2 \sqrt{2\sqrt{2}}}{\log_2 \sqrt{2}} - \frac{\log_2 \frac{1}{\sqrt{2}}}{1 - \log_2 \sqrt{\sqrt{2}}} = \frac{\log_2 2^{\frac{1}{3}} - \log_2 2^{\frac{3}{4}}}{\log_2 2^{\frac{1}{2}}} - \frac{\log_2 2^{-\frac{1}{2}}}{1 - \log_2 2^{\frac{1}{4}}} = \frac{\frac{1}{3} - \frac{3}{4}}{\frac{1}{2}} - \frac{-\frac{1}{2}}{1 - \frac{1}{4}} =$$

$$= \frac{\frac{1}{3} - \frac{3}{4}}{\frac{1}{2}} - \frac{-\frac{1}{2}}{1 - \frac{1}{4}} = -\frac{1}{6}$$

8. Calcula el valor del dato desconocido:

a) $\log_3 81 = x \Rightarrow 3^x = 81 \Rightarrow 3^x = 3^4 \Rightarrow x = 4$

b) $\log_5 a = 4 \Rightarrow 5^4 = a \Rightarrow a = 625$

c) $\log_a 1000 = 4 \Rightarrow a^4 = 10000 \Rightarrow a^4 = 10^4 \Rightarrow a = 10$

d) $\log_{32} \sqrt{2} = x \Rightarrow 32^x = \sqrt{2} \Rightarrow 2^{5x} = 2^{\frac{1}{2}} \Rightarrow 5x = \frac{1}{2} \Rightarrow x = \frac{1}{10}$

e) $\log_9 \frac{1}{81} = x \Rightarrow 9^x = \frac{1}{81} \Rightarrow 9^x = 9^{-2} \Rightarrow x = -2$

f) $\log_x 2 = \left(\frac{1}{5}\right)^{-1} \Rightarrow \log_x 2 = 5 \Rightarrow x^5 = 2 \Rightarrow (x^5)^{\frac{1}{5}} = 2^{\frac{1}{5}} \Rightarrow x^{\frac{5}{5}} = 2^{\frac{1}{5}} \Rightarrow x = \sqrt[5]{2}$

g) $\log_{\frac{1}{5}} x = -3 \Rightarrow \left(\frac{1}{5}\right)^{-3} = x \Rightarrow x = 5^3 = 125$

9. Escribe las siguientes expresiones como un solo logaritmo:

$$\text{a) } \log(xy) - 2\log\left(\frac{x}{y}\right) = \log(xy) - \log\left(\frac{x}{y}\right)^2 = \log\left(\frac{xy}{\frac{x^2}{y^2}}\right) = \log\left(\frac{y^3}{x}\right)$$

$$\begin{aligned} \text{b) } 2\ln(a-b) - \ln(a^2 - b^2) &= \ln(a-b)^2 - \ln[(a+b)(a-b)] = \\ &= \ln(a-b)^2 - \ln[(a+b)(a-b)] = \ln \frac{(a-b)^2}{(a+b)(a-b)} = \ln\left(\frac{a-b}{a+b}\right) \end{aligned}$$

$$\begin{aligned} \text{c) } 4\log_2 \frac{\sqrt{a-b}}{a} - \frac{1}{2}\log_2 \left(\frac{a-b}{a}\right)^4 &= \log_2 \left(\frac{\sqrt{a-b}}{a}\right)^4 - \log_2 \left[\left(\frac{a-b}{a}\right)^4\right]^{\frac{1}{2}} = \\ &= \log_2 \frac{(a-b)^2}{a^4} - \log_2 \left(\frac{a-b}{a}\right)^2 = \log_2 \left[\frac{(a-b)^2}{a^4} \cdot \frac{a^4}{(a-b)^2}\right] = \log_2 \left[\frac{a^2(a-b)^2}{a^4(a-b)^2}\right] = \log_2 \left(\frac{1}{a^2}\right) = \\ &= \log_2(a^{-2}) \end{aligned}$$
