

1. Realiza la operación:

1. $\frac{x^3(x^3)^3x}{x(-3x)^3(x^3)^3}$
2. $\frac{-2 \cdot 3x^2(-2)}{-2(3x^3)^3(-1)}$
3. $\frac{-3(x^3)^3x}{-3(-2x^3)^2(-x)}$
4. $\frac{x^2(-2x^3)x^2}{-3(2x^2)^2(-3)}$
5. $\frac{(-x^2)^3(x^3)^3(-2)}{(-2x)^2(-1)x^2}$
6. $\frac{(x^2)^3(-1)(3x^2)^2}{2(x^2)^2(x^2)^2}$
7. $\frac{(-x^3)^2(-1)(2x^3)^2}{3(-2x)(-2)}$
8. $\frac{(-x^2)^3(2x)^3(-2x)}{3(-3x^2)(-3x)}$
9. $\frac{2x^2(-x^3)^2(-2x)^2}{(3x^2)^32(-3)}$
10. $\frac{2x^3(-3x^2)^2(-2x)}{(3x)^2(-x^2)^2(-2x)}$
11. $\frac{(-3x^2)^33x^3(x^2)^3}{(2x)^2(3x^2)^3(-3)}$
12. $\frac{(x^3)^3(-2x^3)^23x^2}{-3(-2x^2)^3(2x^2)^2}$
13. $\frac{2(-2x^3)^3(-2x^2)}{(-2x^2)^3(-1)(-3x)^2}$
14. $\frac{3(-x)^3(2x)^2}{(-2x^2)^3(2x^2)^22x}$
15. $\frac{x(-x)x^3}{(-3x)^2(x^3)^2(2x^3)^3}$
16. $\frac{(-2x^2)^2(-2)(x^2)^2}{(-2x)^3(3x^3)^2(3x)^3}$
17. $\frac{(-2x)^2(-x^2)^32x}{(2x^2)^3(-2x)^3(-x^2)^3}$
18. $\frac{(-3x^2)^3(2x^3)^32x^3}{(3x^2)^2(-2x^2)(-3x^2)^3}$
19. $\frac{(-3x^3)^2x(-3x)^3}{(3x)^3(-2x)^3(2x^2)^2}$
20. $\frac{-3(3x^2)^3(-3x^2)}{(-2x^2)^3(x^2)^2(-3x^2)^3}$

2. Realiza la operación:

1. $\frac{2y^3(3y^2)^2(3y)^3}{y^2(2xy^2)^23xy}$
2. $\frac{2xy^3(3xy^2)^23y}{-3x \cdot 2xy^2(-3)}$
3. $\frac{(y^3)^2(-3y^3)(-x)^3}{-y(2x)^3(y^3)^2}$
4. $\frac{(2x)^3(-xy^3)(-y^3)}{(-y)^3x(-2x)}$
5. $\frac{(2xy)^33xy^2(2y^3)^2}{(3x)^2(-xy)(-xy^3)}$
6. $\frac{(-xy)^2(xy^2)^2(-2y^3)}{(2y^3)^3(-3)(-3y)^3}$
7. $\frac{(xy^3)^2(xy)^3(-3xy^2)}{(-2y^3)^2(-y^2)^2(-y^3)}$
8. $\frac{(-3y)^2(-3x)(-3xy^2)^2}{2xy^2(-x)^22y^2}$
9. $\frac{(-2xy^3)^3(-3y)^33xy^2}{-3(-2xy^3)^2(-3y^2)^2}$
10. $\frac{(-2y^3)^2(2y^3)^3(2y^2)^3}{-xy^3(3y^3)^2(-1)}$
11. $\frac{-3(-x)x}{xy^3(-2xy^2)^3(-2xy^2)}$
12. $\frac{(-2xy^3)^2(2y^2)^2(3x)^2}{-3xy^2(2y^3)^32xy^3}$

3. Halla el valor numérico de la siguiente expresión:

1. x^3+2x^2-2 ; $x = -3$
2. $-2x^4-x^2-2x+1$; $x = 2$
3. x^3-2x^2+3x+3 ; $x = -1$
4. $2x^4-x^3-x^2-2x$; $x = 2$
5. $-3x^4-2x^3-3x+2$; $x = -2$
6. $2(2x-3)-2(x+2)$; $x = -1$
7. $-2(3x-2)-3(x+2)$; $x = -3$
8. $-3(3x-1)+2(3x-3)$; $x = 2$
9. $x-x(3x-1)-3(x+1)$; $x = -3$
10. $3x(3x+1)-3(x-2)$; $x = -2$
11. $x+(x-3)^2-(x+2)(x+2)$; $x = 2$
12. $x(2x^2+x-3)-(3x+3)^2$; $x = -2$
13. $x+(x-3)(x+3)-(3x+2)^2$; $x = -1$
14. $-(3x+2)(3x-2)+2x^2(x-2)$; $x = 1$
15. $x-(3x-2)(3x+2)-3x(x+2)$; $x = -2$

4. Realiza la operación:

1. $-3x-(-4x^2+1)-(-3x^2-x)$
2. $-(2x-4)-(-3x+4)+(-4x-4)$
3. $(-3x-2)-(-3x+2)-(-4x-1)$
4. $-(x-2)-(x^3+4x+3)-(3x+4)$
5. $(x-1)-(-3x+3)-(-4x^2-x+4)$
6. $-(x-3)-(x+2)-(-2x^3+2x^2+2x)$
7. $-(3x^2+x+4)-(-x+3)+(x^2+3x+2)$
8. $(-4x^3+3x^2-3)-(-4x-4)-(-3x+1)$
9. $(-2x+3)-(x^3+3x^2+x-2)+(-4x-2)$
10. $-(x^2-2x-1)-(-3x^2+3x-4)-(-2x+1)$
11. $-(-x^3+x^2+4x+4)+(-4x+4)-(-x-4)$
12. $-(-x^3-3x)-(3x^3-3x^2+1)-(-x^3+x^2)$

5. Realiza la operación:

1. $(-2x+3)(-2x^2-2x)$
2. $(x+2)(-4x^3+4x-3)$
3. $(x^2+1)(-2x^2-2x+3)$
4. $(-3x^2-3x)(4x^2-2x-4)$
5. $(-2x^3+x^2)(3x^3-x+4)$
6. $(-3x^2+x+4)(-x^3-4x^2-x)$
7. $(3x+2)(3x-2)$
8. $(4x^2-1)(4x^2+1)$
9. $(x^3-2x^2)(x^3+2x^2)$
10. $(x^2+3x)(3x-x^2)$
11. $(-2x^2+3)(2x^2+3)$
12. $(2x^2-3)^2$
13. $(-x^2+2x)^2$
14. $(-2x-3)^2$
15. $(3x^3-2x)^2$
16. $(3x^3-5x^2)^2$

$$17. \left(x^2 - \frac{1}{3}\right)\left(x^2 + \frac{1}{3}\right)$$

$$18. \left(\frac{1}{2}x - \frac{2}{3}\right)\left(\frac{1}{2}x + \frac{2}{3}\right)$$

$$19. \left(2x^2 - \frac{3}{2}\right)^2$$

$$20. \left(\frac{2}{3}x - \frac{1}{2}\right)^2$$

6. Realiza la operación:

$$1. (x-3)^2 + x(x^2+x)$$

$$2. (x+2)(x-2) - x(3x+3)$$

$$3. (2x+3)(2x-3) + (x-3)^2$$

$$4. -(x+1)^2 - (3x^3-x)(x^2-1)$$

$$5. (2x+2)(2x-2) - (3x+3)^2$$

$$6. -(2x^2+x)(2x^2-x) - (3x-1)^2$$

$$7. -(x-1)(2x^2-2x) + (x+3)(x-3)$$

$$8. -2x + (3x+2)^2 + (3x-1)(3x+1)$$

$$9. 3x^3 - (3x-2)^2 - (3x-3)(3x+3)$$

$$10. 2x(x^2-2x-1) + (3x+3)(3x-3)$$

$$11. -2x(2x-3) + (x^2+2x)(x^2-2x)$$

$$12. -(2x+2)(2x^2-1) + 3(3x^2-x-1)$$

$$13. (x^2-2)(x^3+x+2) + (2x+1)(2x-1)$$

$$14. -x^2(-2x+3)(3x-3) - x(3x-1)^2$$

$$15. -3x^2 + (x+1)(x-1) + (x-2)(3x^2-1)$$

$$16. 2(-x+3)(-x-3) + 2x^2(2x^3+3x)^2$$

$$17. 2x(-2x-1)(-2x+1) + 3x^3(2x-2)^2$$

$$18. -(3x^2-2)^2 + (3x^2-3)(2x^2+3x-2)$$

$$19. 2x + (x^2-2)(x^2+2) - (x-3)(2x^2+3)$$

$$20. (2x+3)(2x-3) + 2x^3(3x^3+2x^2-x)$$

$$21. 2x-3x(3x-1) - (3x^2+3x)(x^2+x-2)$$

$$22. -2x^2(2x^3+2x^2) - (3x^2+x)(3x^2-x)$$

$$23. (2x^2-3)(x^2-2x+3) + (3x+2)(3x-2)$$

$$24. -3x^3-x^2(2x^2-x+2) - (3x-1)(x^2+3)$$

$$25. -(3x+2)(3x-2) - (x-1)(2x^3-3x^2-3x)$$

$$26. -(2x+2)(2x-2) - (x-2)(3x^3-2x^2-3x)$$

$$27. -2x^2-2x^2(3x^2-2x+3) - (x+2)(2x+2)$$

$$28. -2x^3(-x-2)(-x+2) + 2x^3(x+2)(3x+2)$$

$$29. 3x+2x(3x^3+x^2-2) - (2x-2)(2x^2+2x+1)$$

$$30. -3x^3(2x^3-2x-3) - (2x^3+3x)(2x^3-3x-1)$$

7. Realiza la siguiente división:

$$1. \begin{cases} D: -16x-8x^2-7 \\ d: 3+2x \end{cases}$$

$$2. \begin{cases} D: x^3-8 \\ d: -x^2-2x-4 \end{cases}$$

$$3. \begin{cases} D: 3x^3-x-2 \\ d: -1-3x \end{cases}$$

$$4. \begin{cases} D: 8x^3+6x^2+10x-1 \\ d: 1-4x \end{cases}$$

$$5. \begin{cases} D: 9x^3+18x^2+17x+6 \\ d: 3x^2+4x+3 \end{cases}$$

$$6. \begin{cases} D: -12x^3-8x^2-8x-13 \\ d: -4x-4 \end{cases}$$

$$7. \begin{cases} D: 4x^4-2x^3-4x+2 \\ d: -2x-1 \end{cases}$$

$$8. \begin{cases} D: 3x^4+x^3+8x^2+17x+2 \\ d: -4x-3x^2 \end{cases}$$

$$9. \begin{cases} D: -4x^6+15x^5-23x^4+3x^3-7x+3 \\ d: x^3-3x^2+3x+3 \end{cases}$$

$$10. \begin{cases} D: 14-13x-12x^2+4x^3+2x^4+4x^5 \\ d: 4-2x-4x^2 \end{cases}$$

$$11. \begin{cases} D: 4x^5+16x^4+11x^3+11x^2-2x-6 \\ d: 1-3x-4x^3 \end{cases}$$

$$12. \begin{cases} D: 1+8x-4x^3-5x^4-2x^5-x^6 \\ d: 2+4x+x^2+x^3 \end{cases}$$

8. Realiza, usando la regla de Ruffini, la división:

$$1. \begin{cases} D: -3x^3-10x^2-6x \\ d: x+2 \end{cases}$$

$$2. \begin{cases} D: x^4+2x^3+3x-1 \\ d: x+2 \end{cases}$$

$$3. \begin{cases} D: -4x^3+4x^2+7x-2 \\ d: x-2 \end{cases}$$

$$4. \begin{cases} D: 2x^4+5x^3-12x^2-x \\ d: x+4 \end{cases}$$

$$5. \begin{cases} D: -x^4+x^3+9x^2-6x-9 \\ d: x-3 \end{cases}$$

$$6. \begin{cases} D: 2x^4-2x^3-7x-7 \\ d: x+1 \end{cases}$$

$$7. \begin{cases} D: -2x^4+3x^2+8x-3-6x^3 \\ d: x+3 \end{cases}$$

$$8. \begin{cases} D: 4x^3-12x-8x^2+3x^4-6 \\ d: x+2 \end{cases}$$

$$9. \begin{cases} D: -2x^2+8x^3+4x-2x^4+13 \\ d: x-4 \end{cases}$$

$$10. \begin{cases} D: x^6+3x^5-3x^4-8x^3-9x-1 \\ d: x+3 \end{cases}$$

$$11. \begin{cases} D: x^3 - \frac{19}{4}x^2 + 3x + \frac{1}{2} \\ d: x-4 \end{cases}$$

$$12. \begin{cases} D: \frac{4}{3}x^4 + \frac{4}{3}x^3 - \frac{1}{2}x^2 - \frac{1}{2}x \\ d: x+1 \end{cases}$$

9. Dado el polinomio $2x^3-6mx^2-x+3m$, determina el valor de m , sabiendo que es divisible por $x+1$.

10. Calcula el valor de m , sabiendo que el polinomio $3mx^3-4mx^2+2x$ es divisible por $x-1$.

11. Halla m para que el resto de la división del polinomio $-mx^3+x^2-6x+m$ por $x+3$ sea -1 .

12. Halla el valor de m para que el polinomio $3x^3+3x^2-7x-2m^2$ sea divisible por $x+2$.

13. Dado el polinomio $2m^2x^3-5x^2+5mx+2$, determina m, sabiendo que el resto de su división por $x-1$ es 4.

14. Calcula m, sabiendo que el resto de la división del polinomio $6x^3-2m^2x^2-6x-m$ por $x-2$ es 2.

15. Dados los polinomios $-nx^2-2x-2m$ y $-2mx^2+n$, calcula los valores de m y n, sabiendo que el resto de sus divisiones por $x-1$ es 2.

16. Dado el polinomio $x^3+nx^2-mx+21$, determina m y n para que el resto de las divisiones por $x+2$ y $x+3$ sea 3.

17. Determina m y n para que el polinomio $x^3+nx^2-2mx+3$ sea divisible por $x-1$ y $x-3$.

18. Halla los valores de m y n para que los polinomios $x^2+3nx+m$ y $-3nx^2-x-m$ sean divisibles por $x-1$.

19. Descompón, al máximo, en producto de factores:

- | | | | | | |
|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|
| 1. $x-x^3$ | 2. x^3-9x | 3. $4x^3-x$ | 4. x^2-x^4 | 5. x^6-x^2 | 6. $3-3x^4$ |
| 7. $3x^4-48$ | 8. $x-81x^5$ | 9. x^6-81x^2 | 10. $48-3x^4$ | 11. $2x^6-2x^2$ | 12. $4x^3+4x^2+x$ |
| 13. $18x^2-12x+2$ | 14. $12x^2-12x+3$ | 15. $3x^2+18x+27$ | 16. $12x^2+36x+27$ | 17. $3x^3-18x^2+27x$ | 18. $8x^4+24x^3+18x^2$ |
| 19. $27x^3+36x^2+12x$ | 20. x^8-2x^4+1 | 21. $3x^2-6x-9$ | 22. $3x^2-3x-6$ | 23. $2x^2+2x-4$ | 24. $2x^2+4x-6$ |
| 25. $3x^5-243x$ | 26. $-x^5+2x^3-x$ | 27. $x^6-2x^4+x^2$ | 28. $162x^6-2x^2$ | 29. $2x^4-4x^2+2$ | 30. $-3x^6+48x^2$ |
| 31. x^3+x^2-4x-4 | 32. x^3+x^2-9x-9 | 33. x^3-x^2-4x+4 | 34. $2x^2+10x+12$ | 35. x^9-8x^5+16x | 36. $3x^3-9x^2+6x$ |
| 37. $3x^5-6x^3+3x$ | 38. $x^4+4x^3+3x^2$ | 39. $2x^3+8x^2+6x$ | 40. x^3-5x^2+8x-4 | 41. x^3+5x^2+8x+4 | 42. $3x^6-6x^4+3x^2$ |
| 43. $3x^4+6x^3-9x^2$ | 44. $3x^4-24x^2+48$ | 45. $3x^4+9x^3+6x^2$ | 46. x^8-32x^4+256 | 47. $x^3-2x^2-9x+18$ | 48. $3x^4-3x^3-18x^2$ |
| 49. $4x^3+4x^2-9x-9$ | 50. $9x^3-9x^2-4x+4$ | 51. $x^3+7x^2+15x+9$ | 52. $2x^5-16x^3+32x$ | 53. $3x^8-54x^4+243$ | 54. $-x^6+18x^4-81x^2$ |
| 55. $x^3-8x^2+21x-18$ | 56. $x^3+3x^2-16x-48$ | 57. $4x^3-8x^2-9x+18$ | 58. $81x^9-72x^5+16x$ | 59. $-3x^5+24x^3-48x$ | 60. $3x^5-54x^3+243x$ |

20. Descompón, al máximo, en producto de factores:

- | | | | |
|-------------------------|--------------------------------|-------------------------------|------------------------------------|
| 1. x^4y^5-16y | 2. $32x^4y^4-2$ | 3. $3y^6-3x^4y^2$ | 4. $48xy^6-3x^5y^2$ |
| 5. $2xy^2+6x^2y+4x^3$ | 6. $x^2y^3-x^2y^2-4y+4$ | 7. $x^5y^6-2x^3y^4+xy^2$ | 8. $y^8-32x^4y^4+256x^8$ |
| 9. $3xy^5-54xy^3+243xy$ | 10. $xy^3+2x^2y^2-2xy^2-4x^2y$ | 11. $48x^5y^8-216x^3y^4+243x$ | 12. $y^3+4xy^2-y^2+4x^2y-4xy-4x^2$ |

21. Halla el M.c.d. y m.c.m. de los siguientes polinomios:

- | | | | | | |
|--|---|--|--|---|---|
| 1. $\begin{cases} 9x^2-1 \\ 9x^2-3x \\ 9x^2+6x+1 \end{cases}$ | 2. $\begin{cases} x^2-16 \\ x^2+x-12 \\ x^2-7x+12 \end{cases}$ | 3. $\begin{cases} 2x-4 \\ 4-x^2 \\ 2x^2+8x+8 \end{cases}$ | 4. $\begin{cases} x^2-4 \\ x^2-x-2 \\ x^2+3x+2 \end{cases}$ | 5. $\begin{cases} 3x^2-3 \\ x^2-2x+1 \\ 3x^2+6x+3 \end{cases}$ | 6. $\begin{cases} x^2-1 \\ x^2+2x+1 \\ 3x^4-6x^2+3 \end{cases}$ |
| 7. $\begin{cases} 1-x^2 \\ x^2+2x+1 \\ x^4-2x^3+x^2 \end{cases}$ | 8. $\begin{cases} 9x+9 \\ 9x+27 \\ 3x^2+12x+9 \end{cases}$ | 9. $\begin{cases} 2x^2-2 \\ 2x^4-2 \\ 2x^4-4x^2+2 \end{cases}$ | 10. $\begin{cases} 4x+8 \\ 4x+12 \\ 2x^2+10x+12 \end{cases}$ | 11. $\begin{cases} 2x^2-4x-6 \\ 2x^2-12x+18 \\ x^3-5x^2+3x+9 \end{cases}$ | 12. $\begin{cases} 9x+18 \\ 3x^3+9x^2 \\ 3x^2+15x+18 \end{cases}$ |
| 13. $\begin{cases} 6x^2-9 \\ 27-12x^4 \\ 4x^4-12x^2+9 \end{cases}$ | 14. $\begin{cases} x^4-9x^2 \\ x^2+6x+9 \\ x^5-18x^3+81x \end{cases}$ | 15. $\begin{cases} x^2+2x \\ x^4-x^3-2x^2 \\ x^3+x^2-4x-4 \end{cases}$ | 16. $\begin{cases} 2x^2+2x \\ x^3-2x^2-3x \\ x^3+x^2-9x-9 \end{cases}$ | 17. $\begin{cases} 2x^3-2x \\ 2x^5-2x \\ 2x^3+4x^2+2x \end{cases}$ | 18. $\begin{cases} 2x-2x^5 \\ 2x^3+4x^2+2x \\ 2x^5-4x^3+2x \end{cases}$ |

$$\begin{array}{llll}
 19. \begin{cases} 4x+2 \\ 4x^2-4x+1 \\ 32x^4-16x^2+2 \end{cases} & 20. \begin{cases} 3x^2+9x+6 \\ 3x^2+12x+12 \\ x^3+5x^2+8x+4 \end{cases} & 21. \begin{cases} x^2-9 \\ x^2-x-6 \\ x^3+2x^2-9x-18 \end{cases} & 22. \begin{cases} x^4-4 \\ x^4-4x^2+4 \\ 2x^6+8x^4+8x^2 \end{cases} \\
 23. \begin{cases} x^2-x-2 \\ x^2+3x+2 \\ x^3+x^2-4x-4 \end{cases} & 24. \begin{cases} x^2-4 \\ 16x^2-x^6 \\ x^8-32x^4+256 \end{cases} & 25. \begin{cases} x^2-x \\ x^3-3x^2+2x \\ x^3-5x^2+8x-4 \end{cases} & 26. \begin{cases} 1-x^2 \\ 2x^3+4x^2+2x \\ 2x^6-4x^4+2x^2 \end{cases} \\
 27. \begin{cases} 2x^3+3x^2-1 \\ x^3-x^2-5x-3 \\ 2x^3-x^2-2x+1 \end{cases} & 28. \begin{cases} 3x^2+9x \\ x^2-6x+9 \\ 3x^3-18x^2+27x \end{cases} & 29. \begin{cases} -3x^2 \\ 3x^2+6x \\ 3x^3+15x^2+18x \end{cases} & 30. \begin{cases} 2x^3-7x^2+2x+3 \\ 2x^3-3x^2-8x-3 \\ 4x^3+8x^2+5x+1 \end{cases}
 \end{array}$$

22. Halla el M.c.d. y m.c.m. de los siguientes polinomios:

$$\begin{array}{llll}
 1. \begin{cases} y^4-x^2 \\ y^5-x^2y \\ y^9-x^4y \end{cases} & 2. \begin{cases} 3x^2y+3x \\ 3x^5y^4-3x \\ x^2y^2-2xy+1 \end{cases} & 3. \begin{cases} 9x^2y^2-4 \\ 3x^2y^2-2xy \\ 81x^5y^5-16xy \end{cases} & 4. \begin{cases} y^2-9x^2 \\ y^2+3xy-2y-6x \\ y^2-3xy-2y+6x \end{cases} \\
 5. \begin{cases} xy-3x^2 \\ 2xy-6x^2 \\ xy^2-5x^2y+6x^3 \end{cases} & 6. \begin{cases} xy^2+xy \\ y^3-xy^2+y^2-xy \\ y^4-xy^3+y^3-xy^2 \end{cases} & 7. \begin{cases} 3y^2+3xy-6x^2 \\ xy^2+4x^2y+4x^3 \\ 3y^3+9xy^2-12x^3 \end{cases} & 8. \begin{cases} y^2-x^2 \\ y^2-2xy+x^2 \\ xy^5-2x^3y^3+x^5y \end{cases} \\
 9. \begin{cases} 9y+18 \\ 9y+18x \\ 3y^2+6xy+6y+12x \end{cases} & 10. \begin{cases} 2y^3+4xy^2-6x^2y \\ 2y^3-4xy^2+2x^2y \\ y^3+xy^2-5x^2y+3x^3 \end{cases}
 \end{array}$$

23. Simplifica, al máximo, la fracción:

$$\begin{array}{llll}
 1. \frac{x^2+2x-3}{x^2+6x+9} & 2. \frac{9x^2-6x+1}{9x^2-1} & 3. \frac{3-x}{x^2-5x+6} & 4. \frac{4x^2+4x+1}{4x^2-1} \\
 5. \frac{x^2-9}{x^2+4x+3} & 6. \frac{2x^2-7x+6}{x^2-4x+4} & 7. \frac{x^2+4x+4}{-2x^2-8x-8} & 8. \frac{3x^2+11x+6}{x^2-9} \\
 9. \frac{4x^2-9}{4x^2-12x+9} & 10. \frac{3x^2+12x+9}{x^2+6x+9} & 11. \frac{x^2-4x+3}{x^2-9} & 12. \frac{3x^3-27x}{243x-3x^5} \\
 13. \frac{2x^2+6x}{9-x^2} & 14. \frac{3x^2+10x+3}{x^2+2x-3} & 15. \frac{2x^2+12x+18}{-x^2-6x-9} & 16. \frac{9x^2-15x-6}{3x^2-12x+12} \\
 17. \frac{12x^2-36x+27}{12x^2-27} & 18. \frac{x^2-4x+4}{x^3-3x^2+4} & 19. \frac{x^4+x^3-x^2-x}{x^4-2x^2+1} & 20. \frac{3x^3-6x^2}{3x^3-9x^2+6x} \\
 21. \frac{3x^3-5x^2-2x}{x^3-3x^2+2x} & 22. \frac{6x^2-3x^3}{3x^3-3x^2-6x} & 23. \frac{2x^3+7x^2+6x}{x^3+5x^2+6x} & 24. \frac{12x^2+32x-12}{4x^2-36} \\
 25. \frac{4x^3+4x^2}{2x^3+8x^2+6x} & 26. \frac{2x^4-2x^3-4x^2}{x^4-4x^2} & 27. \frac{6x^3+10x^2-4x}{2x^3+8x^2+8x} & 28. \frac{6x^3+18x^2+12x}{2x^3+8x^2+8x} \\
 29. \frac{6x+4x^2-2x^3}{x^3-5x^2+3x+9} & 30. \frac{x^3+4x^2-3x-18}{x^3-7x+6} & 31. \frac{6x^3+24x^2+18x}{2x^3+10x^2+12x} & 32. \frac{2x^3-18x}{x^3+3x^2-9x-27} \\
 33. \frac{2x^3+3x^2-8x+3}{x^3+3x^2-x-3} & 34. \frac{x^3-2x^2-9x+18}{x^3-7x^2+16x-12} & 35. \frac{x^3-5x^2+6x}{x^3-7x^2+16x-12} & 36. \frac{3x^3-x^2-20x-12}{x^3-4x^2-3x+18}
 \end{array}$$

24. Halla el valor numérico de la siguiente fracción:

$$\begin{array}{llll}
 1. \frac{x^4-x^3}{x^4-x^2}; x=1 & 2. \frac{3x}{x^2-3x}; x=0 & 3. \frac{3x+9}{x^2-9}; x=-3 & 4. \frac{6x^2-6x}{2x^2-6x}; x=1 \\
 5. \frac{27x+81}{9x^2-81}; x=-3 & 6. \frac{2x^2-6x}{x^2-9}; x=3 & 7. \frac{3x-3}{x^2-3x+2}; x=-1 & 8. \frac{2x^2-4x-6}{x^2-2x-3}; x=-1 \\
 9. \frac{x^2+x-6}{x^2-9}; x=-3 & 10. \frac{2x^2+x-1}{x^2-1}; x=-1 & 11. \frac{2x^2+x-3}{x^2-1}; x=-1 & 12. \frac{2x^2-5x-3}{x^2-2x-3}; x=3 \\
 13. \frac{9x^2+18x}{3x^2+12x+12}; x=-2 & 14. \frac{3x^2-12x+9}{3x^2+6x-9}; x=1 & 15. \frac{2x^3-5x^2+3x}{x^3+x^2-2x}; x=\frac{3}{2}
 \end{array}$$

25. Realiza la operación:

$$\begin{array}{llll}
 1. \frac{x-1}{x} + \frac{x^2+x-3}{x^2-3x} - 1 & 2. \frac{x^2+x+1}{x^2+x} + \frac{x-3}{3x} - \frac{1}{3} & 3. \frac{x^2-x-1}{x^2-x-6} - 1 + \frac{x-4}{x-3} & 4. \frac{x^2+x+6}{x^2+2x-3} + \frac{x-3}{x-1} - 1 \\
 5. \frac{x^2+x-10}{x^2-5x+6} - 2 + \frac{x-6}{x-2} & 6. \frac{x^2+4x-3}{x^2-4x+3} + \frac{x}{x-1} + 1 & 7. \frac{x}{x-2} + \frac{2}{3} - \frac{x+4}{6-3x} & 8. \frac{x^2+3x-4}{x^2-x-2} - 2 + \frac{x-1}{x+1}
 \end{array}$$

$$\begin{array}{llll}
9. \frac{x+4}{x+1} + \frac{x^2-2x-9}{x^2+4x+3} - 1 & 10. \frac{x^2+3x-8}{x^2-2x} + \frac{x-4}{x} - 2 & 11. \frac{x+3}{x-1} + 1 + \frac{x^2+x+6}{x^2-4x+3} & 12. \frac{2x+1}{4x+6} + \frac{2x+5}{8x+12} - \frac{1}{4} \\
13. \frac{x^2+x+30}{2x^2+4x} + \frac{x-15}{2x} + 2 & 14. 4 - \frac{x^2+x-6}{x^2+3x+2} - \frac{x+7}{x+1} & 15. \frac{x+1}{x^2-9} + \frac{7}{3x-9} - \frac{1}{3x+9} & 16. \frac{x^2+x-3}{x^2-5x+6} - \frac{x}{x-3} + \frac{x+1}{x-2} \\
17. \frac{2x-1}{2x^2+2x} + \frac{1}{2x} + \frac{1}{2x+2} & 18. 4 - \frac{x+5}{x+2} - \frac{x^2+x+1}{x^2+3x+2} & 19. \frac{x+6}{x+2} + \frac{x^2-2x-12}{x^2+5x+6} - 1 & 20. 1 + \frac{x^2+2x+9}{x^2+2x-3} - \frac{x+6}{-x-3} \\
21. \frac{x^2+2x+9}{4x-x^2-3} - \frac{x-15}{x-3} + 5 & 22. \frac{x^2+x+2}{4x^2-4} - \frac{x}{4x+4} + \frac{x+1}{x-1} & 23. \frac{5}{4} - \frac{2x-5}{4x-6} - \frac{4x^2+4x+9}{16x^2-36} & 24. \frac{x+3}{x+1} - \frac{x^2+x+4}{1-x^2} + \frac{x+1}{x-1} \\
25. \frac{x+17}{3x+6} - \frac{4}{3} - \frac{x^2+x-7}{-x^2-5x-6} & 26. \frac{x^2+x-18}{x^2-4} - \frac{8}{x+2} - \frac{x+1}{2-x} & 27. \frac{x^2-8x+1}{2x^2-2} - \frac{1}{2} - \frac{x^2+x+1}{1-x^2} & 28. \frac{x^2+x-5}{3x^2+9x+6} - \frac{x+6}{-3x-3} - \frac{2}{3} \\
29. \frac{x^2+x+1}{x^2-1} + \frac{2x+3}{2x+2} - \frac{2x-5}{2x-2} & 30. \frac{1}{2x-2} + \frac{2x+3}{2x+2} + \frac{x^2+x+1}{x^2-1} & 31. \frac{7}{2x} - \frac{3}{2x+4} + \frac{x^2+x+7}{x^3+x^2-2x} & 32. -\frac{3}{-x-3} - \frac{x}{x^2+5x+6} - \frac{1}{x+2}
\end{array}$$

26. Realiza la operación:

$$\begin{array}{lll}
1. \frac{x^2-1}{x^2+x-2} : \frac{x+1}{x+2} & 2. \frac{3x^2-3x}{x^2+3x+2} : \frac{x-1}{x+2} & 3. \frac{12x^2-6x}{x^4-9x^2} : \frac{x^2+3x}{4x-2} \\
4. \frac{4x^2-4}{x^3-7x-6} : \frac{x-3}{2} & 5. \left(\frac{12x^2-6x}{18x^2-9x} \right)^{-2} \frac{4x}{27} & 6. \left(\frac{x^2+x}{2x^2+2x} \right)^{-2} : 8 \\
7. \left(\frac{2x^2-2x}{3x^2-3x} \right)^2 \frac{27x}{4x+12} & 8. \frac{6x+6}{4x^2+6x} : \frac{3x+3}{2x^2+3x} & 9. \frac{2x^2-2x}{4x^4-16x^2} : \frac{4x^3-8x^2}{x-1} \\
10. \frac{4x^2-6x}{x^3-6x^2+9x} : \frac{4x-6}{x-3} & 11. \left(\frac{18x-12}{6x^2-4x} \right)^2 \frac{x^3-2x^2}{27} & 12. \frac{12x^2+6x}{12x+12} : \frac{2x+1}{3x+3} \\
13. \frac{2x^2-4x+2}{3x^2+12x+12} : \frac{3x+6}{2x-2} & 14. \frac{4x^2-36}{x^3-x^2-2x} : \frac{2x^2-18}{x^2-2x} & 15. \frac{3x+9}{3x^2-18x+27} : \frac{x-3}{x+3} \\
16. \frac{9x^2+36x+36}{2x^2-12x+18} : \frac{2x-6}{9x+18} & 17. \frac{9x^4-18x^3+9x^2}{9x^2-81} : \frac{x^3-2x^2+x}{3x+9} & 18. \frac{3x^2-12x+12}{x^2-4} \left(\frac{3x}{x^2-5x+6} : \frac{3x}{x-3} \right) \\
19. \left(\frac{2x^2+6x}{x^2+6x+9} \right)^2 \frac{(x+3)^2}{4x^2} & 20. \left(\frac{3x+3}{x^2+2x+1} \right)^{-2} : \frac{x^3+x^2-x-1}{18x} & 21. \frac{x^2-4}{2x-2} \left(\frac{2}{x^2-5x+6} : \frac{1}{x-3} \right) \\
22. \left(\frac{2x^2}{2x^2-6x} \right)^{-2} \frac{2x^3-x^2}{x^3-9x^2+27x-27} & 23. \left(\frac{8x^3-24x^2+18x}{16x^2-48x+36} \right)^{-2} \frac{x^3-2x^2}{4x+8} & 24. \frac{9x^2-1}{x^2-4} : \left(\frac{9x^2-3x}{x^2-x-6} : \frac{3x}{x-3} \right) \\
25. \frac{4x^2-8x}{x^3-4x} : \left(\frac{x^2-2x}{x^2+5x+6} : \frac{x^2}{2x+6} \right) & 26. \frac{x-3}{2x^2-6x} : \frac{6x}{x^2-6x+9} : \frac{3x}{x^2-3x} & 27. \left(\frac{x+2}{x^2+4x+4} \right)^2 \left(\frac{2}{x-2} : \frac{6x^2+6x-12}{x-2} \right) \\
28. \left(\frac{3x^3-12x^2+9x}{3x^3+6x^2-9x} \right)^2 \frac{x^3+4x^2-3x-18}{x^3-5x^2+3x+9} & 29. \left(\frac{x^2+6x+9}{x^3+7x^2+15x+9} \right)^2 \frac{2x^3+7x^2+8x+3}{x-1} & 30. \left(\frac{2x}{2x-2} \right)^{-2} : \left(\frac{x^3+2x^2-3x}{x^2+2x} : \frac{3x^3-2x^2}{x^2+x-2} \right)
\end{array}$$

27. Realiza la operación:

$$\begin{array}{lll}
1. \left(\frac{x}{x-2} + \frac{x-2}{x+2} + 1 \right) \frac{x+2}{x} & 2. \left(\frac{1}{x} - \frac{1}{2x} - \frac{1}{2x+4} \right) : \frac{1}{2x} & 3. \left(\frac{x^2+x-8}{x^2-4} - 1 + \frac{x-2}{x+2} \right) \frac{x+2}{x-3} \\
4. \left(\frac{x-12}{x+1} + 2 + \frac{x^2+x+26}{x^2+4x+3} \right) \frac{1}{2} & 5. \left(4 - \frac{x-4}{x-3} - \frac{x^2+x-22}{x^2-9} \right) \frac{x-3}{x-1} & 6. \left(\frac{x+4}{x-1} + \frac{x^2+x+12}{x^2-3x+2} - 2 \right) : \frac{3}{x-2}
\end{array}$$

$$\begin{aligned}
7. & \left(\frac{x+8}{x+1} + \frac{x^2+x+21}{x^2-x-2} + 7 \right) : \frac{3x+3}{x-2} & 8. & \left(\frac{4x+9}{x+1} - \frac{x^2+x-10}{x^2-1} - \frac{x+1}{x-1} \right) \frac{x-1}{2x} & 9. & \left(\frac{x^2+x+16}{x^2+4x+3} + \frac{2x-15}{x+3} + 1 \right) \frac{x+3}{2x-2} \\
10. & \left(\frac{x-12}{x-1} + 1 + \frac{x^2+x+39}{x^2+2x-3} \right) \left(2 - \frac{x-3}{x-2} \right) & 11. & \left(\frac{x+1}{x+2} + \frac{x^2+x+14}{x^2-4} - \frac{6}{x-2} \right) : \frac{2x-6}{x+2} & 12. & \left(\frac{x+22}{4x-4} + \frac{11}{2} + \frac{x}{x-1} \right) \left(\frac{8}{9x+9} - \frac{4}{9x} \right) \\
13. & \left(\frac{11}{x+3} - \frac{7}{x+1} - \frac{x-1}{x^2+4x+3} \right) \left(2 - \frac{x-7}{x-3} \right) & 14. & \left(\frac{13}{5x+10} + \frac{2}{5x-15} + \frac{x+1}{x^2-x-6} \right) \frac{x+2}{2} & 15. & \left(3 + \frac{x^2+x+3}{2x^2-4x} + \frac{2x-3}{2x} \right) : \left(\frac{1}{2} + \frac{2x-3}{2x} \right) \\
16. & \left(\frac{x}{x-2} - 1 \right) : \frac{1}{x-2} - \frac{x+1}{x+2} \left(2 - \frac{x-2}{x+1} \right) & 17. & \left(\frac{x+2}{x+1} - 1 \right) : \frac{1}{x+1} + \frac{x+3}{x-3} \left(2 - \frac{x+1}{x+3} \right) & 18. & \left(\frac{x+4}{x+3} - 1 \right) (x+3) + \left(2 + \frac{x+7}{-x-2} \right) : \frac{x+3}{x+2} \\
19. & \left(\frac{4}{3} + \frac{x-11}{3x+9} \right) \frac{x+3}{x-1} + \frac{x-3}{3} \left(-1 - \frac{x-2}{3-x} \right) & 20. & \left(1 - \frac{x+7}{3-x} \right) : \frac{x+1}{x-3} + \frac{x-2}{x-1} : \left(2 - \frac{x^2+3x+10}{x^2+x+4} \right) & 21. & \left(\frac{x+5}{x+1} + \frac{x^2+2x+3}{x^2-1} - 2 \right) : \left(\frac{x-1}{x+1} + 1 \right) \\
22. & \left(\frac{14x-3}{2x+2} + \frac{2x-5}{2x-2} + \frac{x^2+x+1}{x^2-1} \right) : \frac{3x-3}{x+1} & 23. & \left(\frac{5}{-6x-18} + \frac{6x-7}{6x-18} + \frac{x^2+x+1}{x^2-9} \right) \frac{x+3}{x^2+x} & 24. & \left(\frac{x^2+x+6}{x-3} - 8+4x + \frac{x^2+x+30}{x+3} \right) \frac{x+3}{2x^2-2x} \\
25. & \left(\frac{2x^2+3x-2}{6x^2-6x} + \frac{2x+13}{6x+18} - \frac{2}{3} \right) \left(2 + \frac{x^2+x-2}{x+1} \right) & 26. & \left(\frac{x^2-2x-21}{x^2+3x} - 2 + \frac{x+7}{x} \right) \frac{x^2+6x+9}{4} & 27. & \left(\frac{x-5}{x} + 1 + \frac{x^2+3x+5}{x^2+x} \right) \frac{x^3+x^2-x-1}{9x^3+18x^2} \\
28. & \left(3 + \frac{x^2-x+1}{3x^2+3x} - \frac{x+1}{3x} \right)^2 : \frac{9x^2+12x+4}{3x^2+6x+3} & 29. & \left(4 - \frac{x^2+x-48}{x^2-9} - \frac{x-4}{x+3} \right)^2 : \frac{4x^2+8x}{3x^2-18x+27} & 30. & \left(\frac{x+1}{x-1} - \frac{x^2+x-6}{1-x^2} - \frac{6}{x+1} \right)^{-2} : \frac{x^3-3x-2}{8x^2-8x+2}
\end{aligned}$$

28. Realiza la operación:

$$\begin{aligned}
1. & \frac{x+2}{x} - \frac{xy-2x^2-6}{x^2y+3x} - 1 & 2. & \frac{y-9x^2+1}{6x^2y+6x^2} + 1 - \frac{3x^2+1}{6x^2} \\
3. & \frac{y^2+y-x^2}{y^2-x^2} + \frac{2y+2x-1}{2y+2x} - 2 & 4. & 4 - \frac{y^2+y-3x^2-x}{y^2-xy} - \frac{y-3x-1}{y} \\
5. & 4 - \frac{y^2+y-9x-18}{y^2-y-2} - \frac{y-3x-5}{y+1} & 6. & \frac{1}{y+1} + \frac{y+x+2}{y^2-xy+y-x} - \frac{1}{y-x} \\
7. & \frac{y+2x-4}{y+3x} + \frac{y^2+y-10x^2-x}{y^2+5xy+6x^2} + 1 & 8. & \frac{y+3x-2}{y-1} - 2 + \frac{y^2+y-9x^2-1}{y^2+3xy-y-3x} \\
9. & \frac{y^2+y+4x-24}{y^2-xy-3y+3x} - \frac{y-x+8}{y-x} + \frac{y+1}{y-3} & 10. & 4 - \frac{y^2+y-27x^2+9x}{y^2+3xy+y+3x} - \frac{y+9x+1}{y+1} \\
11. & \left(1 - \frac{x-1}{x} \right) \frac{x}{y+3x} + \frac{x}{y-3x} \left(\frac{2}{x} - \frac{y-3x}{xy+3x^2} \right) & 12. & \left(\frac{y-2}{2y} + \frac{y^2+y-x}{y^2-xy} - \frac{1}{2} \right)^2 : \frac{3y^3}{xy^2-2x^2y+x^3} \\
13. & \left(y+1 - \frac{xy-y-3x+2}{x} \right) : \frac{y-2}{x} + 2x \left(\frac{2x-3}{2x} + \frac{1}{x} \right) & 14. & \left(1 + \frac{y^2+y+39x^2+3x}{y^2+3xy+2y+6x} + \frac{y-14x-3}{y+2} \right) : \frac{y-3x}{y+2} \\
15. & \left(\frac{y^2+y+x^2-6}{y^2-xy+2y-2x} + \frac{y+x}{y+2} + \frac{y+1}{y-x} \right)^2 : \frac{9y^2-12y+4}{y-x} & 16. & \left(\frac{3xy+y-15x-9}{3xy+y+9x^2+3x} + \frac{3xy+y-3x+3}{3xy+y-3x-1} - 1 \right) \left(2 - \frac{y-5}{y-3} \right)
\end{aligned}$$

— Soluciones —

$$\begin{aligned}
1.1. & \frac{-1}{27} & 1.2. & \frac{2}{9x^7} & 1.3. & \frac{-x^3}{4} & 1.4. & \frac{-x^3}{18} & 1.5. & \frac{-x^{11}}{2} & 1.6. & \frac{-9x^2}{2} & 1.7. & \frac{-x^{11}}{3} & 1.8. & \frac{16x^7}{27} & 1.9. & \frac{-4x^4}{81} & 1.10. & 2x & 1.11. & \frac{x^7}{4} & 1.12. & \frac{x^7}{8} & 1.13. & \frac{4x^3}{9} & 1.14. & \frac{3}{16x^6} & 1.15. & \\
\frac{-1}{72x^{12}} & 1.16. & \frac{1}{243x^4} & 1.17. & \frac{-1}{8x^6} & 1.18. & \frac{-8x^6}{9} & 1.19. & \frac{9}{32} & 1.20. & \frac{9}{8x^8} & 2.1. & \frac{81y^3}{2x^3} & 2.2. & 3xy^6 & 2.3. & \frac{-3y^2}{8} & 2.4. & 4x^2y^3 & 2.5. & \frac{32y^7}{3} & 2.6. & \frac{-x^4}{324y^3} & 2.7. & \frac{3x^6}{4y^2} & 2.8. & \\
\frac{-243y^2}{4} & 2.9. & -6x^2y^4 & 2.10. & \frac{256y^{12}}{9x} & 2.11. & \frac{3}{16x^3y^{11}} & 2.12. & \frac{-3x^2}{y^4} & 3.1. & -11 & 3.2. & -39 & 3.3. & -3 & 3.4. & 16 & 3.5. & -24 & 3.6. & -12 & 3.7. & 25 & 3.8. & -9 & 3.9. & -27 & 3.10. & 42 \\
3.11. & 3 & 3.12. & -15 & 3.13. & -10 & 3.14. & -7 & 3.15. & -34 & 4.1. & 7x^2-2x-1 & 4.2. & -9x-4 & 4.3. & 4x-3 & 4.4. & -x^3-8x-5 & 4.5. & 4x^2+5x-8 & 4.6. & 2x^3-2x^2-4x+1 & 4.7. & -2x^2+3x-5
\end{aligned}$$

4.8. $-4x^3+3x^2+7x$ **4.9.** $-x^3-3x^2-7x+3$ **4.10.** $-4x^2+x+4$ **4.11.** x^3-x^2-7x+4 **4.12.** $-3x^3+2x^2+3x-1$ **5.1.** $4x^3-2x^2-6x$ **5.2.** $-4x^4-8x^3+4x^2+5x-6$ **5.3.** $-2x^4-2x^3+x^2-2x+3$ **5.4.** $-12x^4-6x^3+18x^2+12x$ **5.5.** $-6x^6+3x^5+2x^4-9x^3+4x^2$ **5.6.** $3x^5+11x^4-5x^3-17x^2-4x$ **5.7.** $9x^2-4$ **5.8.** $16x^4-1$ **5.9.** x^6-4x^4 **5.10.** $9x^2-x^4$ **5.11.** $-4x^4+9$ **5.12.** $4x^4-12x^2+9$ **5.13.** $x^4-4x^3+4x^2$ **5.14.** $4x^2+12x+9$ **5.15.** $9x^6-12x^4+4x^2$ **5.16.** $9x^6-30x^5+25x^4$ **5.17.** $x^4 - \frac{1}{9}$ **5.18.** $\frac{1}{4}x^2 - \frac{4}{9}$ **5.19.** $4x^4-6x^2+\frac{9}{4}$

5.20. $\frac{4}{9}x^2 - \frac{2}{3}x + \frac{1}{4}$ **6.1.** x^3+2x^2-6x+9 **6.2.** $-2x^2-3x-4$ **6.3.** $5x^2-6x$ **6.4.** $-3x^5+4x^3-x^2-3x-1$ **6.5.** $-5x^2-18x-13$ **6.6.** $-4x^4-8x^2+6x-1$ **6.7.** $-2x^3+5x^2-2x-9$ **6.8.** $18x^2+10x+3$ **6.9.** $3x^3-18x^2+12x+5$ **6.10.** $2x^3+5x^2-2x-9$ **6.11.** x^4-8x^2+6x **6.12.** $-4x^3+5x^2-x-1$ **6.13.** $x^5-x^3+6x^2-2x-5$ **6.14.** $6x^4-24x^3+15x^2-x$ **6.15.** $3x^3-8x^2-x+1$ **6.16.** $8x^8+24x^6+18x^4+2x^2-18$ **6.17.** $12x^5-24x^4+20x^3-2x$ **6.18.** $-3x^4+9x^3-9x+2$ **6.19.** $x^4-2x^3+6x^2-x+5$ **6.20.** $6x^6+4x^5-2x^4+4x^2-9$ **6.21.** $-3x^4-6x^3-6x^2+11x$ **6.22.** $-4x^5-13x^4+x^2$ **6.23.** $2x^4-4x^3+12x^2+6x-13$ **6.24.** $-2x^4-5x^3-x^2-9x+3$ **6.25.** $-2x^4+5x^3-9x^2-3x+4$ **6.26.** $-3x^4+8x^3-5x^2-6x+4$ **6.27.** $-6x^4+4x^3-10x^2-6x-4$ **6.28.** $4x^5+16x^4+16x^3$ **6.29.** $6x^4-2x^3+x+2$ **6.30.** $-10x^6+6x^4+11x^3+9x^2+3x$

7.1. $\begin{cases} C: -4x-2 \\ r: -1 \end{cases}$ **7.2.** $\begin{cases} C: -x+2 \\ r: 0 \end{cases}$ **7.3.** $C: -x^2+\frac{1}{3}x+\frac{2}{9}; r: \frac{-16}{9}$ **7.4.** $\begin{cases} C: -2x^2-2x-3 \\ r: 2 \end{cases}$ **7.5.** $\begin{cases} C: 3x+2 \\ r: 0 \end{cases}$ **7.6.** $\begin{cases} C: 3x^2-x+3 \\ r: -1 \end{cases}$ **7.7.** $C: -2x^3+2x^2-x+\frac{5}{2}; r: \frac{9}{2}$ **7.8.** $\begin{cases} C: -x^2+x-4 \\ r: x+2 \end{cases}$ **7.10.** $\begin{cases} C: -x^3-2x+4 \\ r: 3x-2 \end{cases}$ **7.11.** $\begin{cases} C: -x^2-4x-2 \\ r: -4x-4 \end{cases}$ **7.12.** $\begin{cases} C: -x^3-x^2+2 \\ r: -3 \end{cases}$ **8.1.** $\begin{cases} C: -3x^2-4x+2 \\ r: -4 \end{cases}$ **8.2.** $\begin{cases} C: x^3+3 \\ r: -7 \end{cases}$ **8.3.** $\begin{cases} C: -4x^2-4x-1 \\ r: -4 \end{cases}$ **8.4.** $\begin{cases} C: 2x^3-3x^2-1 \\ r: 4 \end{cases}$ **8.5.** $\begin{cases} C: -x^3-2x^2+3x+3 \\ r: 0 \end{cases}$ **8.6.** $\begin{cases} C: 2x^3-4x^2+4x-11 \\ r: 4 \end{cases}$ **8.7.** $\begin{cases} C: -2x^3+3x-1 \\ r: 0 \end{cases}$ **8.8.** $\begin{cases} C: 3x^3-2x^2-4x-4 \\ r: 2 \end{cases}$ **8.9.** $\begin{cases} C: -2x^3-2x-4 \\ r: -3 \end{cases}$ **8.10.** $\begin{cases} C: x^5-3x^3+x^2-3x \\ r: -1 \end{cases}$ **8.11.** $C: x^2-\frac{3}{4}; r: \frac{1}{2}$ **8.12.** $C: \frac{4}{3}x^3-\frac{1}{2}x; r: 0$ **9.** $\frac{-1}{3}$ **10.** 2 **11.** -1 **12.** $1, -1$ **13.** $\frac{-7}{2}, 1$ **14.** $\frac{-17}{8}, 2$ **15.** $\left(\frac{-3}{2}, -1\right)$ **16.** $(-21, 8)$ **17.** $\left(\frac{1}{2}, -3\right)$ **18.** $\left(-3, \frac{2}{3}\right)$

19.1. $-x(x+1)(x-1)$ **19.2.** $x(x-3)(x+3)$ **19.3.** $x(2x+1)(2x-1)$ **19.4.** $-x^2(x+1)(x-1)$ **19.5.** $x^2(x^2+1)(x+1)(x-1)$ **19.6.** $-3(x^2+1)(x+1)(x-1)$ **19.7.** $3(x^2+4)(x+2)(x-2)$ **19.8.** $-x(9x^2+1)(3x+1)(3x-1)$ **19.9.** $x^2(x^2+9)(x+3)(x-3)$ **19.10.** $-3(x^2+4)(x+2)(x-2)$ **19.11.** $2x^2(x^2+1)(x+1)(x-1)$ **19.12.** $x(2x+1)^2$ **19.13.** $2(3x-1)^2$ **19.14.** $3(2x-1)^2$ **19.15.** $3(x+3)^2$ **19.16.** $3(2x+3)^2$ **19.17.** $3x(x-3)^2$ **19.18.** $2x^2(2x+3)^2$ **19.19.** $3x(3x+2)^2$ **19.20.** $(x^2+1)^2(x+1)^2(x-1)^2$ **19.21.** $3(x-3)(x+1)$ **19.22.** $3(x-2)(x+1)$ **19.23.** $2(x+2)(x-1)$ **19.24.** $2(x+3)(x-1)$ **19.25.** $3x(x^2+9)(x+3)(x-3)$ **19.26.** $-x(x+1)^2(x-1)^2$ **19.27.** $x^2(x+1)^2(x-1)^2$ **19.28.** $2x^2(9x^2+1)(3x+1)(3x-1)$ **19.29.** $2(x+1)^2(x-1)^2$ **19.30.** $-3x^2(x^2+4)(x+2)(x-2)$ **19.31.** $(x+1)(x+2)(x-2)$ **19.32.** $(x+1)(x+3)(x-3)$ **19.33.** $(x-1)(x+2)(x-2)$ **19.34.** $2(x+2)(x+3)$ **19.35.** $x(x^2+2)^2(x^2-2)^2$ **19.36.** $3x(x-1)(x-2)$ **19.37.** $3x(x+1)^2(x-1)^2$ **19.38.** $x^2(x+1)(x+3)$ **19.39.** $2x(x+1)(x+3)$ **19.40.** $(x-1)(x-2)^2$ **19.41.** $(x+1)(x+2)^2$ **19.42.** $3x^2(x+1)^2(x-1)^2$ **19.43.** $3x^2(x+3)(x-1)$ **19.44.** $3(x+2)^2(x-2)^2$ **19.45.** $3x^2(x+2)(x+1)$ **19.46.** $(x^2+4)^2(x+2)^2(x-2)^2$ **19.47.** $(x-2)(x+3)(x-3)$ **19.48.** $3x^2(x+2)(x-3)$ **19.49.** $(x+1)(2x+3)(2x-3)$ **19.50.** $(x-1)(3x+2)(3x-2)$ **19.51.** $(x+1)(x+3)^2$ **19.52.** $2x(x+2)^2(x-2)^2$ **19.53.** $3(x^2+3)^2(x^2-3)^2$ **19.54.** $-x^2(x+3)^2(x-3)^2$ **19.55.** $(x-2)(x-3)^2$ **19.56.** $(x+3)(x+4)(x-4)$ **19.57.** $(x-2)(2x+3)(2x-3)$ **19.58.** $x(3x^2+2)^2(3x^2-2)^2$ **19.59.** $-3x(x+2)^2(x-2)^2$ **19.60.** $3x(x+3)^2(x-3)^2$ **20.1.** $y(x^2y^2+4)(xy+2)(xy-2)$ **20.2.** $2(4x^2y^2+1)(xy+1)(xy-1)$ **20.3.** $3y^2(y^2+x^2)(y+x)(y-x)$ **20.4.** $3xy^2(4y^2+x^2)(2y+x)(2y-x)$ **20.5.** $2x(y+x)(y+2x)$ **20.6.** $(y-1)(xy+2)(xy-2)$ **20.7.** $xy^2(xy+1)^2(xy-1)^2$ **20.8.** $(y^2+4x^2)^2(y+2x)^2(y-2x)^2$ **20.9.** $3xy(y+3)^2(y-3)^2$ **20.10.** $xy(y-2)(y+2x)$ **20.11.** $3x(2xy^2+3)^2(2xy^2-3)^2$ **20.12.** $(y-1)(y+2x)^2$ **21.1.** $1; 3x(3x-1)(3x+1)^2$ **21.2.** $1; (x-3)(x-4)(x+4)$ **21.3.** $1; 2(x-2)(x+2)^2$ **21.4.** $1; (x+1)(x-2)(x+2)$ **21.5.** $1; 3(x-1)^2(x+1)^2$ **21.6.** $x+1; 3(x-1)^2(x+1)^2$ **21.7.** $1; x^2(x+1)^2(x-1)^2$ **21.8.** $3; 9(x+1)(x+3)$ **21.9.** $2(x+1)(x-1); 2(x+1)^2(x-1)^2(x^2+1)$ **21.10.** $2; 4(x+2)(x+3)$ **21.11.** $x-3; 2(x+1)(x-3)^2$ **21.12.** $2; 9x^2(x+2)(x+3)$ **21.13.** $2x^2-3; 3(2x^2+3)(2x^2-3)^2$ **21.14.** $x+3; x^2(x-3)^2(x+3)^2$ **21.15.** $1; x^2(x+1)(x-2)(x+2)$ **21.16.** $x+1; 2x(x+1)(x-3)(x+3)$ **21.17.** $2x(x+1); 2x(x-1)(x^2+1)(x+1)^2$ **21.18.** $2x(x+1); 2x(x^2+1)(x-1)^2(x+1)^2$ **21.19.** $1; 2(2x-1)^2(2x+1)^2$ **21.20.** $x+2; 3(x+1)(x+2)^2$ **21.21.** $x-3; (x+2)(x+3)(x-3)$ **21.22.** $1; 2x^2(x^2+2)^2(x^2-2)^2$ **21.23.** $x+1; (x+1)(x-2)(x+2)$ **21.24.** $(x+2)(x-2); x^2(x+2)^2(x-2)^2(x^2+4)^2$ **21.25.** $x-1; x(x-1)(x-2)^2$ **21.26.** $x+1; 2x^2(x+1)^2(x-1)^2$ **21.27.** $x+1; (x-1)(x-3)(2x-1)(x+1)^2$ **21.28.** $1; 3x(x+3)(x-3)^2$ **21.29.** $3x; 3x^2(x+2)(x+3)$ **21.30.** $2x+1; (x+1)(x-1)(x-3)(2x+1)^2$ **22.1.** $(y^2+x)(y^2-x); y(y^2+x)(y^2-x)(y^4+x^2)$ **22.2.** $1; 3x(xy+1)(xy-1)^2(x^2y^2+1)$ **22.3.** $3xy-2; xy(3xy-2)(3xy+2)(9x^2y^2+4)$ **22.4.** $1; (y-2)(y+3x)(y-3x)$ **22.5.** $x(y-3x); 2x(y-2x)(y-3x)$ **22.6.** $y(y+1); xy^2(y+1)(y-x)$ **22.7.** $y+2x; 3x(y-x)(y+2x)^2$ **22.8.** $y-x; xy(y+x)^2(y-x)^2$ **22.9.** $3; 9(y+2)(y+2x)$ **22.10.** $y-x; 2y(y+3x)(y-x)^2$ **23.1.** $\frac{x-1}{x+3}$ **23.2.** $\frac{3x-1}{3x+1}$ **23.3.** $\frac{-1}{x-2}$ **23.4.** $\frac{2x+1}{2x-1}$ **23.5.** $\frac{x-3}{x+1}$ **23.6.** $\frac{2x-3}{x-2}$ **23.7.** $\frac{-1}{2}$ **23.8.** $\frac{3x+2}{x-3}$ **23.9.** $\frac{2x+3}{2x-3}$ **23.10.** $\frac{3x+3}{x+3}$ **23.11.** $\frac{x-1}{x+3}$ **23.12.** $\frac{-1}{x^2+9}$ **23.13.** $\frac{-2x}{x-3}$ **23.14.** $\frac{3x+1}{x-1}$ **23.15.** -2 **23.16.** $\frac{3x+1}{x-2}$ **23.17.** $\frac{2x-3}{2x+3}$ **23.18.** $\frac{1}{x+1}$ **23.19.** $\frac{x}{x-1}$ **23.20.** $\frac{x}{x-1}$ **23.21.** $\frac{3x+1}{x-1}$ **23.22.** $\frac{-x}{x+1}$ **23.23.** $\frac{2x+3}{x+3}$ **23.24.** $\frac{3x-1}{x-3}$ **23.25.** $\frac{2x}{x+3}$ **23.26.** $\frac{2x+2}{x+2}$ **23.27.** $\frac{3x-1}{x+2}$ **23.28.** $\frac{3x+3}{x+2}$ **23.29.** $\frac{-2x}{x-3}$ **23.30.** $\frac{x+3}{x-1}$ **23.31.** $\frac{3x+3}{x+2}$ **23.32.** $\frac{2x}{x+3}$ **23.33.** $\frac{2x-1}{x+1}$ **23.34.** $\frac{x+3}{x-2}$ **23.35.** $\frac{x}{x-2}$ **23.36.** $\frac{3x+2}{x-3}$ **24.1.** $\frac{1}{2}$ **24.2.** -1 **24.3.** $\frac{-1}{2}$ **24.4.** 0 **24.5.** $\frac{-1}{2}$ **24.6.** 1 **24.7.** -1 **24.8.** 2 **24.9.** $\frac{5}{6}$ **24.10.** $\frac{3}{2}$ **24.11.** no tiene **24.12.** $\frac{7}{4}$ **24.13.** no tiene **24.14.** $\frac{-1}{2}$ **24.15.** 0 **25.1.** $\frac{x}{x-3}$ **25.2.** $\frac{x}{x+1}$ **25.3.** $\frac{x+1}{x+2}$ **25.4.** $\frac{x}{x+3}$ **25.5.** $\frac{2}{x-3}$ **25.6.** $\frac{3x}{x-3}$ **25.7.** $\frac{2x}{x-2}$ **25.8.** $\frac{2}{x-2}$ **25.9.** $\frac{x}{x+3}$ **25.10.** $\frac{1}{x-2}$ **25.11.** $\frac{3x}{x-3}$ **25.12.** $\frac{x+1}{2x+3}$ **25.13.** $\frac{3x-2}{x+2}$ **25.14.** $\frac{2x}{x+2}$ **25.15.** $\frac{3}{x-3}$ **25.16.** $\frac{x+3}{x-3}$ **25.17.** $\frac{2}{x+1}$ **25.18.** $\frac{2x+1}{x+1}$ **25.19.** $\frac{x}{x+3}$ **25.20.** $\frac{3x}{x-1}$ **25.21.** $\frac{3x+3}{x-1}$ **25.22.** $\frac{2x+3}{2x-2}$ **25.23.** $\frac{x+2}{2x+3}$ **25.24.** $\frac{3x+2}{x-1}$ **25.25.** $\frac{1}{x+3}$ **25.26.** $\frac{2x}{x+2}$ **25.27.** $\frac{x-2}{x+1}$ **25.28.** $\frac{1}{x+2}$ **25.29.** $\frac{x+2}{x-1}$ **25.30.** $\frac{2x}{x-1}$ **25.31.** $\frac{3}{x-1}$ **25.32.** $\frac{1}{x+2}$ **26.1.** 1 **26.2.** $\frac{3x}{x+1}$ **26.3.** $\frac{3}{x-3}$ **26.4.** $\frac{2x-2}{x+2}$ **26.5.** $\frac{x}{3}$ **26.6.** $\frac{1}{2}$ **26.7.** $\frac{3x}{x+3}$ **26.8.** 1 **26.9.** $\frac{2x}{x+2}$ **26.10.** $\frac{1}{x-3}$ **26.11.** $\frac{x-2}{3}$ **26.12.** $\frac{3x}{2}$ **26.13.** $\frac{x-1}{x+2}$ **26.14.** $\frac{2}{x+1}$ **26.15.** $\frac{1}{x-3}$ **26.16.** $\frac{x+2}{x-3}$ **26.17.** $\frac{3x}{x-3}$ **26.18.** $\frac{3}{x+2}$ **26.19.** 1 **26.20.** $\frac{2x}{x-1}$ **26.21.** $\frac{x+2}{x-1}$ **26.22.** $\frac{2x-1}{x-3}$ **26.23.** $\frac{x-2}{x+2}$ **26.24.** $\frac{3x+1}{x-2}$ **26.25.** $\frac{2x}{x-2}$ **26.26.** $\frac{1}{x-3}$ **26.27.** $\frac{3x-3}{x+2}$ **26.28.** $\frac{x-2}{x+1}$ **26.29.** $\frac{2x+3}{x-1}$ **26.30.** $\frac{3x-2}{x+3}$ **27.1.** $\frac{3x-2}{x-2}$ **27.2.** $\frac{2}{x+2}$ **27.3.** $\frac{x}{x-2}$ **27.4.** $\frac{2x-2}{x+3}$ **27.5.** $\frac{2x+2}{x+3}$ **27.6.** $\frac{3x}{x-1}$ **27.7.** $\frac{3x-3}{x+1}$ **27.8.** 1 **27.9.** $\frac{2x-2}{x+1}$ **27.10.** $\frac{3x}{x+3}$ **27.11.** $\frac{x}{x-2}$ **27.12.** $\frac{3}{x+1}$ **27.13.** $\frac{3}{x+3}$ **27.14.** $\frac{2x-3}{x-3}$ **27.15.** $\frac{3x-3}{x-2}$ **27.16.** $\frac{x}{x+2}$ **27.17.** $\frac{2x+2}{x-3}$ **27.18.** $\frac{2x}{x+3}$ **27.19.** $\frac{2x}{x-1}$ **27.20.** $\frac{3x}{x-1}$ **27.21.** $\frac{3}{x-1}$ **27.22.** $\frac{3x}{x-1}$ **27.23.** $\frac{2}{x-3}$ **27.24.** $\frac{3x}{x-3}$ **27.25.** $\frac{2x-1}{x-1}$ **27.26.** 1 **27.27.** $\frac{x-1}{x+2}$ **27.28.** 3 **27.29.** $\frac{3x}{x+2}$ **27.30.** $\frac{2}{x-2}$ **28.1.** $\frac{y+2x}{xy+3}$ **28.2.** $\frac{y-2}{2y+2}$ **28.3.** $\frac{1}{2y-2x}$ **28.4.** $\frac{2y}{y-x}$ **28.5.** $\frac{2y+3x}{y-2}$ **28.6.** $\frac{1}{y-x}$ **28.7.** $\frac{3y-3}{y+2x}$ **28.8.** $\frac{1}{y+3x}$ **28.9.** $\frac{y}{y-x}$ **28.10.** $\frac{2y}{y+3x}$ **28.11.** $\frac{2}{y-3x}$ **28.12.** $\frac{x}{3y}$ **28.13.** $\frac{2xy}{y-2}$ **28.14.** $\frac{3y+x}{3x+y}$ **28.15.** $\frac{1}{y-x}$ **28.16.** $\frac{y-3}{y+3x}$