

En todos los cálculos se trabajará con GRADOS SEXAGESIMALES y he redondeado a la 2ª cifra decimal. Para hacerlo en radianes bastaría seleccionar el MODE RAD de la calculadora.

Problema 1: $\operatorname{sen} \alpha = \frac{2}{3}, \quad 0^\circ < \alpha < 90^\circ$

Problema 2: $\operatorname{sen} \alpha = \frac{1}{4}, \quad 90^\circ < \alpha < 180^\circ$

Problema 3: $\operatorname{sen} \alpha = -\frac{4}{5}, \quad 180^\circ < \alpha < 270^\circ$ **Problema 4:** $\operatorname{sen} \alpha = -\frac{2}{5}, \quad 270^\circ < \alpha < 360^\circ$

Problema 5: $\cos \alpha = \frac{3}{7}, \quad 0^\circ < \alpha < 90^\circ$

Problema 6: $\cos \alpha = -\frac{2}{5}, \quad 90^\circ < \alpha < 180^\circ$

Problema 7: $\cos \alpha = -\frac{1}{3}, \quad 180^\circ < \alpha < 270^\circ$ **Problema 8:** $\cos \alpha = \frac{4}{5}, \quad 270^\circ < \alpha < 360^\circ$

Problema 9: $\operatorname{tg} \alpha = 2, \quad 0^\circ < \alpha < 90^\circ$

Problema 10: $\operatorname{tg} \alpha = -3, \quad 90^\circ < \alpha < 180^\circ$

Problema 11: $\operatorname{tg} \alpha = \frac{1}{2}, \quad 180^\circ < \alpha < 270^\circ$ **Problema 12:** $\operatorname{tg} \alpha = -\frac{5}{3}, \quad 270^\circ < \alpha < 360^\circ$

SOLUCIONES

Problema 1: $\operatorname{sen} \alpha = \frac{2}{3}, \quad 0^\circ < \alpha < 90^\circ$

Con calculadora:

$$\alpha = \arcsen \frac{2}{3} \approx 41,81^\circ \Rightarrow \begin{cases} \cos 41,81^\circ \approx 0,75 \\ \operatorname{tg} 41,81^\circ \approx 0,89 \end{cases}$$

Sin calculadora:

$$\operatorname{sen}^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \left(\frac{2}{3}\right)^2 + \cos^2 \alpha = 1 \Leftrightarrow \cos^2 \alpha = 1 - \left(\frac{2}{3}\right)^2 = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\Rightarrow \cos \alpha = \pm \sqrt{\frac{5}{9}} = \pm \frac{\sqrt{5}}{3} \approx 0,75$$

$$\Rightarrow \operatorname{tg} \alpha = \frac{\operatorname{sen} \alpha}{\cos \alpha} = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \approx 0,89$$

(*) Se ha tomado la RAÍZ POSITIVA en el coseno porque α pertenece al primer cuadrante.

Problema 2: $\operatorname{sen} \alpha = \frac{1}{4}$, $90^\circ < \alpha < 180^\circ$

Con calculadora:

$$\arcsen \frac{1}{4} \approx 14,48^\circ \Rightarrow \alpha = 180^\circ - 14,48^\circ = 165,52^\circ \Rightarrow \begin{cases} \cos 165,52^\circ \approx -0,97 \\ \operatorname{tg} 165,52^\circ \approx -0,26 \end{cases}$$

Sin calculadora:

$$\operatorname{sen}^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \left(\frac{1}{4}\right)^2 + \cos^2 \alpha = 1 \Leftrightarrow \cos^2 \alpha = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\Rightarrow \cos \alpha = \pm \sqrt{\frac{15}{16}} = \pm \frac{\sqrt{15}}{4} \approx -0,97$$

$$\Rightarrow \operatorname{tg} \alpha = \frac{\operatorname{sen} \alpha}{\cos \alpha} = \frac{\frac{1}{4}}{-\frac{\sqrt{15}}{4}} = -\frac{1}{\sqrt{15}} = -\frac{\sqrt{15}}{15} \approx -0,26$$

(*) Se ha tomado la RAÍZ NEGATIVA en el coseno porque α pertenece al segundo cuadrante.

Problema 3: $\operatorname{sen} \alpha = -\frac{4}{5}$, $180^\circ < \alpha < 270^\circ$

Con calculadora:

$$\arcsen \frac{-4}{5} \approx -53,13^\circ \Rightarrow \alpha = 180^\circ - (-53,13^\circ) = 233,13^\circ \Rightarrow \begin{cases} \cos 233,13^\circ \approx -0,60 \\ \operatorname{tg} 233,13^\circ \approx 1,33 \end{cases}$$

Sin calculadora:

$$\operatorname{sen}^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \left(-\frac{4}{5}\right)^2 + \cos^2 \alpha = 1 \Leftrightarrow \cos^2 \alpha = 1 - \left(-\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\Rightarrow \cos \alpha = \pm \sqrt{\frac{9}{25}} = \pm \frac{3}{5} = -0,60$$

$$\Rightarrow \operatorname{tg} \alpha = \frac{\operatorname{sen} \alpha}{\cos \alpha} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3} \approx 1,33$$

(*) Se ha tomado la RAÍZ NEGATIVA en el coseno porque α pertenece al tercer cuadrante.

Problema 4: $\operatorname{sen} \alpha = -\frac{2}{5}$, $270^\circ < \alpha < 360^\circ$

Con calculadora:

$$\operatorname{arcsen} \frac{-2}{5} \approx -23,58^\circ = -23,58^\circ + 360^\circ = 336,42^\circ \Rightarrow \begin{cases} \cos 336,42^\circ \approx 0,92 \\ \operatorname{tg} 336,42^\circ \approx -0,44 \end{cases}$$

Sin calculadora:

$$\operatorname{sen}^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \left(-\frac{2}{5}\right)^2 + \cos^2 \alpha = 1 \Leftrightarrow \cos^2 \alpha = 1 - \left(-\frac{2}{5}\right)^2 = 1 - \frac{4}{25} = \frac{21}{25}$$

$$\Rightarrow \cos \alpha = \pm \sqrt{\frac{21}{25}} = \frac{\sqrt{21}}{5} \approx 0,92$$

$$\operatorname{tg} \alpha = \frac{\operatorname{sen} \alpha}{\cos \alpha} = \frac{-\frac{2}{5}}{\frac{\sqrt{21}}{5}} \stackrel{(*)}{=} -\frac{2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21} \approx -0,44$$

(*) Se ha tomado la RAÍZ NEGATIVA en el coseno porque α pertenece al tercer cuadrante.

Problema 5: $\cos \alpha = \frac{3}{7}$, $0^\circ < \alpha < 90^\circ$

Con calculadora:

$$\alpha = \arccos \frac{3}{7} \approx 64,62^\circ \Rightarrow \begin{cases} \operatorname{sen} 64,62^\circ \approx 0,90 \\ \operatorname{tg} 64,62^\circ \approx 2,11 \end{cases}$$

Sin calculadora:

$$\cos^2 \alpha + \operatorname{sen}^2 \alpha = 1 \Rightarrow \left(\frac{3}{7}\right)^2 + \operatorname{sen}^2 \alpha = 1 \Leftrightarrow \operatorname{sen}^2 \alpha = 1 - \left(\frac{3}{7}\right)^2 = 1 - \frac{9}{49} = \frac{40}{49}$$

$$\Rightarrow \operatorname{sen} \alpha = \pm \sqrt{\frac{40}{49}} \stackrel{(*)}{=} +\frac{\sqrt{40}}{7} \approx 0,90$$

$$\Rightarrow \operatorname{tg} \alpha = \frac{\operatorname{sen} \alpha}{\cos \alpha} = \frac{\frac{\sqrt{40}}{7}}{\frac{3}{7}} = \frac{\sqrt{40}}{3} \approx 2,11$$

(*) Se ha tomado la RAÍZ POSITIVA en el seno porque α pertenece al primer cuadrante.

Problema 6: $\cos \alpha = -\frac{2}{5}$, $90^\circ < \alpha < 180^\circ$

Con calculadora:

$$\alpha = \arccos -\frac{2}{5} \approx 113,58^\circ \Rightarrow \begin{cases} \text{sen} 113,58^\circ \approx 0,92 \\ \text{tg} 113,58^\circ \approx -2,29 \end{cases}$$

Sin calculadora:

$$\cos^2 \alpha + \sin^2 \alpha = 1 \Rightarrow \left(-\frac{2}{5}\right)^2 + \sin^2 \alpha = 1 \Leftrightarrow \sin^2 \alpha = 1 - \left(-\frac{2}{5}\right)^2 = 1 - \frac{4}{25} = \frac{21}{25}$$

$$\Rightarrow \sin \alpha = \pm \sqrt{\frac{21}{25}} = \pm \frac{\sqrt{21}}{5} \approx 0,92$$

$$\Rightarrow \text{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{\sqrt{21}}{5}}{-\frac{2}{5}} = -\frac{\sqrt{21}}{2} \approx -2,29$$

(*) Se ha tomado la RAÍZ POSITIVA en el seno porque α pertenece al segundo cuadrante.

Problema 7: $\cos \alpha = -\frac{1}{3}$, $180^\circ < \alpha < 270^\circ$

Con calculadora:

$$\arccos \left(-\frac{1}{3}\right) \approx 109,47^\circ \Rightarrow \alpha = 360 - 109,47^\circ = 250,53^\circ \begin{cases} \text{sen} 250,53^\circ \approx -0,94 \\ \text{tg} 250,53^\circ \approx 2,83 \end{cases}$$

Sin calculadora:

$$\cos^2 \alpha + \sin^2 \alpha = 1 \Rightarrow \left(-\frac{1}{3}\right)^2 + \sin^2 \alpha = 1 \Leftrightarrow \sin^2 \alpha = 1 - \left(-\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\Rightarrow \sin \alpha = \pm \sqrt{\frac{8}{9}} = \pm \frac{\sqrt{8}}{3} \approx -0,94$$

$$\Rightarrow \text{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{-\frac{\sqrt{8}}{3}}{-\frac{1}{3}} = \sqrt{8} \approx 2,83$$

(*) Se ha tomado la RAÍZ NEGATIVA en el seno porque α pertenece al tercer cuadrante.

Problema 8: $\cos \alpha = \frac{4}{5}$, $270^\circ < \alpha < 360^\circ$

Con calculadora:

$$\arccos \frac{4}{5} \approx 36,87^\circ \Rightarrow \alpha = 360 - 36,87^\circ = 323,13^\circ \begin{cases} \text{sen} 323,13^\circ \approx -0,60 \\ \text{tg} 323,13^\circ \approx -0,75 \end{cases}$$

Sin calculadora:

$$\cos^2 \alpha + \sin^2 \alpha = 1 \Rightarrow \left(\frac{4}{5}\right)^2 + \sin^2 \alpha = 1 \Leftrightarrow \sin^2 \alpha = 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\Rightarrow \sin \alpha = \pm \sqrt{\frac{9}{25}} \stackrel{(*)}{=} -\frac{3}{5} = -0,60$$

$$\Rightarrow \text{tg} \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4} = -0,75$$

(*) Se ha tomado la RAÍZ NEGATIVA en el seno porque α pertenece al cuarto cuadrante.

Problema 9: $\text{tg} \alpha = 2$, $0^\circ < \alpha < 90^\circ$

Con calculadora:

$$\alpha = \arctg 2 \approx 63,43^\circ \Rightarrow \begin{cases} \text{sen} 63,43^\circ \approx 0,89 \\ \cos 63,43^\circ \approx 0,45 \end{cases}$$

Sin calculadora:

$$\left. \begin{array}{l} \cos^2 \alpha + \sin^2 \alpha = 1 \\ \frac{\sin \alpha}{\cos \alpha} = 2 \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} \cos^2 \alpha + \sin^2 \alpha = 1 \\ \sin \alpha = 2 \cdot \cos \alpha \end{array} \right\} \Rightarrow \cos^2 \alpha + (2 \cdot \cos \alpha)^2 = 1 \Rightarrow 5 \cdot \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = \frac{1}{5} \Rightarrow \cos \alpha = \pm \sqrt{\frac{1}{5}} \stackrel{(*)}{=} +\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5} \approx 0,45$$

$$\Rightarrow \sin \alpha = 2 \cdot \frac{\sqrt{5}}{5} = \frac{2\sqrt{5}}{5} \approx 0,89$$

(*) Se ha tomado la RAÍZ POSITIVA en el coseno porque α pertenece al primer cuadrante.

Problema 10: $\text{tg} \alpha = -3$, $90^\circ < \alpha < 180^\circ$

Con calculadora:

$$\arctg(-3) \approx -71,57^\circ \Rightarrow \alpha = 180^\circ + (-71,57^\circ) = 108,43^\circ \Rightarrow \begin{cases} \text{sen} 108,43^\circ \approx 0,95 \\ \cos 108,43^\circ \approx -0,32 \end{cases}$$

Sin calculadora:

$$\left. \begin{array}{l} \cos^2 \alpha + \sin^2 \alpha = 1 \\ \frac{\sin \alpha}{\cos \alpha} = -3 \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} \cos^2 \alpha + \sin^2 \alpha = 1 \\ \sin \alpha = -3 \cdot \cos \alpha \end{array} \right\} \Rightarrow \cos^2 \alpha + (-3 \cdot \cos \alpha)^2 = 1 \Rightarrow 10 \cdot \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = \frac{1}{10} \Rightarrow \cos \alpha = \pm \sqrt{\frac{1}{10}} \stackrel{(*)}{=} -\frac{1}{\sqrt{10}} = -\frac{\sqrt{10}}{10} \approx -0,32$$

$$\Rightarrow \sin \alpha = -3 \cdot \frac{-\sqrt{10}}{10} = \frac{3\sqrt{10}}{10} \approx 0,95$$

(*) Se ha tomado la RAÍZ NEGATIVA en el coseno porque α pertenece al segundo cuadrante.

Problema 11: $\operatorname{tg} \alpha = \frac{1}{2}$, $180^\circ < \alpha < 270^\circ$

Con calculadora:

$$\operatorname{arc} \operatorname{tg} \frac{1}{2} \approx 26,57^\circ \Rightarrow \alpha = 180^\circ + 26,57^\circ = 206,57^\circ \Rightarrow \begin{cases} \operatorname{sen} 206,57^\circ \approx -0,45 \\ \cos 206,57^\circ \approx -0,89 \end{cases}$$

Sin calculadora:

$$\left. \begin{aligned} \cos^2 \alpha + \operatorname{sen}^2 \alpha &= 1 \\ \frac{\operatorname{sen} \alpha}{\cos \alpha} &= \frac{1}{2} \end{aligned} \right\} \Leftrightarrow \left. \begin{aligned} \cos^2 \alpha + \operatorname{sen}^2 \alpha &= 1 \\ \operatorname{sen} \alpha &= \frac{1}{2} \cdot \cos \alpha \end{aligned} \right\} \Rightarrow \cos^2 \alpha + \left(\frac{1}{2} \cdot \cos \alpha \right)^2 = 1 \Rightarrow \frac{5}{4} \cdot \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = \frac{4}{5} \Rightarrow \cos \alpha = \pm \sqrt{\frac{4}{5}} = \pm \frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5} \approx -0,89$$

$$\Rightarrow \operatorname{sen} \alpha = \frac{1}{2} \cdot \frac{-2\sqrt{5}}{5} = \frac{-\sqrt{5}}{5} \approx -0,45$$

(*) Se ha tomado la RAÍZ NEGATIVA en el coseno porque α pertenece al tercer cuadrante.

Problema 12: $\operatorname{tg} \alpha = -\frac{5}{3}$, $270^\circ < \alpha < 360^\circ$

Con calculadora:

$$\operatorname{arc} \operatorname{tg} \left(\frac{-5}{3} \right) \approx -59,04^\circ \Rightarrow \alpha = 360^\circ + (-59,04^\circ) = 300,96^\circ \Rightarrow \begin{cases} \operatorname{sen} 300,96^\circ \approx -0,86 \\ \cos 300,96^\circ \approx 0,51 \end{cases}$$

Sin calculadora:

$$\left. \begin{aligned} \cos^2 \alpha + \operatorname{sen}^2 \alpha &= 1 \\ \frac{\operatorname{sen} \alpha}{\cos \alpha} &= \frac{-5}{3} \end{aligned} \right\} \Leftrightarrow \left. \begin{aligned} \cos^2 \alpha + \operatorname{sen}^2 \alpha &= 1 \\ \operatorname{sen} \alpha &= \frac{-5}{3} \cdot \cos \alpha \end{aligned} \right\} \Rightarrow \cos^2 \alpha + \left(\frac{-5}{3} \cdot \cos \alpha \right)^2 = 1 \Rightarrow \frac{34}{9} \cdot \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = \frac{9}{34} \Rightarrow \cos \alpha = \pm \sqrt{\frac{9}{34}} = \pm \frac{3}{\sqrt{34}} = \frac{3\sqrt{34}}{34} \approx 0,51$$

$$\Rightarrow \operatorname{sen} \alpha = \frac{-5}{3} \cdot \frac{3\sqrt{34}}{34} = \frac{-5\sqrt{34}}{34} \approx -0,86$$

(*) Se ha tomado la RAÍZ POSITIVA en el coseno porque α pertenece al cuarto cuadrante.