

POTENCIAS Y RADICALES

Notas teóricas

- Operaciones con potencias:

I. $a^m : a^n = \frac{a^m}{a^n} = a^{m-n}$	VII. $a^{-1} = \frac{1}{a}$
II. $(a^m)^n = a^{m \cdot n}$	VIII. $a^{-b} = \frac{1}{a^b}$
III. $a^p \cdot b^p = (a \cdot b)^p$	IX. $\left(\frac{a}{b}\right)^{-1} = \frac{1}{\frac{a}{b}} = \frac{b}{a}$
IV. $(a^p \cdot b^q)^m = a^{p \cdot m} \cdot b^{q \cdot m}$	X. $\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \left(\frac{b}{a}\right)^n$
V. $a^0 = 1$	
VI. $a^1 = a$	

- Operaciones con radicales:

XI. $\sqrt{a} = a^{\frac{1}{2}}$	XIV. $\sqrt[n]{a^m} \cdot \sqrt[p]{a^q} = a^{\frac{m}{n}} \cdot a^{\frac{p}{q}} =$ $= \sqrt[nq]{a^{mq+np}} = \sqrt[nq]{a^{mq} \cdot a^{np}}$
XII. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$	
XIII. $\sqrt[n]{\sqrt[m]{a^p}} = \left(a^p\right)^{\frac{1}{m}}^{\frac{1}{n}} = a^{\frac{p}{mn}}$	

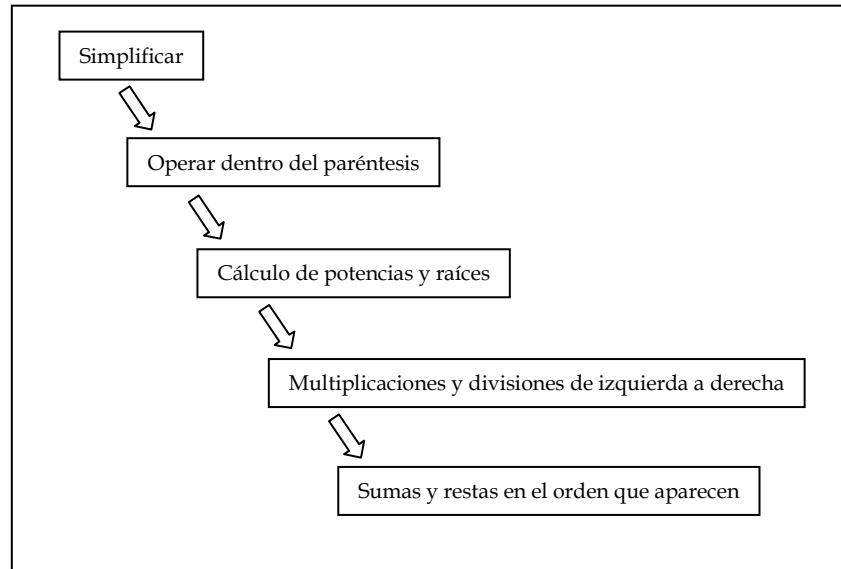
- Racionalizar:

Racionalizar es quitar del denominador las raíces. Se pueden presentar dos casos:

- a) En el denominador hay sólo una raíz. en este caso, la raíz se elimina multiplicando el numerador y el denominador el mismo número de veces que el radical de la raíz.

- b) En el denominador hay una raíz y otro término que la suma o la resta.
En este caso, las raíz o raíces se eliminan multiplicando el numerador y el denominador por el conjugado del denominador.

- La jerarquía que hay que seguir a la hora de operar con radicales :



Ejercicios resueltos

Opera con las siguientes potencias y raíces

$$1. \quad 16^{-2} \cdot 4^3 = (2^4)^{-2} \cdot (2^2)^3 = 2^{-8} \cdot 2^6 = 2^{-8+6} = 2^{-2} = \frac{1}{4}$$

$$2. \quad (7^2)^{-3} \cdot 7^3 = 7^{2 \cdot (-3)} \cdot 7^3 = 7^{-6} \cdot 7^3 = 7^{-6+3} = 7^{-3} = \frac{1}{7^3}$$

$$3. \quad (3^{-2} : 3^3) \cdot 3^{-2} = 3^{-2-3} \cdot 3^{-2} = 3^{-5} \cdot 3^{-2} = 3^{-5+(-2)} = 3^{-5-2} = 3^{-7} = \frac{1}{3^7}$$

$$4. \quad \frac{4^2 \cdot 12^3 \cdot 15^2}{9^3 \cdot 8^2 \cdot 3^3} = \frac{(2^2)^2 \cdot (2^2 \cdot 3)^3 \cdot (3 \cdot 5)^2}{(3^2)^3 \cdot (2^3)^2 \cdot 3^3} = \frac{2^4 \cdot 2^6 \cdot 3^3 \cdot 3^2 \cdot 5^2}{3^6 \cdot 2^6 \cdot 3^3} = \frac{2^{10} \cdot 3^5 \cdot 5^2}{2^6 \cdot 3^9} = \\ = 2^4 \cdot 3^{-4} \cdot 5^2$$

$$5. \quad \frac{8^4 \cdot 15^3 \cdot 18^2 \cdot 12^{-3}}{20^3 \cdot 27^2 \cdot 3^{-3}} = \frac{(2^3)^4 \cdot (3 \cdot 5)^3 \cdot (2 \cdot 3^2)^2 \cdot (2^2 \cdot 3)^{-3}}{(2^2 \cdot 5)^3 \cdot (3^3)^2 \cdot 3^{-3}} =$$

$$= \frac{2^{12} \cdot 3^3 \cdot 5^3 \cdot 2^2 \cdot 3^4 \cdot 2^{-6} \cdot 3^{-3}}{2^6 \cdot 5^3 \cdot 3^6 \cdot 3^{-3}} = \frac{2^8 \cdot 3^4 \cdot 5^3}{2^6 \cdot 3^3 \cdot 5^3} = 2^2 \cdot 3 = 12$$

$$6. \frac{27^{-1} \cdot 81 \cdot 3^4 \cdot \left(\frac{2^3}{3}\right)^{-1} \cdot 2^3}{36 \cdot \left(\frac{1}{3}\right)^{-2} \cdot \frac{4}{3} \cdot \frac{27}{16} \cdot (2^0)^{-2}} = \frac{(3^3)^{-1} \cdot 3^4 \cdot 3^4 \cdot \frac{3}{2^3} \cdot 2^3}{3^2 \cdot 2^2 \cdot 3^2 \cdot \frac{2^2}{3} \cdot \frac{3^3}{2^4} \cdot 1} = \frac{3^6}{3^6} = 1$$

$$7. \frac{(-27)^3 \cdot 32^{-5} \cdot (-8)^5 \cdot (25^2)^{-6}}{(-72)^4 \cdot (-50^3)^4} = \frac{(3^3)^3 \cdot (2^5)^{-5} \cdot (2^3)^5 \cdot (5^4)^{-6}}{(3^2 \cdot 2^3)^4 \cdot [(5^2 \cdot 2)^3]^4} =$$

$$= \frac{3^9 \cdot 2^{-25} \cdot 2^{15} \cdot 5^{-24}}{3^8 \cdot 2^{12} \cdot 5^{24} \cdot 2^{12}} = \frac{3}{2^{34} \cdot 5^{48}}$$

$$8. \quad 2^{\frac{3}{2}} \cdot 2^{\frac{1}{5}} = 2^{\frac{3}{2} + \frac{1}{5}} = 2^{\frac{3 \cdot 5 + 1 \cdot 2}{10}} = 2^{\frac{15 + 2}{10}} = 2^{\frac{15+2}{10}} = 2^{\frac{17}{10}} = 2^{\frac{17}{10}} = \sqrt[10]{2^{17}}$$

$$9. \quad \sqrt[3]{19^5} : \sqrt[4]{19^3} = 19^{\frac{5}{3}} : 19^{\frac{3}{4}} = 19^{\frac{5}{3} - \frac{3}{4}} = 19^{\frac{5 \cdot 4 - 3 \cdot 3}{12}} = 19^{\frac{20 - 9}{12}} = 19^{\frac{20-9}{12}} =$$

$$= 19^{\frac{11}{12}} = \sqrt[12]{19^{11}}$$

$$10. \quad \frac{5^5 \cdot 5^{\frac{1}{2}}}{\sqrt{5} \cdot 5^{-3}} = \frac{5^5 \cdot \cancel{\sqrt{5}}}{\cancel{\sqrt{5}} \cdot 5^{-3}} = 5^{5-(-3)} = 5^{5+3} = 5^8$$

$$11. \quad \frac{2^{\frac{1}{5}} \cdot 2^3 \cdot 2^{-\frac{1}{2}}}{2^3 \cdot 2^{\frac{25}{125}}} = \frac{\cancel{2^{\frac{1}{5}}} \cdot \cancel{2^3} \cdot 2^{-\frac{1}{2}}}{\cancel{2^3} \cdot \cancel{2^{\frac{1}{5}}}} = 2^{-\frac{1}{2}} = \frac{1}{2^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}$$

$$12. \quad \frac{2^{\frac{1}{2}} \cdot 2^{-\frac{1}{3}} \cdot 2^2}{2^2 \cdot 2^{\frac{1}{2}}} = 2^{-\frac{1}{3}} = \frac{1}{2^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{2}}$$

$$13. \quad \frac{\sqrt[4]{27}}{\sqrt[3]{18}} = \frac{\sqrt[4]{3^3}}{\sqrt[3]{2 \cdot 3^2}} = \sqrt[12]{\frac{(3^3)^3}{(2 \cdot 3^2)^4}} = \sqrt[12]{\frac{3^9}{2^4 \cdot 3^8}} = \sqrt[12]{\frac{3}{2^4}} = \sqrt[12]{\frac{3}{16}}$$

$$14. \quad \sqrt[4]{-80} : \sqrt[3]{18} = \frac{-\sqrt[4]{2^4 \cdot 5}}{\sqrt[3]{2 \cdot 3^2}} = -\frac{2\sqrt[4]{5}}{\sqrt[3]{2 \cdot 3^2}} = \frac{2\sqrt[4]{5^3}}{\sqrt[4]{(2 \cdot 3^2)^4}} = 2 \cdot \sqrt[4]{\frac{5^3}{2^4 \cdot 3^8}} =$$

$$= \frac{\cancel{2}}{\cancel{2} \cdot 3^2} \cdot \sqrt[4]{5^3} = \frac{\sqrt[4]{75}}{9}$$

$$15. \left(\sqrt[15]{-\frac{1}{243}} \right)^3 = \left(-\sqrt[15]{\frac{1}{3^5}} \right)^3 = -\sqrt[15]{\left(\frac{1}{3^5} \right)^3} = -\sqrt[5]{\frac{1}{3^{15}}} = -\frac{1}{3^3} = -\frac{1}{27}$$

$$16. \sqrt[3]{\sqrt{2}} \cdot \sqrt[3]{16} = \sqrt[6]{2} \cdot \sqrt[3]{16} = \sqrt[6]{2 \cdot 16^2} = \sqrt[6]{2 \cdot (2^4)^2} = \sqrt[6]{2^9} = \sqrt[6]{2^6 \cdot 2^3} = 2 \cdot \sqrt[6]{2^3} = 2 \cdot \sqrt{2}$$

$$17. \sqrt[3]{\sqrt{2}} \cdot \sqrt[3]{16} = \sqrt[6]{2} \cdot \sqrt[3]{16} = \sqrt[6]{2 \cdot 16^2} = \sqrt[6]{2 \cdot (2^4)^2} = \sqrt[6]{2^9} = \sqrt[6]{2^6 \cdot 2^3} = 2 \cdot \sqrt[6]{2^3} = 2 \cdot \sqrt{2}$$

$$18. \sqrt{\sqrt[3]{\sqrt[4]{64^4}}} = \sqrt{\sqrt[3]{\sqrt[4]{(2^6)^4}}} = \sqrt{\sqrt[3]{4} \sqrt[4]{2^6}} = \sqrt[2 \cdot 3 \cdot 4]{2^{24}} = \sqrt[24]{2^{24}} = 2$$

$$19. \sqrt{\frac{3\sqrt{2}}{8}} = \sqrt{\frac{\sqrt{3^2 \cdot 2}}{2 \cdot 2^2}} = \frac{1}{2} \sqrt{\sqrt{\frac{3^2 \cdot 2}{2^2}}} = \frac{1}{2} \sqrt[4]{\frac{3^2 \cdot 2}{2^2}} = \frac{1}{2} \sqrt[4]{\frac{9}{2}}$$

$$20. \frac{\left(\sqrt[4]{3^2} \right)^2 \cdot \left(\sqrt[3]{3} \right)^6}{\left(\sqrt[12]{3^4} \right)^6} = \frac{\sqrt[4]{3^4} \cdot \sqrt[3]{3^6}}{\sqrt[12]{3^{24}}} = \frac{3 \cdot 3^2}{3^2} = 3$$

$$21. \frac{\left(\sqrt[5]{3} \right)^4 \cdot \left(\sqrt[3]{3} \right)^2}{\left(\sqrt{3^4} \right)^3} = \frac{\sqrt[5]{3^4} \cdot \sqrt[3]{3^2}}{\sqrt{3^{12}}} = \frac{\sqrt[15]{(3^4)^3 \cdot (3^2)^5}}{3^2} = \frac{\sqrt[15]{3^{12} \cdot 3^{10}}}{3^2} = \frac{\sqrt[15]{3^{22}}}{3^2} = \sqrt[15]{\frac{3^{22}}{3^{30}}} = \sqrt[15]{\frac{1}{3^8}}$$

$$22. \frac{\left(\sqrt[4]{3^4} \right)^2 \cdot \sqrt[4]{\sqrt[5]{3^{25}}}}{\left[\sqrt[9]{\sqrt[5]{3}} \right]^{15} \cdot 3} = \frac{\left[(3^4)^{\frac{1}{4}} \right]^4 \cdot \left[(3^{25})^{\frac{1}{5}} \right]^4}{\left[\left(3^{\frac{1}{5}} \right)^{\frac{1}{9}} \right]^{15} \cdot 3} = \frac{3^{4 \cdot \frac{1}{4} \cdot 4} \cdot 3^{25 \cdot \frac{1}{5} \cdot 4}}{3^{\frac{1}{5} \cdot \frac{1}{9} \cdot 15} \cdot 3} = \frac{3^4 \cdot \cancel{3^4}}{\cancel{3^4} \cdot 3} = 3^5$$

$$23. \frac{\left(\sqrt[9]{2^3} \right)^2 \cdot 2}{\sqrt{\left(\sqrt[4]{2} \right)^4}} = \frac{\left(2^3 \right)^{\frac{2}{9}} \cdot 2}{\left(\left(2^{\frac{1}{4}} \right)^4 \right)^{\frac{1}{2}}} = \frac{2^{\frac{6}{9}} \cdot 2}{2^{\frac{1}{2}}} = \frac{2^{\frac{2}{3}+1}}{2^{\frac{1}{2}}} = \frac{2^{\frac{5}{3}}}{2^{\frac{1}{2}}} = 2^{\frac{5}{3} - \frac{1}{2}} = 2^{\frac{10-3}{6}} = 2^{\frac{7}{6}} = \sqrt[6]{2^7} = 2\sqrt[6]{2}$$

$$24. \frac{\left(\sqrt[4]{5^2}\right)^4 \cdot \sqrt[4]{5\sqrt{5^{20}}}}{\left[\sqrt[3]{5\sqrt{5}}\right]^{15} \cdot 25} = \frac{\left(\left(5^2\right)^{\frac{1}{4}}\right)^4 \cdot \left(\left(5^{20}\right)^{\frac{1}{5}}\right)^{\frac{1}{4}}}{\left[\left(5^{\frac{1}{5}}\right)^{\frac{1}{3}}\right]^{15} \cdot 5^2} = \frac{5^2 \cdot 5}{5 \cdot 5^2} = 1$$

$$25. \frac{\sqrt{\frac{a}{b}} \sqrt[3]{2a^{-2}} \sqrt{\frac{b^3}{a}}}{2\sqrt{ab^2}} = \frac{\sqrt[3]{2a^{-2}} \left(\frac{a}{b}\right)^3 \sqrt{\frac{b^3}{a}}}{\sqrt{4ab^2}} = \frac{\sqrt[3]{\left[2a^{-2} \left(\frac{a}{b}\right)^3\right]^2 \cdot \frac{b^3}{a}}}{\sqrt{4ab^2}} =$$

$$= \frac{\sqrt[12]{\left[2a^{-2} \left(\frac{a}{b}\right)^3\right]^2 \cdot \frac{b^3}{a}}}{\sqrt{4ab^2}} = \frac{\sqrt[12]{\frac{4a}{b^3}}}{2b\sqrt{a}} = \frac{1}{2b} \sqrt[12]{\frac{4a}{a^6b^3}} = \frac{1}{2b} \sqrt[12]{\frac{4}{a^5b^3}}$$

$$26. \sqrt{8} - \sqrt{50} - \frac{1}{2}\sqrt{98} = \sqrt{2^2 \cdot 2} - \sqrt{2 \cdot 5^2} - \frac{1}{2}\sqrt{7^2 \cdot 2} = 2\sqrt{2} - 5\sqrt{2} - 7\sqrt{2} = -10\sqrt{2}$$

$$27. \frac{1}{2}\sqrt{3} - \sqrt{12} - \frac{3}{4}\sqrt{75} = \frac{1}{2}\sqrt{3} - \sqrt{2^2 \cdot 3} - \frac{3}{4}\sqrt{5^2 \cdot 3} = \frac{1}{2}\sqrt{3} - 2\sqrt{3} - \frac{3 \cdot 5}{4}\sqrt{3} =$$

$$= \frac{1}{2}\sqrt{3} - 2\sqrt{3} - \frac{15}{4}\sqrt{3} = -\frac{21}{4}\sqrt{3}$$

$$28. \sqrt{9xy} + \frac{xy}{\sqrt{4xy}} + \frac{\sqrt[6]{(xy)^{21}}}{x^3y^3} = 3\sqrt{xy} - \frac{xy}{2\sqrt{xy}} - \sqrt[6]{\frac{(xy)^{21}}{(x^3y^3)^6}} = 3\sqrt{xy} - \frac{1}{2}\sqrt{\frac{(xy)^2}{xy}} - \sqrt[6]{(xy)^3} =$$

$$= 3\sqrt{xy} - \frac{1}{2}\sqrt{xy} - \sqrt{xy} = \frac{3}{2}\sqrt{xy}$$

$$29. \sqrt{256x^2y} + \frac{1}{3}\sqrt[4]{\frac{81y^2}{x^{-4}}} - x\sqrt{225y} = x \cdot \sqrt{2^8y} + \frac{3}{3}x \cdot \sqrt[4]{y^2} - x \cdot \sqrt{3^2 \cdot 5^2y} =$$

$$= 16x \cdot \sqrt{y} + x \cdot \sqrt{y} - 15x \cdot \sqrt{y} = 2x \cdot \sqrt{y}$$

Racionaliza

$$30. \frac{1}{2 \cdot \sqrt[3]{5}} = \frac{1}{2 \cdot \sqrt[3]{5}} \frac{\sqrt[3]{5} \sqrt[3]{5}}{\sqrt[3]{5} \sqrt[3]{5}} = \frac{\sqrt[3]{25}}{2 \cdot 5} = \frac{\sqrt[3]{25}}{10}$$

$$31. \frac{1}{\sqrt[5]{x^4}} = \frac{1}{\sqrt[5]{x^4}} \left(\frac{\sqrt[5]{x^4}}{\sqrt[5]{x^4}} \right)^4 = \frac{\left(\sqrt[5]{x^4} \right)^4}{\left(\sqrt[5]{x^4} \right)^5} = \frac{\left(x^4 \right)^{\frac{1}{5}}}{x^4} = \frac{x^{\frac{16}{5}}}{x^4} = \frac{\sqrt[5]{x^{16}}}{x^4} = \frac{\sqrt[5]{x^{15} \cdot x}}{x^4} = \frac{x^3 \cdot \sqrt[5]{x}}{x^4} = \frac{\sqrt[5]{x}}{x}$$

$$32. \frac{\sqrt[3]{x}}{\sqrt[6]{x^5}} = \frac{\sqrt[3]{x}}{\sqrt[6]{x^5}} \left(\frac{\sqrt[6]{x^5}}{\sqrt[6]{x^5}} \right)^5 = \frac{\sqrt[3]{x} \left(\sqrt[6]{x^5} \right)^5}{\left(\sqrt[6]{x^5} \right)^6} = \frac{x^{\frac{1}{3}} \left(x^5 \right)^{\frac{1}{6} \times 5}}{x^5} = \frac{x^{\frac{1}{3}} \cdot x^{\frac{25}{6}}}{x^5} = \frac{x^{\frac{2+25}{6}}}{x^5} = \frac{x^{\frac{27}{6}}}{x^5} = \frac{x^{\frac{27}{6}}}{x^5} = \frac{\sqrt[6]{x^{27}}}{x^5} = \frac{\sqrt[6]{x^3}}{x}$$

$$33. \frac{\sqrt{2}}{\sqrt{3}+1} = \frac{\sqrt{2} \cdot (\sqrt{3}-1)}{(\sqrt{3}+1) \cdot (\sqrt{3}-1)} = \frac{\sqrt{2} \cdot (\sqrt{3}-1)}{(\sqrt{3})^2 - 1^2} = \frac{\sqrt{2} \cdot (\sqrt{3}-1)}{3-1} = \frac{\sqrt{2} \cdot (\sqrt{3}-1)}{2}$$

$$34. \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \cdot \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})^2}{(\sqrt{2})^2 - (\sqrt{3})^2} = \frac{(\sqrt{2}+\sqrt{3})^2}{2-3} = -(\sqrt{2}+\sqrt{3})^2$$

$$35. \frac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}} = \frac{(2\sqrt{3}+\sqrt{2}) \cdot (2\sqrt{3}+\sqrt{2})}{(2\sqrt{3}-\sqrt{2}) \cdot (2\sqrt{3}+\sqrt{2})} = \frac{(2\sqrt{3})^2 + 2 \cdot 2\sqrt{3} + (\sqrt{2})^2}{(2\sqrt{3})^2 - (\sqrt{2})^2} = \frac{4 \cdot 3 + 4\sqrt{3} + 2}{4 \cdot 3 - 2} = \frac{7+2\sqrt{3}}{5}$$