

1º) Sin usar la calculadora, calcula:

$$a) \log_2 \frac{1}{32} = \boxed{-5}$$

$$b) \log_3 81 = \boxed{4}$$

$$c) \log^3 \sqrt{10} = \log 10^{1/3} = \frac{1}{3} \log 10 = \boxed{\frac{1}{3}}$$

$$d) \log_2 \sqrt{8} = \log_2 2^{3/2} = \boxed{\frac{3}{2}}$$

$$e) \log_2 128 = \boxed{7}$$

$$f) \log^3 \sqrt{100} = \log 10^{2/3} = \boxed{\frac{2}{3}}$$

2º) Sabiendo que $\log 2 = 0,301$ y $\log 7 = 0,845$, calcula:

$$a) \log 196 = \log (2^2 \cdot 7^2) = \log 2^2 + \log 7^2 = 2 \log 2 + 2 \log 7 = 2 \cdot 0,301 + 2 \cdot 0,845 = 0,602 + 1,69 = \boxed{2,292}$$

$$b) \log \sqrt{0,07} = \log \left(\frac{7}{100} \right)^{1/2} = \frac{1}{2} \log \left(\frac{7}{100} \right) = \frac{1}{2} (\log 7 - \log 100) = \frac{1}{2} (0,845 - 2) = \boxed{0,5725}$$

$$c) \log^3 \sqrt{\frac{16}{343}} = \frac{1}{3} \log \left(\frac{16}{343} \right) = \frac{1}{3} (\log 16 - \log 343) = \frac{1}{3} (\log 2^4 - \log 7^3) =$$

3º) Calcula el valor de x en las siguientes igualdades:

$$a) \log_x 121 = 2$$

$$\boxed{x = 11}$$

$$b) \log_2 4^3 = x$$

$$\boxed{x = 6}$$

$$c) \log e^3 = x$$

$$\frac{1}{3} (4 \log 2 - 3 \log 7) = \boxed{-0,44}$$

4º) Simplifica las siguientes expresiones a un solo logaritmo:

ERRATA

$$\ln e^3 = x \Rightarrow \boxed{x = 3}$$

$$a) 2 \log x + \log y + \log(m+n) - \log m - \log n =$$

$$\log x^2 + \log y + \log(m+n) - \log m - \log n = \log x^2 \cdot y \cdot (m+n) - \log m - \log n =$$

$$= \log \left(\frac{x^2 y \cdot (m+n)}{m \cdot n} \right) = \boxed{\log \left(\frac{x^2 y (m+n)}{m \cdot n} \right)}$$

$$b) 3 \log 2 + \log 5 + \log \frac{1}{25} - \log 4 =$$

$$\log 2^3 + \log 5 + \log \frac{1}{25} - \log 4 = \log (2^3 \cdot 5 \cdot \frac{1}{25}) - \log 4 = \log \left(\frac{8}{5} \right) - \log 4 =$$

5º) Calcula, utilizando la definición de logaritmo:

$$\log_2 64 + \log_2 \frac{1}{4} - \log_3 9 - \log_2 \sqrt{2} = 6 + (-2) - 2 - \frac{1}{2} =$$

$$= 4 - 2 - \frac{1}{2} = 2 - \frac{1}{2} = \boxed{\frac{3}{2}}$$

$$= \log \left(\frac{8}{5} \right) = \log \left(\frac{8}{20} \right) =$$

$$= \log \left(\frac{4}{10} \right) = \log \left(\frac{2}{5} \right)$$

6º) Halla:

$$a) \log \left(\frac{0,01 \cdot \sqrt[3]{100}}{10^{-1} \cdot 0,01} \right) = \log (0,01 \cdot \sqrt[3]{100}) - \log (10^{-1} \cdot 0,01) = \log 0,01 + \log \sqrt[3]{100} -$$

$$- (\log 10^{-1} + \log 0,01) = -2 + \frac{2}{3} - (-1 + (-2)) = -2 + \frac{2}{3} + 3 = 1 + \frac{2}{3} =$$

$$\boxed{\frac{5}{3}}$$

$$b) \log_2 \left(\frac{\sqrt{64} \cdot 2^3}{32 \cdot \sqrt{8}} \right) = \log_2 (\sqrt{64} \cdot 2^3) - \log_2 (32 \cdot \sqrt{8}) =$$

$$= \log_2 \sqrt{64} + \log_2 2^3 - (\log_2 32 + \log_2 \sqrt{8}) = \log_2 8 + 3 \log_2 2 - (\log_2 2^5 + \log_2 2^{3/2}) +$$

$$+ \log_2 2^{3/2} = 3 + 3 - (5 + \frac{3}{2}) = 6 - 5 + \frac{3}{2} = 1 + \frac{3}{2} = \boxed{\frac{5}{2}}$$