RESOLUCIÓN DE ECUACIONES IRRACIONALES

Resuelve las siguientes ecuaciones irracionales:

1) $\sqrt{1+\sqrt{x+3}}=2$

2) $\sqrt{x-\sqrt{2x-3}}=1$

3) $\sqrt{2+\sqrt{x-5}}=\sqrt{13-x}$

4) $\sqrt{2x-\sqrt{x+5}}=\sqrt{1+x}$

5) $\sqrt{12+\sqrt{x+7}}=\sqrt{25-x}$

6) $\sqrt{\sqrt{x+16}-\sqrt{x}}=2$

7) $\dfrac{\sqrt{x+2}}{\sqrt{x-1}}=\dfrac{4}{\sqrt{x+2}}$

8) $\sqrt{x-3}=\dfrac{\sqrt{x-1}}{\sqrt{x-3}}$

9) $\sqrt{x+4}-\sqrt{x-4}=\dfrac{x+1}{\sqrt{x+4}}$

10) $\sqrt{x+3}+\sqrt{x+6}=\dfrac{3}{\sqrt{x+3}}$

11) $\sqrt{2x+1}+2\sqrt{x}=\dfrac{21}{\sqrt{2x+1}}$

12) $\sqrt{2x+10}+\sqrt{3x}=\dfrac{28}{\sqrt{2x+10}}$

13) $\dfrac{\sqrt{4x+20}}{4+\sqrt{x}}=\dfrac{4-\sqrt{x}}{\sqrt{x}}$

14) $\dfrac{\sqrt{96+12x}}{4+\sqrt{x}}=\dfrac{4-\sqrt{x}}{\sqrt{x-3}}$

Soluciones:

1) $\sqrt{1+\sqrt{x+3}}=2$

$1+\sqrt{x+3}=4 \Rightarrow \sqrt{x+3}=3 \Rightarrow x+3=9 \Rightarrow \boxed{x=6}$

Comprobación:

Si x = 6 $\Rightarrow \sqrt{1+\sqrt{6+3}}=2 \Rightarrow \sqrt{1+3}=2$

2) $\sqrt{x-\sqrt{2x-3}}=1$

$x-\sqrt{2x-3}=1 \Rightarrow x-1=\sqrt{2x-3} \Rightarrow x^2-2x+1=2x-3 \Rightarrow x^2-4x+4=0 \Rightarrow (x-2)^2=0 \Rightarrow \boxed{x=2}$

Comprobación:

Si x = 2 $\Rightarrow \sqrt{2-\sqrt{4-3}}=\sqrt{2-1}=1$

3) $\sqrt{2+\sqrt{x-5}}=\sqrt{13-x}$

$2+\sqrt{x-5}=13-x \Rightarrow \sqrt{x-5}=11-x \Rightarrow x-5=(11-x)^2 \Rightarrow x-5=121-22x+x^2 \Rightarrow x^2-23x+126=0$

$$x=\frac{23\pm\sqrt{23^2-4\cdot 126}}{2}=\frac{23\pm 5}{2} \Rightarrow \begin{cases} x_1=\dfrac{23+5}{2}=14 \Rightarrow \boxed{\cancel{x=14}} \\ x_2=\dfrac{23-5}{2}=9 \Rightarrow \boxed{x=9} \end{cases}$$

Comprobación:

Si x = 14 $\Rightarrow \sqrt{2+\sqrt{14-5}}=\sqrt{13-14}$!!!

Si x = 9 $\Rightarrow \sqrt{2+\sqrt{9-5}}=\sqrt{13-9} \Rightarrow \sqrt{2+2}=\sqrt{4}$

4) $\sqrt{2x-\sqrt{x+5}}=\sqrt{1+x}$

$2x-\sqrt{x+5}=1+x \Rightarrow x-1=\sqrt{x+5} \Rightarrow (x-1)^2=x+5 \Rightarrow x^2-2x+1=x+5 \Rightarrow x^2-3x-4=0$

$$x=\frac{3\pm\sqrt{3^2-4\cdot(-4)}}{2}=\frac{3\pm 5}{2} \Rightarrow \begin{cases} x_1=\dfrac{3+5}{2}=4 \Rightarrow \boxed{x=4} \\ x_2=\dfrac{3-5}{2}=-1 \Rightarrow \boxed{\cancel{x=-1}} \end{cases}$$

Comprobación:

Si x = 4 $\Rightarrow \sqrt{8-\sqrt{4+5}}=\sqrt{1+4} \Rightarrow \sqrt{8-3}=\sqrt{5}$

Si x = -1 $\Rightarrow \sqrt{-2-\sqrt{-1+5}}=\sqrt{1-1} \Rightarrow \sqrt{-2-2}\neq 0$

5) $\sqrt{12+\sqrt{x+7}}=\sqrt{25-x}$

$12+\sqrt{x+7}=25-x \Rightarrow \sqrt{x+7}=13-x \Rightarrow x+7=169-26x+x^2 \Rightarrow x^2-27x+162=0$

$$x=\frac{27\pm\sqrt{27^2-4\cdot 162}}{2}=\frac{27\pm 9}{2} \Rightarrow \begin{cases} x_1=\dfrac{27+9}{2}=18 \Rightarrow \boxed{\cancel{x=18}} \\ x_2=\dfrac{27-9}{2}=9 \Rightarrow \boxed{x=9} \end{cases}$$

Comprobación:

Si x = 18 $\Rightarrow \sqrt{12+\sqrt{18+7}}=\sqrt{25-18} \Rightarrow \sqrt{12+5}\neq\sqrt{7}$

Si x = 9 $\Rightarrow \sqrt{12+\sqrt{9+7}}=\sqrt{25-9} \Rightarrow \sqrt{12+4}=\sqrt{16}$

6) $\sqrt{\sqrt{x+16}-\sqrt{x}}=2$

$\sqrt{x+16}-\sqrt{x}=4 \Rightarrow \sqrt{x+16}=\sqrt{x}+4 \Rightarrow x+16=x+16+8\sqrt{x} \Rightarrow 8\sqrt{x}=0 \Rightarrow \boxed{x=0}$

Comprobación:

Si x = 0 $\Rightarrow \sqrt{\sqrt{16}-\sqrt{0}}=2 \Rightarrow \sqrt{4}=2$

7) $\dfrac{\sqrt{x+2}}{\sqrt{x-1}} = \dfrac{4}{\sqrt{x+2}}$

$x+2 = 4\sqrt{x-1} \Rightarrow x^2 + 4x + 4 = 16x - 16 \Rightarrow x^2 - 12x + 20 = 0$

$$x = \frac{12 \pm \sqrt{12^2 - 4\cdot 20}}{2} = \frac{12 \pm 8}{2} \Rightarrow \begin{cases} x_1 = \dfrac{12+8}{2} = 10 \Rightarrow \boxed{x = 10} \\ x_2 = \dfrac{12-8}{2} = 2 \Rightarrow \boxed{x = 2} \end{cases}$$

Comprobación:

Si $x = 10 \Rightarrow \dfrac{\sqrt{12}}{\sqrt{9}} = \dfrac{4}{\sqrt{12}} \Rightarrow 12 = 3 \cdot 4$

Si $x = 2 \Rightarrow \dfrac{\sqrt{4}}{\sqrt{1}} = \dfrac{4}{\sqrt{4}} \Rightarrow 4 = 4$

8) $\sqrt{x-3} = \dfrac{\sqrt{x-1}}{\sqrt{x-3}}$

$x - 3 = \sqrt{x-1} \Rightarrow x^2 + 9 - 6x = x - 1 \Rightarrow x^2 - 7x + 10 = 0$

$$x = \frac{7 \pm \sqrt{7^2 - 4\cdot 10}}{2} = \frac{7 \pm 3}{2} \Rightarrow \begin{cases} x_1 = \dfrac{7+3}{2} = 5 \Rightarrow \boxed{x = 5} \\ x_2 = \dfrac{7-3}{2} = 2 \Rightarrow \boxed{\cancel{x = 2}} \end{cases}$$

Comprobación:

Si $x = 5 \Rightarrow \sqrt{5-3} = \dfrac{\sqrt{5-1}}{\sqrt{5-3}} \Rightarrow 2 = \sqrt{4}$

Si $x = 2 \Rightarrow \sqrt{2-3} = \dfrac{\sqrt{2-1}}{\sqrt{2-3}}$!!!

9) $\sqrt{x+4} - \sqrt{x-4} = \dfrac{x+1}{\sqrt{x+4}}$

$x + 4 - \sqrt{x^2 - 16} = x + 1 \Rightarrow 3 = \sqrt{x^2 - 16} \Rightarrow 9 = x^2 - 16 \Rightarrow x^2 = 25 \Rightarrow \boxed{x = 5}$

Comprobación:

Si $x = 5 \Rightarrow \sqrt{5+4} - \sqrt{5-4} = \dfrac{5+1}{\sqrt{5+4}} \Rightarrow 3 - 1 = \dfrac{6}{3}$

Si $x = -5 \Rightarrow \sqrt{-5+4} - \sqrt{-5-4} = \dfrac{-5+1}{\sqrt{-5+4}}$!!!

10) $\sqrt{x+3} + \sqrt{x+6} = \dfrac{3}{\sqrt{x+3}}$

$x + 3 + \sqrt{x^2 + 3x + 6x + 18} = 3 \Rightarrow x^2 + 3x + 6x + 18 = x^2 \Rightarrow 9x = -18 \Rightarrow \boxed{x = -2}$

Comprobación:

Si $x = -2 \Rightarrow \sqrt{-2+3} + \sqrt{-2+6} = \dfrac{3}{\sqrt{-2+3}} \Rightarrow 1 + 2 = \dfrac{3}{1}$

11) $\sqrt{2x+1} + 2\sqrt{x} = \dfrac{21}{\sqrt{2x+1}}$

$2x + 1 + 2\sqrt{2x^2 + x} = 21 \Rightarrow x - 10 = -\sqrt{2x^2 + x} \Rightarrow x^2 - 20x + 100 = 2x^2 + x \Rightarrow x^2 + 21x - 100 = 0$

$$x = \frac{-21 \pm \sqrt{(-21)^2 - 4\cdot(-100)}}{2} = \frac{-21 \pm 29}{2} \Rightarrow \begin{cases} x_1 = \dfrac{-21+29}{2} = 4 \Rightarrow \boxed{x = 4} \\ x_2 = \dfrac{-21-29}{2} = -25 \Rightarrow \boxed{\cancel{x = -25}} \end{cases}$$

Comprobación:

Si x = 4 $\Rightarrow \sqrt{8+1}+2\sqrt{4}=\dfrac{21}{\sqrt{8+1}} \Rightarrow 3+4=\dfrac{21}{3}$

Si x = -25 $\Rightarrow \sqrt{-50+1}+2\sqrt{-25}=\dfrac{21}{\sqrt{-50+1}}$!!!

12) $\sqrt{2x+10}+\sqrt{3x}=\dfrac{28}{\sqrt{2x+10}}$

$2x+10+\sqrt{6x^2+30x}=28 \Rightarrow 2x-18=-\sqrt{6x^2+30x} \Rightarrow 4x^2+324-72x=6x^2+30x$

$2x^2+102x-324=0 \Rightarrow x^2+51x-162=0$

$$x=\frac{-51\pm\sqrt{(-51)^2-4\cdot(-162)}}{2}=\frac{-51\pm 57}{2} \Rightarrow \begin{cases} x_1=\dfrac{-51+57}{2}=3 \Rightarrow \boxed{x=3} \\ x_2=\dfrac{-51-57}{2}=-54 \Rightarrow \boxed{\cancel{x=-54}} \end{cases}$$

Comprobación:

Si x = 3 $\Rightarrow \sqrt{6+10}+\sqrt{9}=\dfrac{28}{\sqrt{6+10}} \Rightarrow 4+3=\dfrac{28}{4}$

13) $\dfrac{\sqrt{4x+20}}{4+\sqrt{x}}=\dfrac{4-\sqrt{x}}{\sqrt{x}}$

$\sqrt{4x^2+20x}=16-x \Rightarrow 4x^2+20x=256-32x+x^2 \Rightarrow 3x^2+52x-256=0$

$$x=\frac{-52\pm\sqrt{(-52)^2-4\cdot 3\cdot(-256)}}{2\cdot 3}=\frac{-52\pm 76}{6} \Rightarrow \begin{cases} x_1=\dfrac{-52+76}{6}=4 \Rightarrow \boxed{x=4} \\ x_2=\dfrac{-52-76}{2}=-64 \Rightarrow \boxed{\cancel{x=-64}} \end{cases}$$

Comprobación:

Si x = 4 $\Rightarrow \dfrac{\sqrt{16+20}}{4+\sqrt{4}}=\dfrac{4-\sqrt{4}}{\sqrt{4}} \Rightarrow \dfrac{6}{4+2}=\dfrac{4-2}{2} \Rightarrow 1=1$

14) $\dfrac{\sqrt{96+12x}}{4+\sqrt{x}}=\dfrac{4-\sqrt{x}}{\sqrt{x-3}}$

$\sqrt{96x-288+12x^2-36x}=16-x \Rightarrow 12x^2+60x-288=256-32x+x^2 \Rightarrow 11x^2+92x-544=0$

$$x=\frac{-92\pm\sqrt{(-92)^2-4\cdot 11\cdot(-544)}}{2\cdot 11}=\frac{-92\pm 180}{22} \Rightarrow \begin{cases} x_1=\dfrac{-92+180}{22}=4 \Rightarrow \boxed{x=4} \\ x_2=\dfrac{-92-180}{22}=-\dfrac{136}{11} \end{cases}$$

Comprobación:

Si x = 4 $\Rightarrow \dfrac{\sqrt{96+48}}{4+\sqrt{4}}=\dfrac{4-\sqrt{4}}{\sqrt{4-3}} \Rightarrow \dfrac{12}{4+2}=\dfrac{4-2}{1}$