Determina centro y radio de las circunferencias:

231. $3x^2+3y^2+6x-12y-48=0$

$$x^2+y^2+2x-4y-16=0$$
D E F

$D=-2a$ $\quad a=\frac{2}{-2}=-1$ $\quad -16=1+4-R^2$

$E=-2b$ $\quad b=\frac{-4}{-2}=2$ $\quad R=\sqrt{21}$

$F=a^2+b^2-r^2$

$\boxed{\text{Centro } (-1,2) \quad \text{Radio}=\sqrt{21}}$

232. $2x^2+2y^2-x+3y+3=0$

$D=-\frac{1}{2}$ $\quad E=\frac{3}{2}$ $\quad F=\frac{3}{2}$

$$x^2+y^2-\frac{1}{2}x+\frac{3}{2}y+\frac{3}{2}=0$$

$-\frac{1}{2}=-2a \Rightarrow a=\frac{1}{4}$

$\frac{3}{2}=-2b \Rightarrow b=-\frac{3}{4}$

$\frac{3}{2}=\frac{1}{16}+\frac{9}{16}-R^2 \Rightarrow R^2=\frac{10}{16}-\frac{24}{16}$

$R=\sqrt{-\frac{14}{16}}$ NO EXISTE

No es una circunferencia

233. $x^2+y^2+y=0$

$D=0$ $\quad 0=-2a \Rightarrow a=0$

$E=1$ $\quad 1=-2b \Rightarrow b=-\frac{1}{2}$

$F=0$

$0=0+\frac{1}{4}-R^2 \Rightarrow R=\frac{1}{2}$

$\boxed{\begin{array}{l}\text{Centro } (0,-\frac{1}{2}) \\ \text{Radio}=\frac{1}{2}\end{array}}$

234. $3x^2+3y^2-2x+3y-6=0$

$D=-2/3$

$x^2+y^2-\frac{2}{3}x+y-2=0$ $\quad E=1$

$F=-2$

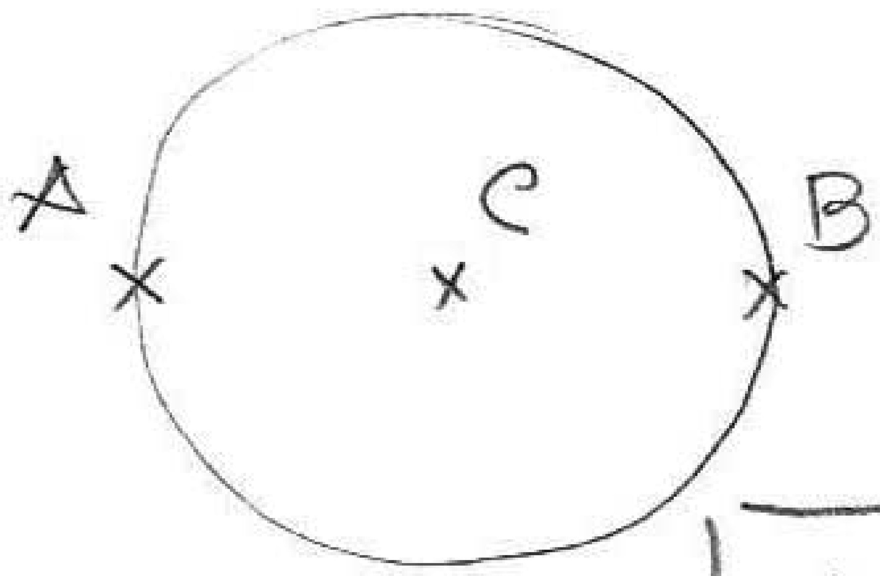
$+2a=+2/3$ $\quad a=\frac{1}{3}$ $\quad -2=\frac{1}{9}+\frac{1}{4}-R^2$

$-2b=1$ $\quad b=-\frac{1}{2}$ $\quad \frac{-2}{1}-\frac{1}{9}-\frac{1}{4}=\frac{-72-4-9}{36}=-\frac{85}{36}$

$\boxed{\begin{array}{l}\text{Centro } (\frac{1}{3},-\frac{1}{2}) \\ \text{Radio}=\sqrt{\frac{85}{36}}\end{array}}$

Halla la ecuación de la circunferencia.

235. De diámetro AB siendo $A(-1,5)$ y $B(3,-7)$

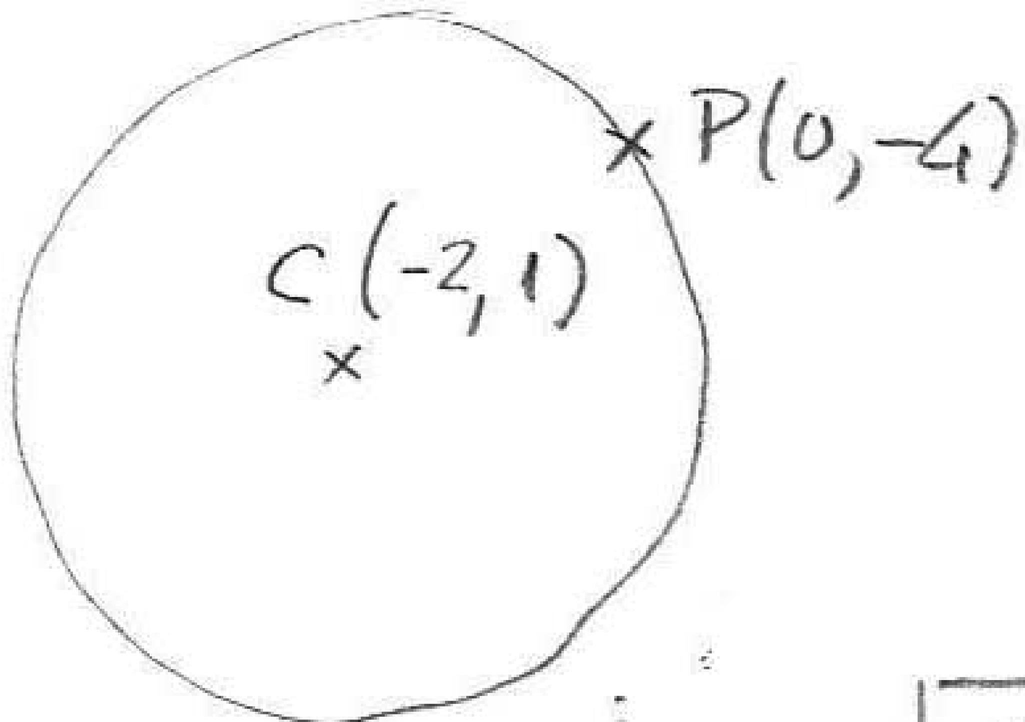


$$C\left(\frac{-1+3}{2}, \frac{5-7}{2}\right) \Rightarrow C(1,-1)$$

$$\text{Radio} = d(AC) = \sqrt{4+36} = \sqrt{40}$$

$$\boxed{(x-1)^2+(y+1)^2=40}$$

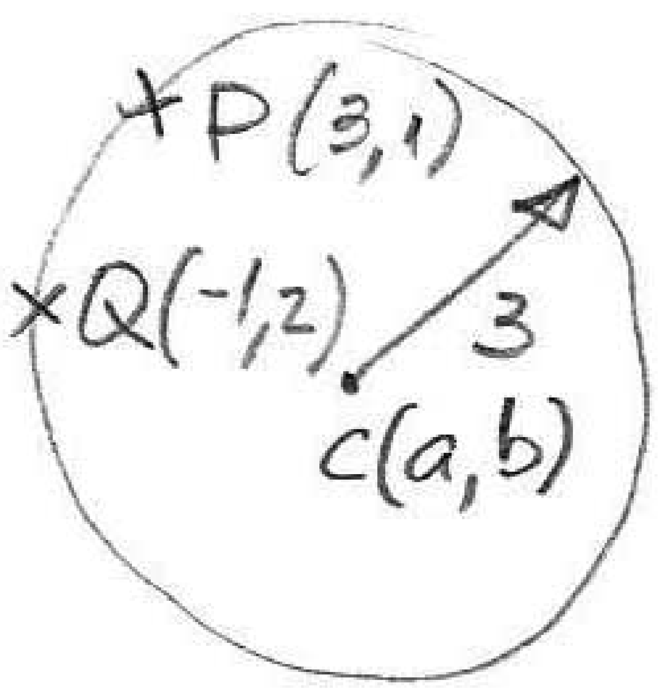
236. Que pasa por $P(0,-4)$ y Centro en $C(-2,1)$



$$(x-a)^2+(y-b)^2=R^2$$
$$(x+2)^2+(y-1)^2=R^2$$
$$(0+2)^2+(-4-1)^2=R^2$$
$$4+25=R^2 \longrightarrow R=\sqrt{29}$$

$$\boxed{(x+2)^2+(y-1)^2=29}$$

237. Que pasa por $P(3,1)$, $Q(-1,2)$ y su radio es 3



Las distancias $\overline{CP}$ y $\overline{CQ}$ tienen que valer 3

$$\left.\begin{array}{l}\sqrt{(3-a)^2+(1-b)^2}=3\\ \sqrt{(-1-a)^2+(2-b)^2}=3\end{array}\right\} \left\{\begin{array}{l}(3-a)^2+(1-b)^2=9\\ (-1-a)^2+(2-b)^2=9\end{array}\right\}$$

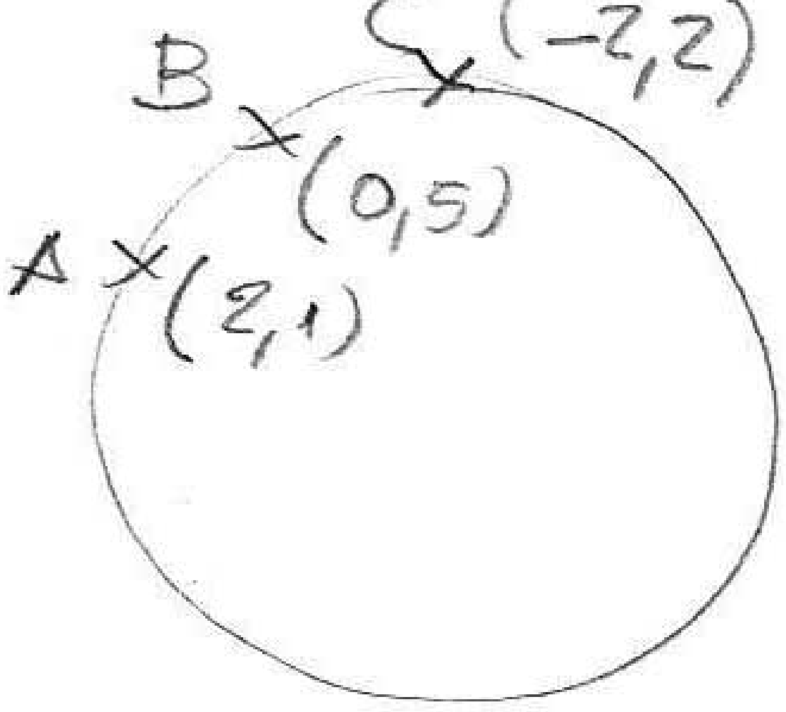
$$\left.\begin{array}{l}\cancel{9}+a^2-6a+1+b^2-2b=\cancel{9}\\ 1+a^2+2a+4+b^2-4b=9\end{array}\right\} \longrightarrow a^2-6a+(b^2-2b+1)=0 \Rightarrow$$

$$a=\frac{6\pm\sqrt{36-4(b^2-2b+1)}}{2}=\frac{6\pm\sqrt{36-4b^2+8b-4}}{2}=\frac{6\pm\sqrt{32+8b-4b^2}}{2}=$$

$$=\frac{6\pm2\sqrt{8+2b-b^2}}{}$$

(...)

238. Que pasa por los puntos $(2,-1)$ $(0,5)$ y $(-2,2)$



$$x^2+y^2+Dx+Ey+F=0$$

$$\left.\begin{array}{l} 2^2+1^2+2D-E+F=0 \\ 5^2+5E+F=0 \\ 4+4-2D+2E+F=0 \end{array}\right\} \quad \left.\begin{array}{l} 2D-E+F=-5 \\ 5E+F=-25 \\ -2D+2E+F=-8 \end{array}\right\}$$

$$\boxed{E=\frac{-25-F}{5}}$$

$$\left.\begin{array}{l} 2D-\left(\frac{-25-F}{5}\right)+F=-5 \\ -2D+2\left(\frac{-25-F}{5}\right)+F=-8 \end{array}\right\} \quad \left.\begin{array}{l} 10D+25+F+5F=-5 \\ -10D-50-2F+5F=-40 \end{array}\right\}$$

$$\left.\begin{array}{l} \cancel{10D}+6F=-30 \\ \cancel{-10D}+3F=10 \end{array}\right\}$$

$$9F=-20 \longrightarrow F=-\frac{20}{9}$$

$$D=\frac{-30+\frac{120}{9}}{10}=\frac{-150}{90}=-\frac{5}{3}$$

$$E=\frac{-25+\frac{20}{9}}{5}=\frac{-205}{45}=\frac{-41}{9}$$

Ecuación $\boxed{x^2+y^2-\frac{5}{3}x-\frac{41}{9}y-\frac{20}{9}=0}$

239. Concéntrica con $2x^2+2y^2-3x+4y-5=0$ y radio $=\sqrt{3}$

$$x^2+y^2-\frac{3}{2}x+2y-\frac{5}{2} \Rightarrow D=-\frac{3}{2} \qquad E=2$$

$$-2a=-\frac{3}{2} \Rightarrow a=\frac{3}{4} \qquad -2b=2 \Rightarrow b=-1$$

Centro $\left(\frac{3}{4},-1\right)$ Radio $\sqrt{3}$

Ecuación: $\boxed{\left(x-\frac{3}{4}\right)^2+(y+1)^2=3}$

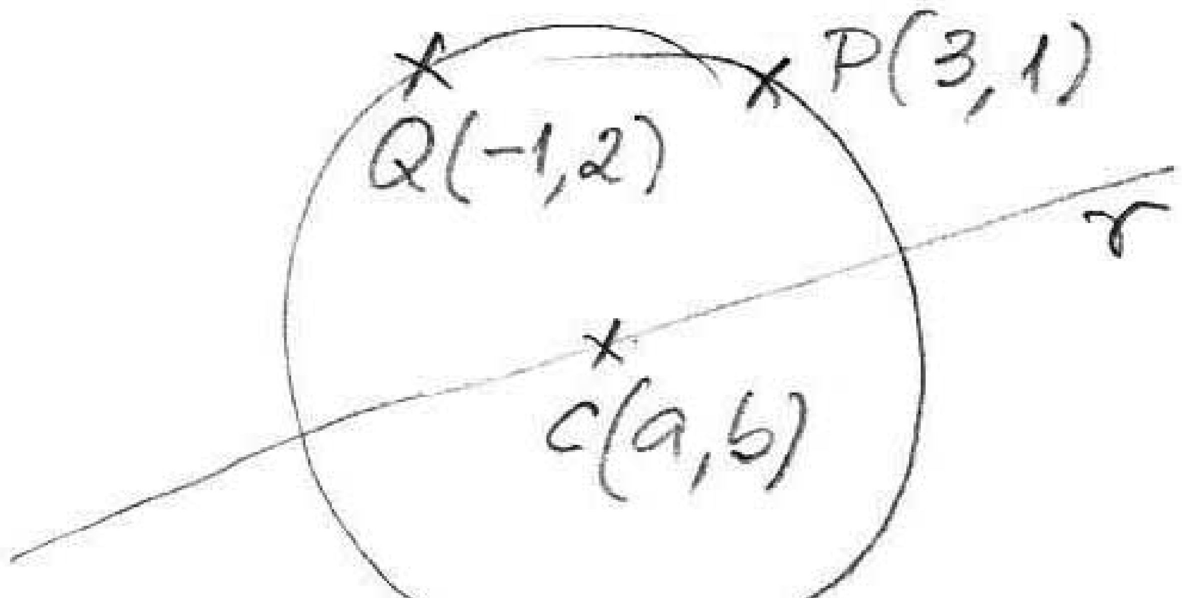
240. Concentrica con $2x^2+2y^2-3x+4y-5=0$ y que pasa por $(4,2)$

Centro $\left(\frac{3}{4},-1\right)$ $\quad \left(x-\frac{3}{4}\right)^2+(y+1)^2=R^2$

$$\left(4-\frac{3}{4}\right)^2+(2+1)^2=R^2 \Rightarrow R^2=\left(\frac{13}{4}\right)^2+3^2=\frac{169}{16}+9=\frac{313}{16}$$

Ecuación $\boxed{\left(x-\frac{3}{4}\right)^2+(y+1)^2=\frac{313}{16}}$

241. Que pasa por $P(3,1)$; $Q(-1,2)$ y cuyo centro está en la recta $x-2y+1=0$



$$(x-a)^2+(y-b)^2=r^2$$

$$\left.\begin{array}{l}(3-a)^2+(1-b)^2=r^2\\(-1-a)^2+(2-b)^2=r^2\\a-2b+1=0\end{array}\right\} \to a=2b-1$$

$$\left.\begin{array}{l}[3-(2b-1)]^2+[1-b]^2=r^2\\{}[-1-(2b-1)]^2+(2-b)^2=r^2\end{array}\right\} \quad \left.\begin{array}{l}(4-2b)^2+(1-b)^2=r^2\\(2b)^2+(2-b)^2=r^2\end{array}\right\}$$

$$\left.\begin{array}{l}16+4b^2-16b+1+b^2-2b=r^2\\4b^2+4+b^2-4b=r^2\end{array}\right\} \quad \left.\begin{array}{l}5b^2-18b+17=r^2\\5b^2-4b+4=r^2\end{array}\right\}$$

$$\cancel{5b^2}-18b+17=\cancel{5b^2}-4b+4 \qquad 13=14b \qquad \boxed{b=\frac{13}{14}}$$

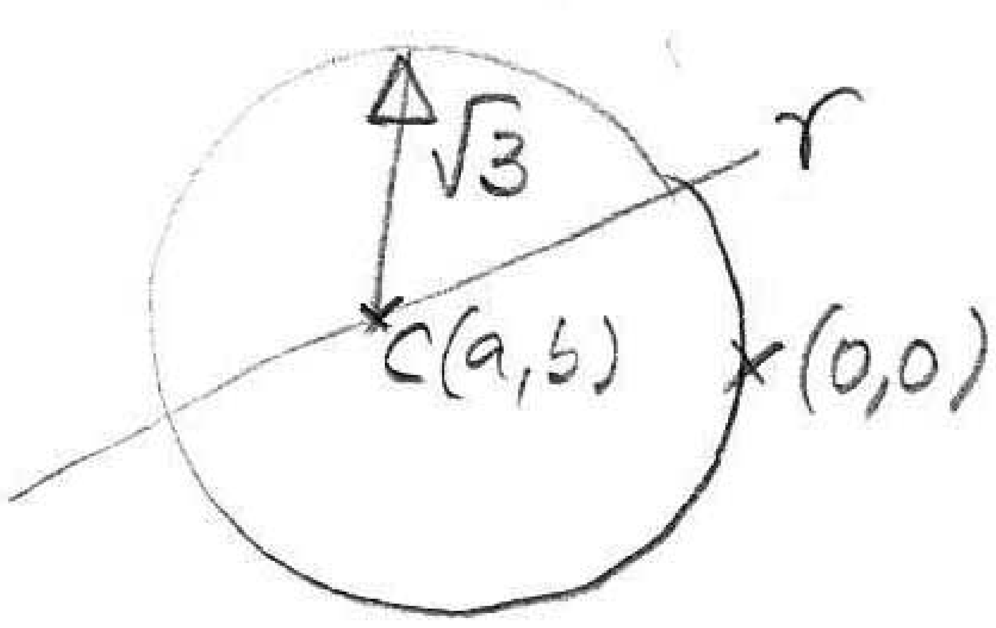
$$r^2=5\left(\frac{13}{14}\right)^2-18\left(\frac{13}{14}\right)+17 \Rightarrow r^2=\frac{845}{196}-\frac{234}{14}+17 \Rightarrow$$

$$r^2=\frac{845}{196}-\frac{3276}{196}+\frac{3332}{196}=\frac{901}{196} \Rightarrow \boxed{r=\frac{\sqrt{901}}{14}}$$

$$a=2b-1=\frac{26}{14}-1=\frac{12}{14}=\frac{6}{7}$$

$$\boxed{\left(x-\frac{6}{7}\right)^2+\left(y-\frac{13}{14}\right)^2=\frac{901}{196}}$$

242. Cuyo radio es $\sqrt{3}$, pasa por $(0,0)$ y su centro está en la recta $x+y=0$



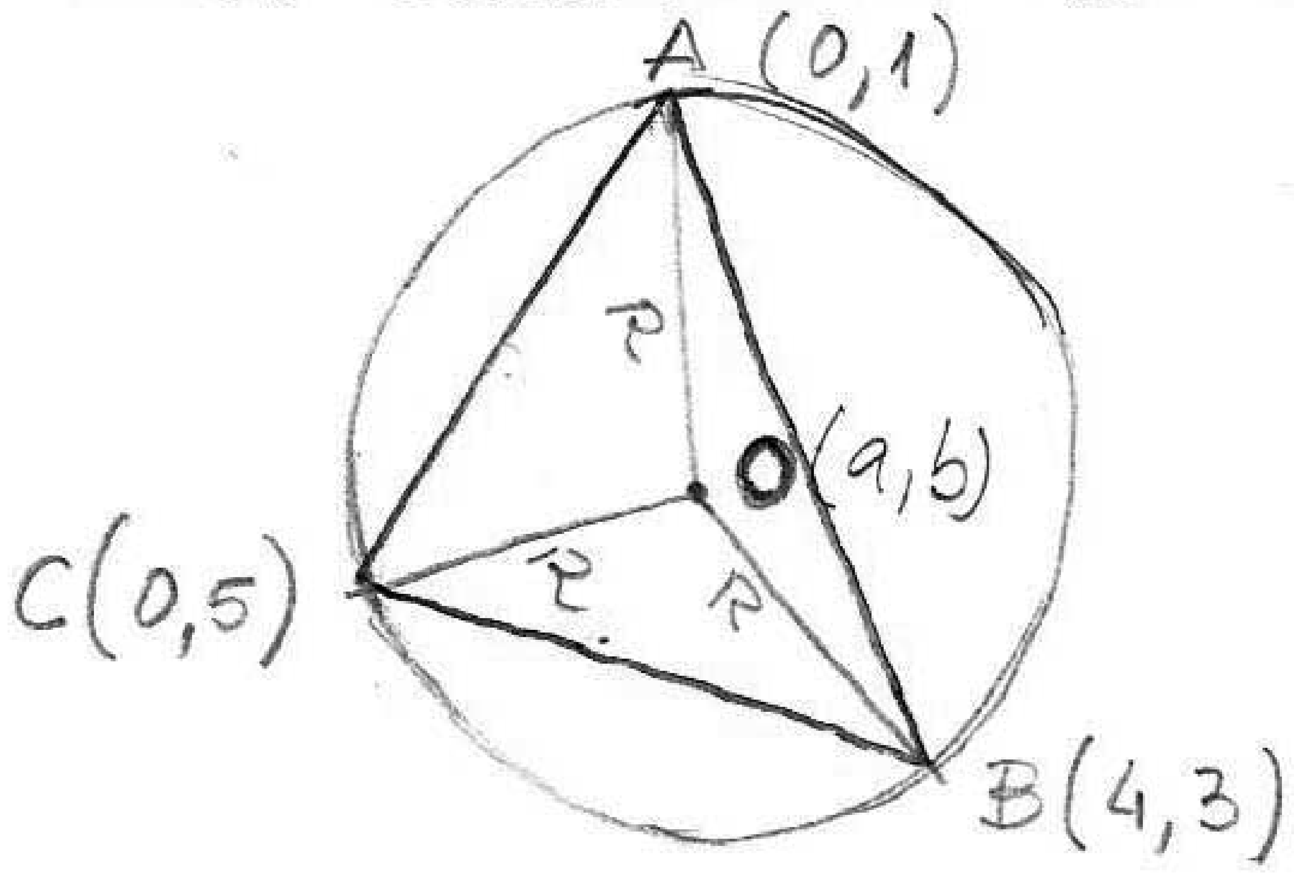
$$(x-a)^2+(y-b)^2=r^2$$

$$\left.\begin{array}{l}(-a)^2+(-b)^2=3\\a+b=0\end{array}\right\} \quad \left.\begin{array}{l}a^2+b^2=3\\a+b=0\end{array}\right\}$$

$$a=-b \Rightarrow (-b)^2+b^2=3 \Rightarrow 2b^2=3 \Rightarrow b=\sqrt{\frac{3}{2}}$$

$$a=-\sqrt{\frac{3}{2}} \qquad \boxed{\left(x+\sqrt{\frac{3}{2}}\right)^2+\left(y-\sqrt{\frac{3}{2}}\right)^2=3}$$

243. Circunscrita en el triángulo de vértices $(0,1)$; $(4,3)$; $(0,5)$



$$x^2+y^2+Dx+Ey+F=0$$

$$\left.\begin{array}{l} 16+9+4D+3E+F=0 \\ 25 \qquad\quad +5E+F=0 \\ 1 \qquad\quad\; +E+F=0 \end{array}\right\} \rightarrow E=-1-F$$

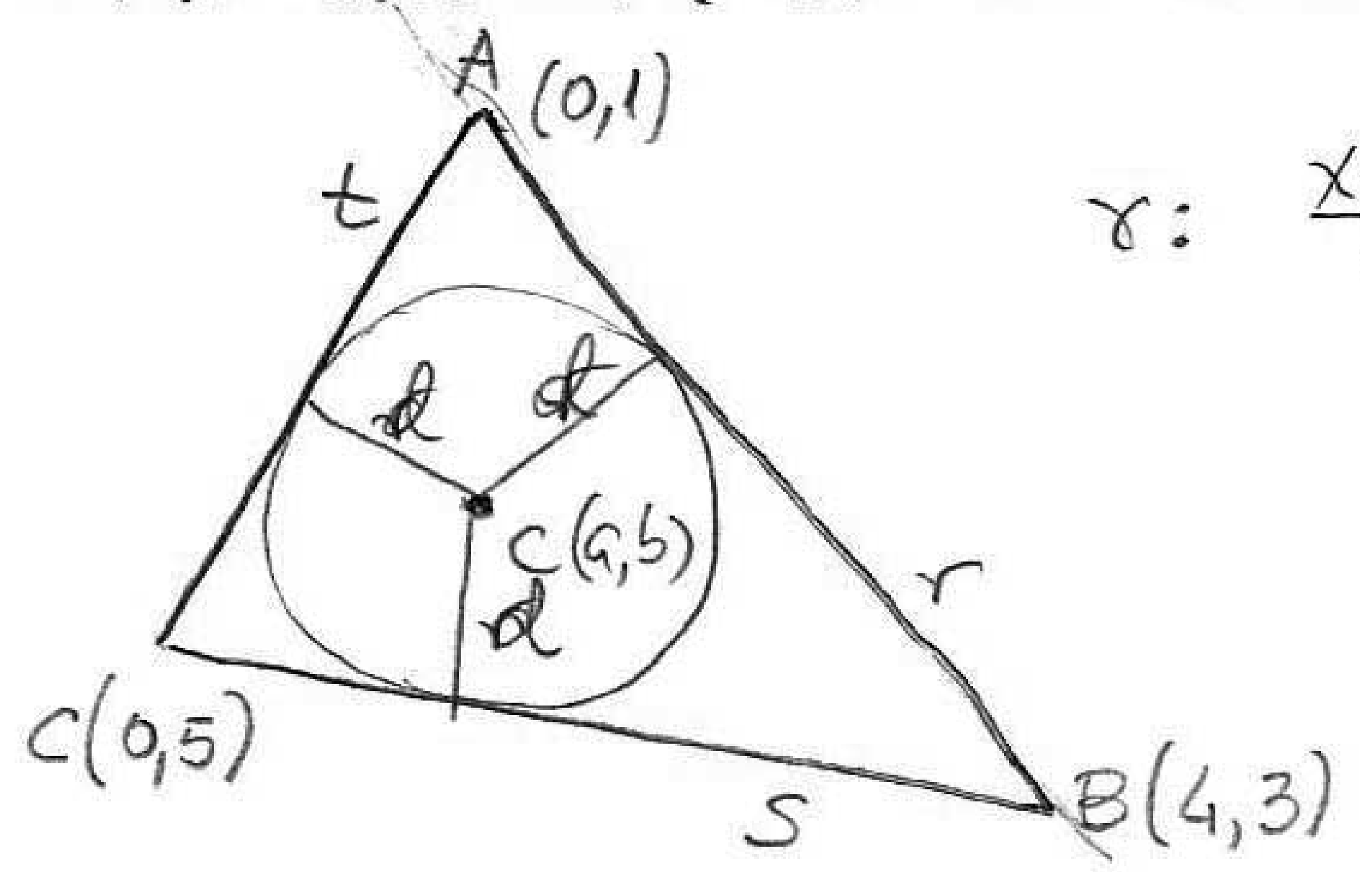
$$25+5(-1-F)+F=0$$

$$25-5-4F=0 \Rightarrow \boxed{F=5} \quad \boxed{E=-6}$$

$$D=\frac{-16-9+18-5}{4}=-3$$

$$\boxed{\text{Circunferencia: } x^2+y^2-3x-6y+5=0}$$

244. Inscrita en el triángulo del ejercicio anterior



$$r: \frac{x-0}{4}=\frac{y-1}{2} \Rightarrow \boxed{2x-4y+4=0} \;(r)$$

$$s: \frac{x-0}{4}=\frac{y-5}{-2} \Rightarrow -2x-4y+20=0$$

$$2x+4y-20=0 \Rightarrow \boxed{x+2y-10=0} \;(s)$$

$$t: \frac{x-0}{0}=\frac{y-1}{4} \Rightarrow 4x=0 \Rightarrow \boxed{x=0} \;(t)$$

distancia del centro a cada recta

$$\frac{|2.a-4b+4|}{\sqrt{4+16}}=\frac{|a+2b-10|}{\sqrt{1+4}}=\frac{|a|}{\sqrt{1}}$$

$$\left.\begin{array}{l} 2a-4b+4=\sqrt{20}\,a \\ a+2b-10=\sqrt{5}\,a \end{array}\right\} \qquad \left.\begin{array}{l} 2a-\cancel{4b}+4=\sqrt{20}\,a \\ 2a+\cancel{4b}-20=2\sqrt{5}\,a \end{array}\right\} \qquad 4a-4\sqrt{5}a=16$$

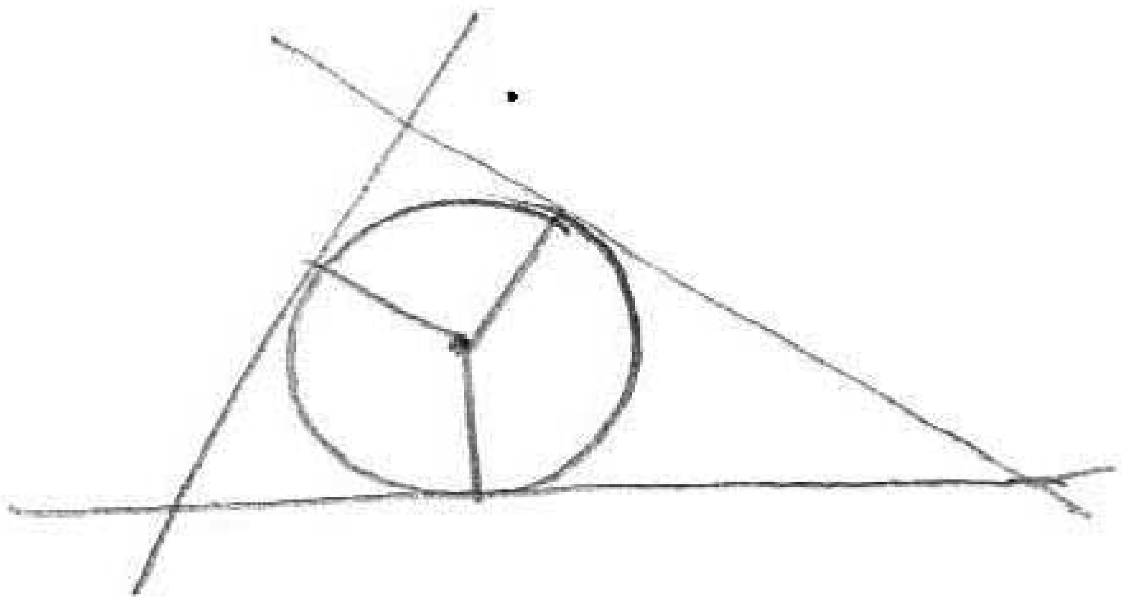
$$4a \quad -16=4\sqrt{5}\,a \qquad a=\frac{16}{4-4\sqrt{5}}=-3'236$$

$$b=\frac{\sqrt{5}\cdot(-3'236)-(-3'236)+10}{2}=3 \qquad \text{Centro } (-3'236,\ 3)$$

Radio $= 3'236$.

$$\boxed{\text{Circunferencia } (x+3'236)^2+(y-3)^2=10'472}$$

245. Inscrita en el triangulo de lados $x-y=0$ $x-2y+4=0$ $x+y+2=0$



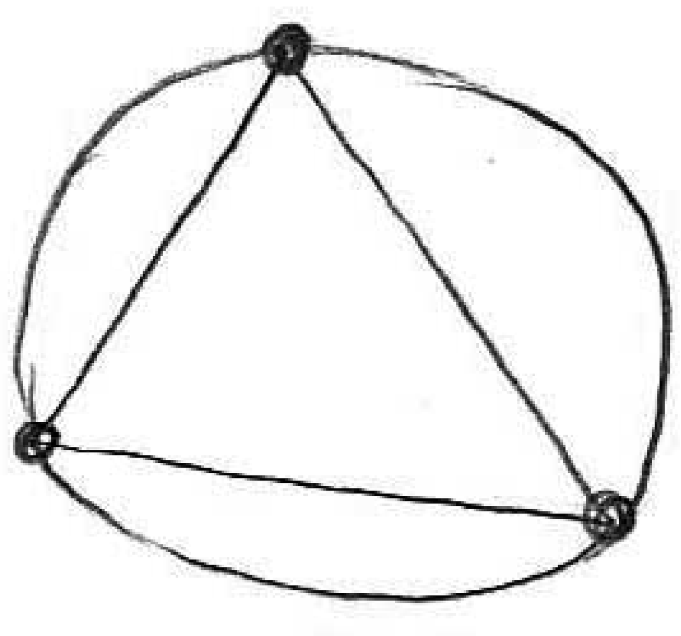
$$\frac{a-b}{\sqrt{2}} = \frac{a-2b+4}{\sqrt{5}} = \frac{a+b+2}{\sqrt{2}}$$

$$\left.\begin{array}{l} \sqrt{5}a-\sqrt{5}b=\sqrt{2}a-2\sqrt{2}b+4\sqrt{2} \\ \cancel{\sqrt{2}a}-\sqrt{2}b=\cancel{\sqrt{2}a}+\sqrt{2}b+2\sqrt{2} \end{array}\right\}$$

$$b=\frac{2\sqrt{2}}{-2\sqrt{2}}=\boxed{-1}$$

$$\sqrt{5}a+\sqrt{5}=\sqrt{2}a+2\sqrt{2}+4\sqrt{2} \Rightarrow a=\frac{6\sqrt{2}-\sqrt{5}}{\sqrt{5}-\sqrt{2}}$$

246. Circunscrita al triangulo del ejercicio anterior



$$x^2+y^2+Dx+Ey+F=0$$

$$\left.\begin{array}{l} x-y=0 \\ x-2y+4=0 \end{array}\right\} \quad x=4 \quad \boxed{A(4,4)}$$

$$y-4=0 \rightarrow y=4$$

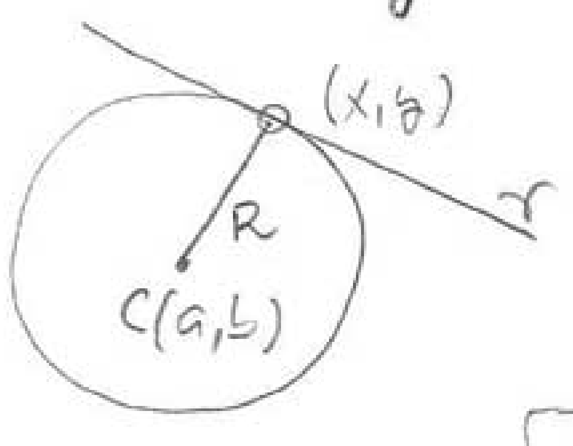
$$\left.\begin{array}{l} x-y=0 \\ x+y+2=0 \end{array}\right\} \quad y=-1 \quad \boxed{B(-1,-1)}$$

$$2x+2=0 \qquad x=-1$$

$$\left.\begin{array}{l} x-2y+4=0 \\ x+y+2=0 \end{array}\right\} \quad x=-\frac{8}{3} \quad y=\frac{2}{3} \quad \boxed{C\left(-\frac{8}{3},\frac{2}{3}\right)}$$

$$-3y+2=0$$

247. Cuyo centro es el punto $C(1,-5)$ y es tangente a la recta $3x-4y+1=0$



$$(x-1)^2+(y+5)^2=R^2$$

$$R=d(C,r)=\frac{|3+20+1|}{\sqrt{9+16}}=\frac{24}{5}$$

$$\boxed{(x-1)^2+(y+5)^2=\frac{576}{25}}$$

—oOo—

¿Cuales de las siguientes ecuaciones corresponde a una circunferencia? Hallar el centro y el radio

$x^2+y^2-3x+5=0$ $\qquad$ $-x^2-y^2+5x-2y+1=0$

$x^2+y^2+2y-3=0$ $\qquad$ $-2x^2-2y^2+6x-4y+1=0$

$3x^2+3y^2-5x+1=0$ $\qquad$ $x^2+y^2-2y+4=0$

$x^2+y^2-3x+5=0 \Rightarrow D=-3 \quad E=0 \quad F=5$

$D=-2a \longrightarrow -3=-2a \Rightarrow a=3/2$

$E=-2b \longrightarrow 0=-2b \Rightarrow b=0$

$F=a^2+b^2-R^2 \rightarrow 5=\frac{9}{4}-R^2 \Rightarrow R=\sqrt{\frac{9}{4}-5}$ $\quad$ NO existe

NO

$x^2+y^2+2y-3=0 \Rightarrow D=0 \quad E=2 \quad F=-3$

$D=-2a \longrightarrow 0=-2a \Rightarrow \underline{a=0}$

$E=-2b \longrightarrow 2=-2b \Rightarrow \underline{b=-1}$

$F=a^2+b^2-R^2 \rightarrow -3=1-R^2 \Rightarrow R^2=4 \rightarrow \underline{R=2}$

SI

$3x^2+3y^2-5x+1=0 \Rightarrow D=-\frac{5}{3} \quad E=0 \quad F=\frac{1}{3}$

$D=-2a \Longrightarrow -5/3=-2a \longrightarrow a=5/6$

$E=-2b \Longrightarrow 0=-2b \longrightarrow b=0$

$F=a^2+b^2-R^2 \Rightarrow 1/3=\frac{25}{36}-R^2 \Rightarrow R=\sqrt{\frac{25}{36}-\frac{1}{3}}$

SI

$-x^2-y^2+5x-2y+1=0 \quad D=-5 \quad E=2 \quad F=-1$

$D=-2a \longrightarrow -5=-2a \quad a=5/2$

$E=-2b \longrightarrow 2=-2b \quad b=-1$

$F=a^2+b^2-R^2 \longrightarrow -1=\frac{25}{4}+1-R^2 \Rightarrow R^2=\frac{25}{4}+2$

SI

$-2x^2-2y^2+6x-4y+1=0 \qquad D=-3 \qquad E=2 \qquad F=-1/2$

$D=-2a \longrightarrow -3=-2a \rightarrow a=3/2$
$E=-2b \longrightarrow 2=-2b \rightarrow b=-1$
$F=a^2+b^2-R^2 \longrightarrow -1/2 = 9/4+1-R^2 \longrightarrow R=\sqrt{\frac{11}{4}+\frac{1}{2}}$

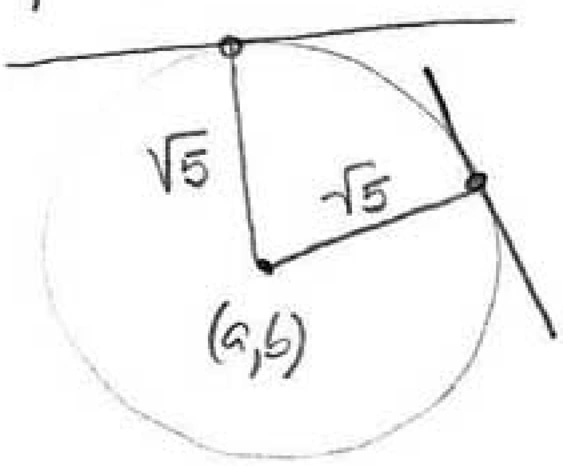
SI

$x^2+y^2-2y+4=0 \Rightarrow D=0 \qquad E=-2 \qquad F=4$

$-2a=0 \longrightarrow a=0$
$-2=-2b \rightarrow b=1$
$4=0+1-R^2 \longrightarrow R=\sqrt{-3}$

NO

Halla una ecuación de la circunferencia de radio $r=\sqrt{5}$ tangente a las dos rectas $2x-y=0$ $\quad 2x+y-2=0$

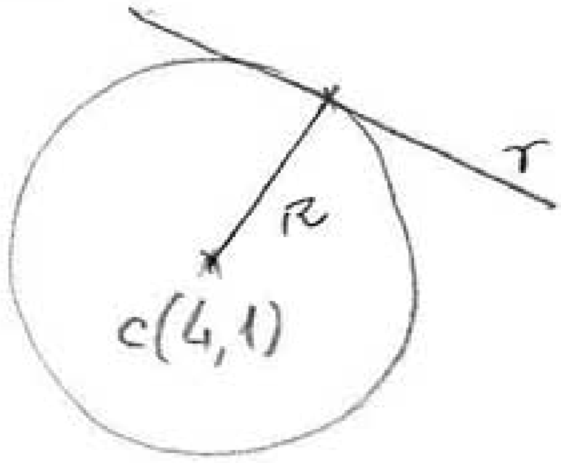


$$\left.\begin{aligned} \frac{2a-b}{\sqrt{4+1}} &= \sqrt{5} \\ \frac{2a+b-2}{\sqrt{5}} &= \sqrt{5} \end{aligned}\right\} \quad \begin{aligned} 2a-\cancel{b} &= 5 \\ 2a+\cancel{b}-2 &= 5 \\ \hline 4a-2 &= 10 \end{aligned}$$

$a=\frac{12}{4}=3 \qquad b=2a-5=1$

$$\boxed{(x-3)^2+(y-1)^2=5}$$

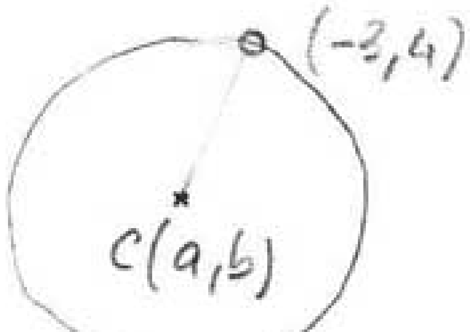
¿Qué circunferencia de centro $c(4,1)$ es tangente a la recta de ecuación $3x-4y+2=0$?



$$d(c,R)=\frac{12-4+2}{\sqrt{9+16}}=\frac{10}{5}=2$$

$$\boxed{(x-4)^2+(y-1)^2=4}$$

Halla la ecuación de la recta normal a la circunferencia de ecuación $x^2+y^2=25$ en el punto $(-3,4)$

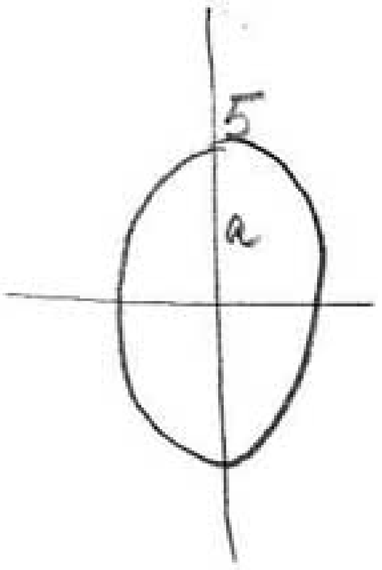


$D=0 \qquad E=0 \qquad D=-2a \rightarrow a=0$
$E=-2b \rightarrow b=0$

$c(0,0)$

$$\frac{x-0}{-3}=\frac{y-0}{4} \Rightarrow \boxed{4x+3y=0}$$

Halla la ecuación reducida de la elipse de semieje mayor vertical $a=5$ y excentricidad $e=0'6$

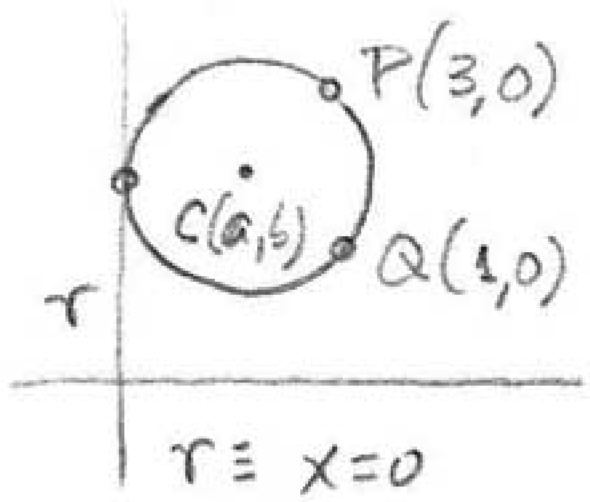


$e = \frac{c}{a}$; $0'6 = \frac{c}{5} \Rightarrow c = 3$

$a^2 = b^2 + c^2 \Rightarrow b = \sqrt{5^2 - 3^2} = 4$

$\boxed{\frac{x^2}{16} + \frac{y^2}{25} = 1}$ ya que el eje mayor es vertical.

248. Ecuación de la circunferencia que pasa por los puntos $P(3,0)$ $Q(1,0)$ y es tangente al eje de ordenadas



$d(C,P) = \sqrt{(3-a)^2 + (0-b)^2} = \sqrt{9 + a^2 - 6a + b^2}$

$d(C,Q) = \sqrt{(1-a)^2 + (0-b)^2} = \sqrt{1 + a^2 - 2a + b^2}$

$d(C,r) = \frac{1a + 0b + 0}{\sqrt{1^2 + 0^2}} = \frac{a}{1} = a$

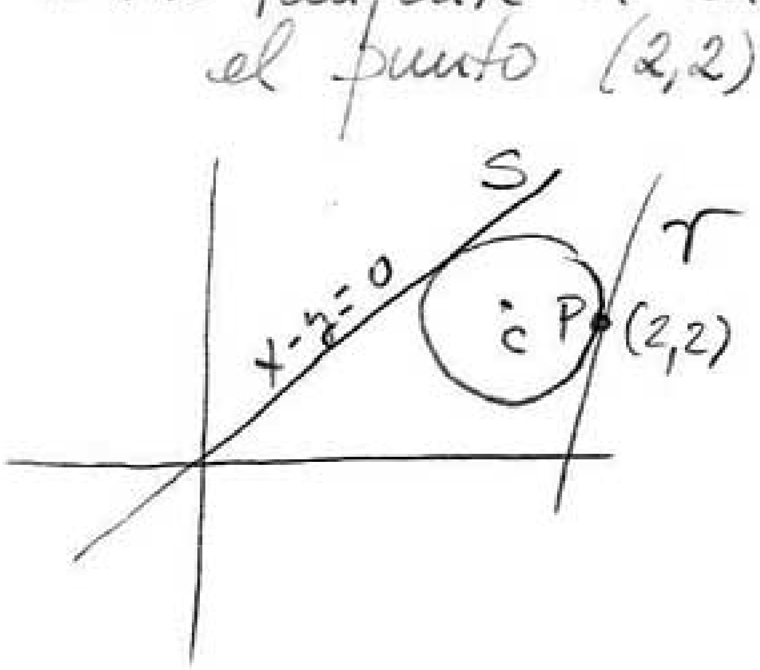
$\sqrt{9 + \cancel{a^2} - 6a + \cancel{b^2}} = \sqrt{1 + \cancel{a^2} - 2a + \cancel{b^2}} \Rightarrow 8 = 4a \Rightarrow a = 2$

$\sqrt{9 + a^2 - 6a + b^2} = a \quad 9 + \cancel{a^2} - 6a + b^2 = \cancel{a^2} \Rightarrow$

$9 - 12 + b^2 = 0 \Rightarrow \underline{b^2 = 3}$ $\underline{b = \sqrt{3}}$ Radio $= \underline{a = 2}$

$\boxed{(x-2)^2 + (y-\sqrt{3})^2 = 4}$

249. tangente a la bisectriz del I y III cuadrante en el punto $(2,2)$ y también tangente a la recta $x+y+1=0$



$d(C,P) = \sqrt{(2-a)^2 + (2-b)^2}$

$d(C,s) = \frac{a-b}{\sqrt{2}}$

$d(C,r) = \frac{a+b+1}{\sqrt{2}}$

$\frac{\cancel{a}-b}{\cancel{\sqrt{2}}} = \frac{\cancel{a}+b+1}{\cancel{\sqrt{2}}} \Rightarrow \boxed{b = -\frac{1}{2}}$

Sigue →