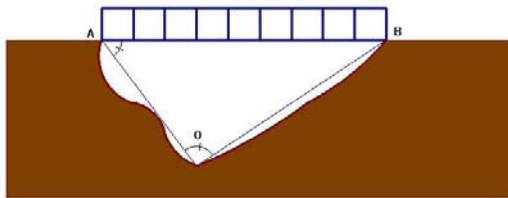


1. Sean a y b dos ángulos tales que $\operatorname{tg} a = \frac{4}{5}$ siendo $a < \frac{\pi}{2}$ y $\cos b = -\frac{3}{7}$ siendo $\frac{\pi}{2} < b < \pi$.

Halla, sin utilizar la calculadora: $\operatorname{sen}(a - b)$, $\cos(a + b)$, $\operatorname{tg} 2a$, $\cot\left(\frac{b}{2}\right)$ **2 puntos**

2. Se quiere construir un puente entre los puntos A y B de la siguiente figura. Se sabe que $\hat{O} = 93^\circ$, $\hat{A} = 48^\circ$ y que la distancia, medida en línea recta entre los puntos A y O es de 75 m. Calcula la longitud del puente. **1,5 puntos**



3. Resuelve las ecuaciones:

- $\operatorname{tg} 5x + \operatorname{tg}^2 5x = 2$
- $\cos 2x = \operatorname{sen} x$

2,5 puntos

4. Comprueba la identidad: $\operatorname{sen} 2x = \frac{2\operatorname{tg} x}{1 + \operatorname{tg}^2 x}$

1,25 puntos

5. Simplifica la expresión: $(\cot a - \operatorname{tg} a) \left[\operatorname{tg}\left(\frac{\pi}{4} + a\right) - \operatorname{tg}\left(\frac{\pi}{4} - a\right) \right]$

1,25 puntos

6. Representa gráficamente la función: $y = \operatorname{sen} \frac{x}{4}$

1,5 puntos

SOLUCIONES

1) Empezamos hallando las restantes razones trigonométricas de a y b:

$$\operatorname{tg} a = \frac{4}{5} \text{ siendo } a < \frac{\pi}{2} \rightarrow 1 + \operatorname{tg}^2 a = \frac{1}{\cos^2 a} \rightarrow 1 + \frac{16}{25} = \frac{1}{\cos^2 a} \rightarrow \frac{41}{25} = \frac{1}{\cos^2 a}$$

$$\cos^2 a = \frac{25}{41} \Rightarrow \cos a = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41} \text{ (1º cuad)} \rightarrow \operatorname{sen} a = \cos a \cdot \operatorname{tg} a = \frac{5}{\sqrt{41}} \cdot \frac{4}{5} = \frac{4\sqrt{41}}{41}$$

$$\cos b = -\frac{3}{7} \text{ siendo } \frac{\pi}{2} < b < \pi \rightarrow \operatorname{sen}^2 b = 1 - \cos^2 b = 1 - \frac{9}{49} = \frac{40}{49}$$

$$\operatorname{sen} b = \pm \sqrt{\frac{40}{49}} = \frac{2\sqrt{10}}{7} \text{ (segundo cuadrante)} \rightarrow \operatorname{tg} b = \frac{\operatorname{sen} b}{\cos b} = -\frac{2\sqrt{10}}{3}$$

$$\operatorname{sen}(a-b) = \operatorname{sen} a \cdot \cos b - \operatorname{sen} b \cdot \cos a = \frac{4\sqrt{41}}{41} \cdot \left(-\frac{3}{7}\right) - \frac{2\sqrt{10}}{7} \cdot \frac{5\sqrt{41}}{41} = \frac{-12\sqrt{41} - 10\sqrt{410}}{287}$$

$$\cos(a+b) = \cos a \cos b - \operatorname{sen} a \operatorname{sen} b = \frac{5\sqrt{41}}{41} \cdot \left(-\frac{3}{7}\right) - \frac{4\sqrt{41}}{41} \cdot \frac{2\sqrt{10}}{7} = -\frac{15\sqrt{41} + 8\sqrt{410}}{287}$$

$$\operatorname{tg} 2a = \frac{2\operatorname{tg} a}{1 - \operatorname{tg}^2 a} = \frac{2 \cdot \left(\frac{4}{5}\right)}{1 - \left(\frac{4}{5}\right)^2} = \frac{\frac{8}{5}}{\frac{9}{25}} = \frac{8 \cdot 25}{5 \cdot 9} = \frac{40}{9}$$

$$\cot\left(\frac{b}{2}\right) = \frac{1}{\operatorname{tg}\left(\frac{b}{2}\right)} = \frac{1}{\pm \sqrt{\frac{1 - \cos b}{1 + \cos b}}} = \sqrt{\frac{1 + \cos b}{1 - \cos b}} = \text{(1º cuad)} = \sqrt{\frac{1 - \frac{3}{7}}{1 + \frac{3}{7}}} = \sqrt{\frac{\frac{4}{7}}{\frac{10}{7}}} = \sqrt{\frac{4}{10}} = \sqrt{\frac{2}{5}}$$

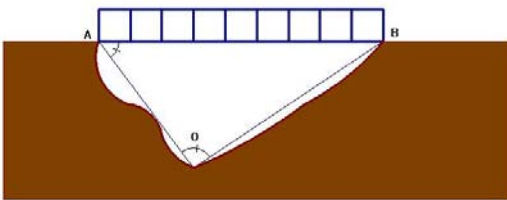
2) $\hat{O} = 93^\circ$, $\hat{A} = 48^\circ$ y que la distancia entre los puntos A y O es de 75 m

el tercer ángulo: $\hat{B} = 180 - (93 + 48) = 39^\circ$

aplicamos el teorema del seno:

$$\frac{AO}{\operatorname{sen} B} = \frac{AB}{\operatorname{sen} O} \rightarrow \frac{75}{\operatorname{sen} 39} = \frac{x}{\operatorname{sen} 93}$$

$$x = \frac{75 \operatorname{sen} 93}{\operatorname{sen} 39} = 119 \text{ m}$$



3) a) $\operatorname{tg} 5x + \operatorname{tg}^2 5x = 2 \rightarrow \operatorname{tg}^2 5x + \operatorname{tg} 5x - 2 = 0 \rightarrow z = \operatorname{tg} 5x$

$$z = \frac{-1 \pm \sqrt{1+8}}{2} = \left\langle \begin{array}{l} 1 \\ -2 \end{array} \right\rangle \rightarrow \left\langle \begin{array}{l} \operatorname{tg} 5x = 1 \\ \operatorname{tg} 5x = -2 \end{array} \right\rangle \rightarrow 5x = \left\langle \begin{array}{l} 45^\circ + 180k \\ 116^\circ 33' + 180k \end{array} \right\rangle \rightarrow \left\langle \begin{array}{l} x = 9^\circ + 36k \\ x = 23^\circ 19' + 36k \end{array} \right\rangle$$

b) $\cos 2x = \operatorname{sen} x \rightarrow \cos^2 x - \operatorname{sen}^2 x = \operatorname{sen} x \rightarrow 1 - \operatorname{sen}^2 x - \operatorname{sen}^2 x = \operatorname{sen} x$

$$2\operatorname{sen}^2 x + \operatorname{sen} x - 1 = 0 \rightarrow z = \operatorname{sen} x \Rightarrow z = \frac{-1 \pm \sqrt{1+8}}{4} = \left\langle \begin{array}{l} \frac{1}{2} \\ -1 \end{array} \right\rangle \rightarrow \left\langle \begin{array}{l} \operatorname{sen} x = \frac{1}{2} \\ \operatorname{sen} x = -1 \end{array} \right\rangle$$

$$x = \begin{cases} 30^\circ + 360k \\ 150^\circ + 360k \end{cases} \text{ y } x = 270^\circ + 360k$$

$$4) \operatorname{sen} 2x = \frac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x} \rightarrow 2 \operatorname{sen} x \cos x = \frac{2 \frac{\operatorname{sen} x}{\cos x}}{\frac{1}{\cos^2 x}} = \frac{2 \operatorname{sen} x}{\cos x} \cdot \frac{1}{\cos^2 x} = \frac{2 \operatorname{sen} x \cos^2 x}{\cos x}$$

$$\rightarrow 2 \operatorname{sen} x \cos x = 2 \operatorname{sen} x \cos x$$

$$\begin{aligned} 5) (\cot a - \operatorname{tg} a) \left[\operatorname{tg} \left(\frac{\pi}{4} + a \right) - \operatorname{tg} \left(\frac{\pi}{4} - a \right) \right] &= \left(\frac{1}{\operatorname{tg} a} - \operatorname{tg} a \right) \left[\frac{\operatorname{tg} \frac{\pi}{4} + \operatorname{tg} a}{1 - \operatorname{tg} \frac{\pi}{4} \operatorname{tg} a} - \frac{\operatorname{tg} \frac{\pi}{4} - \operatorname{tg} a}{1 + \operatorname{tg} \frac{\pi}{4} \operatorname{tg} a} \right] = \\ &= \left(\frac{1 - \operatorname{tg}^2 a}{\operatorname{tg} a} \right) \left[\frac{1 + \operatorname{tg} a}{1 - \operatorname{tg} a} - \frac{1 - \operatorname{tg} a}{1 + \operatorname{tg} a} \right] = \left(\frac{1 - \operatorname{tg}^2 a}{\operatorname{tg} a} \right) \left[\frac{(1 + \operatorname{tg} a)^2 - (1 - \operatorname{tg} a)^2}{1 - \operatorname{tg}^2 a} \right] = \\ &= \left(\frac{1 - \operatorname{tg}^2 a}{\operatorname{tg} a} \right) \left[\frac{1 + \operatorname{tg}^2 a + 2 \operatorname{tg} a - (1 + \operatorname{tg}^2 a - 2 \operatorname{tg} a)}{1 - \operatorname{tg}^2 a} \right] = \left(\frac{1 - \operatorname{tg}^2 a}{\operatorname{tg} a} \right) \frac{4 \operatorname{tg} a}{1 - \operatorname{tg}^2 a} = \frac{4 \operatorname{tg} a}{\operatorname{tg} a} = 4 \end{aligned}$$

6) $y = \operatorname{sen} \frac{x}{4}$ es una senoide, hacemos la tabla de valores (en radianes):

x	0	π	2π	3π	4π
y	0	$\frac{\sqrt{2}}{2}$	1	$-\frac{\sqrt{2}}{2}$	0

