9) $\sqrt{x-4} - 3 = -\sqrt{x-1}$

$\sqrt{x-4} = -\sqrt{x-1} + 3$

$(\cancel{\sqrt{x-4}})^{\cancel{2}} = (3 - \sqrt{x-1})^2$

$\cancel{x} - 4 = 9 + \cancel{x} - 1 - 6\sqrt{x-1}$

$-4 - 8 = -6\sqrt{x-1}$

$\dfrac{-12}{-6} = \sqrt{x-1}$

$2 = \sqrt{x-1}$

$2^2 = (\cancel{\sqrt{x-1}})^{\cancel{2}}$

$4 = x - 1$

$5 = x$

Verificamos:

$x = 5 \qquad \sqrt{5-4} - 3 \overset{?}{=} -\sqrt{5-1}$

$\sqrt{1} - 3 = -\sqrt{4}$

$1 - 3 = -2$

$-2 = -2$ SÍ

$x = 5$. SÍ ES SOLUCIÓN

11) $2\sqrt{2x-7} - \sqrt{2x-15} = \sqrt{6x-23}$

$$\left(2\sqrt{2x-7} - \sqrt{2x-15}\right)^2 = \left(\cancel{\sqrt{6x-23}}\right)^{\cancel{2}}$$

$$4\cdot(2x-7) + 2x - 15 - 4\sqrt{(2x-7)(2x-15)} = 6x-23$$

$$8x - 28 + 2x - 15 - 4\sqrt{4x^2-30x-14x+105} = 6x-23$$

$$10x - 43 - 4\sqrt{4x^2-44x+105} = 6x-23$$

$$-4\sqrt{4x^2-44x+105} = 6x-23-10x+43$$

$$\sqrt{4x^2-44x+105} = \frac{-4x+20}{-4}$$

$$\sqrt{4x^2-44x+105} = \frac{\cancel{-4}(x-5)}{\cancel{-4}}$$

$$\left(\cancel{\sqrt{4x^2-44x+105}}\right)^{\cancel{2}} = (x-5)^2$$

$$4\cancel{x^2} - 44x + 105 = \cancel{x^2} + 25 - 10x$$

$$3x^2 - 34x + 80 = 0$$

$$x = \frac{34 \pm \sqrt{34^2-4\cdot3\cdot80}}{6} = \frac{34 \pm \sqrt{196}}{6} =$$

$$= \frac{34 \pm 14}{6} = \begin{cases} x_1 = 8 \\ x_2 = \frac{20}{6} = \frac{10}{3} \end{cases}$$

Verificamos:

$x = 8$ $\qquad 2\sqrt{2\cdot8-7} - \sqrt{2\cdot8-15} \overset{?}{=} \sqrt{6\cdot8-23}$

$$2\cdot\sqrt{9} - \sqrt{1} = \sqrt{48-23}$$

$$2\cdot3 - 1 = \sqrt{25}$$

$$5 = 5 \quad \text{Sí}$$

$x = 8$ SÍ ES SOLUCIÓN

$$x = \frac{10}{3} \qquad 2\sqrt{2\cdot\frac{10}{3} - 7} - \sqrt{2\cdot\frac{10}{3} - 15} = \sqrt{6\cdot\frac{10}{3} - 23}$$

$$2\sqrt{\frac{20-21}{3}} - \sqrt{\frac{20-45}{3}} = \sqrt{\frac{60-69}{3}}$$

Las raices son números $x \notin \mathbb{R}$.

Por lo tanto la única solución es $\underline{\underline{x = 8}}$

12) $\sqrt{2x+7} + \sqrt{x+8} - 6 = 0$

$$\sqrt{2x+7} = 6 - \sqrt{x+8}$$

$$(\sqrt{2x+7})^2 = (6 - \sqrt{x+8})^2$$

$$\cancel{2}x + 7 = 36 + \cancel{x} + 8 - 12\sqrt{x+8}$$

$$x - 37 = -12\sqrt{x+8}$$

$$\frac{x-37}{-12} = \sqrt{x+8}$$

$$\frac{(x-37)^2}{144} = x+8$$

$$x^2 + 1369 - 74x = 144(x+8)$$

$$x^2 + 1369 - 74x = 144x + 1152$$

$$x^2 - 218x + 217 = 0$$

$$x^2 - 218x + 217 = 0$$

$$x = \frac{218 \pm \sqrt{218^2 - 4 \cdot 217}}{2} = \frac{218 \pm \sqrt{47524 - 868}}{2} =$$

$$= \frac{218 \pm \sqrt{46656}}{2} = \frac{218 \pm 216}{2} = \begin{cases} x_1 = 217 \\ x_2 = 1. \end{cases}$$

Verificamos.

$x = 217$

$$\sqrt{2 \cdot 217 + 7} + \sqrt{217 + 8} - 6 \overset{?}{=} 0$$

$$\sqrt{441} + \sqrt{225} - 6 = 0$$

$$21 + 15 - 6 = 0$$

$$30 \neq 0 \quad \text{NO.}$$

$x = 217$ NO ES SOLUCIÓN.

$x = 1$

$$\sqrt{2 \cdot 1 + 7} + \sqrt{1 + 8} - 6 \overset{?}{=} 0$$

$$\sqrt{9} + \sqrt{9} - 6 = 0$$

$$3 + 3 - 6 = 0$$

$$0 = 0 \quad \text{SÍ}$$

$x = 1$ SI ES SOLUCIÓN.

13) $\sqrt{4x^2+4x-1} = \sqrt{x^2-1} + x$

$$\left(\sqrt{4x^2+4x-1}\right)^2 = \left(\sqrt{x^2-1} + x\right)^2$$

$$4x^2+4x-1 = x^2-1 + x^2 + 2x\sqrt{x^2-1}$$

$$\cancel{4x^2} + 4x \cancel{-1} = \cancel{2x^2} \cancel{-1} + 2x\sqrt{x^2-1}$$

$$\frac{2x^2+4x}{2x} = \sqrt{x^2-1}$$

$$\frac{(2x^2+4x)^2}{4x^2} = x^2-1$$

$$4x^4 + 16x^2 + 16x^3 = 4x^2(x^2-1)$$

$$\cancel{4x^4} + 16x^2 + 16x^3 = \cancel{4x^4} - 4x^2$$

$$16x^2 + 16x^3 + 4x^2 = 0$$

$$16x^3 + 20x^2 = 0$$

$$4x^2(4x+5) = 0 \quad \begin{cases} 4x^2 = 0 \Rightarrow x = 0 \\ 4x+5 = 0 \end{cases}$$

$$4x = -5$$

$$x = -\frac{5}{4}$$

Verificamos.

$x=0$. $\sqrt{4\cdot 0^2 + 4\cdot 0 - 1} \overset{?}{=} \sqrt{0^2-1} + 0$

$$\sqrt{-1} = \sqrt{-1} \quad \text{Sí}$$

$x=0$ SÍ ES SOLUCIÓN.

$x=-\frac{5}{4}$ $\qquad \sqrt{4\cdot\left(-\frac{5}{4}\right)^2+4\left(-\frac{5}{4}\right)-1} \stackrel{?}{=} \sqrt{\left(-\frac{5}{4}\right)^2-1}+\left(-\frac{5}{4}\right)$

$$\sqrt{4\cdot\frac{25}{16}-5-1} = \sqrt{\frac{25-16}{16}}-\frac{5}{4}$$

$$\sqrt{\frac{25}{4}-6} = \sqrt{\frac{9}{16}}-\frac{5}{4}$$

$$\sqrt{\frac{25-24}{4}} = \frac{3}{4}-\frac{5}{4}$$

$$\sqrt{\frac{1}{4}} = -\frac{2}{4}$$

$$\frac{1}{2} \neq -\frac{1}{2} \quad \text{No}$$

$x=-\frac{5}{4}$ NO ES SOLUCIÓN.

10) $\sqrt{x^2-1}+3=2\sqrt{x^2+4x+5}-x$

$$\sqrt{x^2-1}+3+x=2\sqrt{x^2+4x+5}$$

$$\left(\sqrt{x^2-1}+\underbrace{3+x}\right)^2=\left(2\sqrt{x^2+4x+5}\right)^2$$

$$x^2-1+(3+x)^2+\underbrace{2(3+x)\sqrt{x^2-1}}=4(x^2+4x+5)$$

$$x^2-1+9+x^2+6x+2(3+x)\sqrt{x^2-1}=4x^2+16x+20$$

$$2x^2+6x+8+\underbrace{2(3+x)\sqrt{x^2-1}}=4x^2+16x+20$$

$$2(3+x)\sqrt{x^2-1}=4x^2+16x+20-2x^2-6x-8$$

$$2(3+x)\sqrt{x^2-1}=2x^2+10x+12$$

$$\cancel{2}(3+x)\sqrt{x^2-1} = \cancel{2}(x^2+5x+6)$$

$$(3+x)\sqrt{x^2-1} = (x+3)(x+2)$$

Luego: 1º) Si $x+3 \neq 0$ podemos simplificar por $x+3$ los dos miembros

$$\cancel{(x+3)}\sqrt{x^2-1} = \cancel{(x+3)}(x+2)$$

$$\sqrt{x^2-1} = x+2$$

$$\left(\sqrt{x^2-1}\right)^2 = (x+2)^2$$

$$\cancel{x^2}-1 = \cancel{x^2}+4+4x$$

$$4x+5=0$$

$$x = -\frac{5}{4}$$

2º) Si $x+3=0$

$$x=-3$$

Verificamos:

$x=-\frac{5}{4}$ $\quad \sqrt{\left(-\frac{5}{4}\right)^2-1}+3 \overset{?}{=} 2\sqrt{\left(-\frac{5}{4}\right)^2+4\left(-\frac{5}{4}\right)+5}+\frac{5}{4}$

$$\sqrt{\frac{25-16}{16}}+3 = 2\sqrt{\frac{25}{16}-\frac{20}{4}+5}+\frac{5}{4}$$

$$\sqrt{\frac{9}{16}}+3 = 2\sqrt{\frac{25-\cancel{80}+\cancel{80}}{16}}+\frac{5}{4}$$

$$\frac{3}{4}+3 = 2\sqrt{\frac{25}{16}}+\frac{5}{4}$$

$$\frac{3}{4}+3 = 2\cdot\frac{5}{4}+\frac{5}{4}$$

$$\frac{3+12}{4} = \frac{10+5}{4}$$

$$\frac{15}{4} = \frac{15}{4} \quad \text{sí}$$

$x = -\frac{5}{4}$ ES SOLUCIÓN.

$x = -3 \quad \sqrt{(-3)^2-1}+3 \stackrel{?}{=} 2\sqrt{(-3)^2+4\cdot(-3)+5}+3$

$$\sqrt{8}+3 = 2\sqrt{9-12+5}+3$$

$$\sqrt{2^3}+3 = 2\sqrt{2}+3$$

$$2\sqrt{2}+3 = 2\sqrt{2}+3 \quad \text{sí}$$

$x=-3$ SÍ ES SOLUCIÓN

5) $x+2+\sqrt{x^2-2x} = 2\sqrt{x^2+2x+2}$

$$\left(x+2+\sqrt{x^2-2x}\right)^2 = \left(2\sqrt{x^2+2x+2}\right)^2$$

$$(x+2)^2 + x^2-2x + 2(x+2)\left(\sqrt{x^2-2x}\right) = 4(x^2+2x+2)$$

$$\cancel{x^2}+4+\cancel{4x}+\cancel{x^2}-\cancel{2x} + \underbrace{2(x+2)\sqrt{x^2-2x}} = 4x^2+8x+8$$

$$2(x+2)\sqrt{x^2-2x} = 4x^2+8x+8-2x^2-2x-4$$

$$\left(2(x+2)\sqrt{x^2-2x}\right)^2 = (2x^2+\underbrace{6x+4})^2$$

$$4(x+2)^2(x^2-2x) = 4x^4+(6x+4)^2+2\cdot 2x^2\cdot(6x+4)$$

$$4(x^2+4+4x)(x^2-2x) = 4x^4+36x^2+16+48x+4x^2(6x+4)$$

$$(4x^2+16+16x)(x^2-2x) = 4x^4+36x^2+16+48x+24x^3+16x^2$$

$$4x^4-8x^3+16x^2-32x+16x^3-32x^2 = 4x^4+24x^3+52x^2+48x+16$$

$$8x^3-24x^3-16x^2-52x^2-32x-48x-16=0$$

$$-16x^3-68x^2-80x-16=0$$ Saco factor común -4

$$4x^3+17x^2+20x+4=0$$

$$\begin{array}{r|rrrr} & 4 & 17 & 20 & 4 \\ -2 & & -8 & -18 & -4 \\ \hline & 4 & 9 & 2 & \boxed{0} \end{array}$$

T. indp. $=4$

Divisores: $\pm 1, \pm 2, \pm 4$

$$4x^2+9x+2=0$$

$$x = \frac{-9 \pm \sqrt{9^2-4\cdot 4\cdot 2}}{4\cdot 2} = \frac{-9 \pm \sqrt{81-32}}{8} = \frac{-9 \pm \sqrt{49}}{8} =$$

$$= \frac{-9 \pm 7}{8} = \begin{cases} x_1 = -\dfrac{1}{4} \\ x_2 = -2 \end{cases}$$

Verificamos las soluciones:

$x = -\frac{1}{4}$

$$-\frac{1}{4}+2+\sqrt{\left(-\frac{1}{4}\right)^2-2\cdot\left(-\frac{1}{4}\right)} \stackrel{?}{=}$$

$$2\sqrt{\left(-\frac{1}{4}\right)^2+2\cdot\left(-\frac{1}{4}\right)+2}$$

$$\frac{-1+8}{4} + \sqrt{\frac{1}{16}+\frac{1}{2}} = 2\sqrt{\frac{1}{16}-\frac{1}{2}+2}$$

$$\frac{7}{4} + \sqrt{\frac{1+8}{16}} = 2\sqrt{\frac{1-8+32}{16}}$$

$$\frac{7}{4} + \sqrt{\frac{9}{16}} = 2\sqrt{\frac{25}{16}}$$

$$\frac{10}{4} = \frac{10}{4}$$ SÍ $\quad x = -\frac{1}{4}$ SÍ ES SOLUCIÓN.

$x = -2$

$$-2+2+\sqrt{(-2)^2-2\cdot(-2)} \overset{?}{=} 2\sqrt{(-2)^2+2\cdot(-2)+2}$$

$$\sqrt{4+4} = 2\sqrt{\cancel{4}-\cancel{4}+2}$$

$$\sqrt{8} = 2\sqrt{2}$$

$$\sqrt{2^3} = 2\sqrt{2}$$

$$2\sqrt{2} = 2\sqrt{2}$$ SÍ.

$x = -2$ SÍ ES SOLUCIÓN.

Soluciones de la ecuación:

$$x = -\frac{1}{4}$$

$$x = -2$$