Resuelve las siguientes ecuaciones logarítmicas:

1) $2\log x-\log(x-16)=2$

2) $\log x^3=\log 6+2\log x$

3) $\log 8+(x^2-5x+7)\log 3=\log 24$

4) $\log(3x-1)-\log(2x+3)=1-\log 25$

5) $\log x^3=\log 6+2\log x$

6) $\dfrac{\log 2+\log(11-x^2)}{\log(5-x)}=2$

7) $\log(7x-9)^2+\log(3x-4)^2=2$

8) $(x^2-4x+7)\log 5+\log 16=4$

9) $(x^2-x-3)\log 4=3\log\dfrac{1}{4}$

10) $\dfrac{\log(16-x^2)}{\log(3x-4)}=2$

11) $\log x^2-\log\dfrac{10x+11}{10}=1$

12) $\log(5x+4)-\log 2=\dfrac{1}{2}\log(x+4)$

13) $\log x+\log 50=\log 1000$

14) $\log x^2-\log\dfrac{10x+11}{10}=1$

15) $(x^2-5x+9)\log 2+\log 125=3$

16) $\log(2^{2-x})^{2+x}+\log 1250=4$

Resuelve las siguientes ecuaciones logarítmicas:

1) $2 \log x - \log (x-16) = 2$

$$\log x^2 - \log (x-16) = 2$$

$$\log \frac{x^2}{x-16} = 2$$

$$10^2 = \frac{x^2}{x-16}$$

$$10^2(x-16) = x^2$$

$$100x - 1600 = x^2$$

$$x^2 - 100x + 1600 = 0$$

$$x = \frac{100 \pm \sqrt{100^2 - 4 \cdot 1600}}{2} = \frac{100 \pm \sqrt{3600}}{2} =$$

$$= \frac{100 \pm 60}{2} = \begin{cases} x_1 = 80 \\ x_2 = 20 \end{cases}$$

"CANDIDATAS"

$x_1 = 80$ $\quad 2 \log 80 - \log (80-16) = 2$ Sí

$x_2 = 20$ $\quad 2 \log 20 - \log (20-16) = 2$ Sí

Las dos soluciones son válidas.

$x_1 = 80$

$x_2 = 20$.

2) $\log x^3 = \log 6 + 2\log x$

$\log x^3 = \log 6 + \log x^2$

$\log x^3 = \log 6x^2$

$x^3 = 6x^2$

$x^3 - 6x^2 = 0$

$x^2(x-6) = 0$

1) $x^2 = 0 \Rightarrow x = 0$

2) $x - 6 = 0 \Rightarrow x = 6.$

"CANDIDATAS":

$x = 0$ $\quad \log \cancel{0}^3 = \log 6 + 2\log \cancel{0}$ NO VALE

$\underline{\underline{x = 6}}$ $\quad \log 6^3 = \log 6 + 2\log 6.$ $\underline{\text{SÍ}}$

3) $\log 8 + (x^2 - 5x + 7)\log 3 = \log 24.$

$\log 8 + \log 3^{x^2-5x+7} = \log 24$

$\log 8 \cdot 3^{x^2-5x+7} = \log 24$

$8 \cdot 3^{x^2-5x+7} = 24$

$\cancel{8} \cdot 3^{x^2-5x+7} = \cancel{8} \cdot 3$

$3^{x^2-5x+7} = 3$

$x^2 - 5x + 7 = 1$

$x^2 - 5x + 6 = 0$

$x_1 = 3$

$x_2 = 2$

COMPROBACIÓN

$x = 3$ $\quad \log 8 + (3^2 - 5 \cdot 3 + 7) \log 3 = \log 24$

$x = 2$ $\quad \log 8 + (2^2 - 5 \cdot 2 + 7) \log 3 = \log 24$

VALEN LAS DOS.

4) $\log (3x - 1) - \log (2x + 3) = 1 - \log 25$

$$\log \frac{3x-1}{2x+3} = \log 10 - \log 25$$

$$\log \frac{3x-1}{2x+3} = \log \frac{10}{25}$$

$$\log \frac{3x-1}{2x+3} = \log \frac{2}{5}$$

$$\frac{3x-1}{2x+3} = \frac{2}{5}$$

$$5(3x-1) = 2(2x+3)$$

$$15x - 5 = 4x + 6$$

$$11x = 11$$

$x = 1$. "CANDIDATA"

$x = 1$ $\quad \log (3 \cdot 1 - 1) - \log (2 \cdot 1 + 3) = 1 - \log 25$

$\log 2 - \log 5 = 1 - \log 25$.

ES SOLUCIÓN.

6) $$\frac{\log 2 + \log(11-x^2)}{\log(5-x)} = 2$$

$$\frac{\log 2(11-x^2)}{\log(5-x)} = 2$$

$$\log 2(11-x^2) = 2\log(5-x)$$

$$\log 2(11-x^2) = \log(5-x)^2$$

$$2(11-x^2) = (5-x)^2$$

$$22 - 2x^2 = 25 + x^2 - 10x$$

$$3x^2 - 10x + 3 = 0$$

$$x = \frac{10 \pm \sqrt{100-36}}{6} = \frac{10 \pm 8}{6} = \begin{cases} x_1 = 3 \\ x_2 = \frac{1}{3} \end{cases}$$

"CANDIDATAS":

$x_1 = 3$ $$\frac{\log 2 + \log 2}{\log 2} = 2 \quad \underline{\underline{\text{SÍ}}}$$

$x = \frac{1}{3}$ $$\frac{\log 2 + \log\left(11 - \left(\frac{1}{3}\right)^2\right)}{\log\left(5 - \frac{1}{3}\right)} = 2 \quad \underline{\underline{\text{SÍ}}}$$

8) $(x^2-4x+7)\log 5+\log 16=4$

$\log 5^{x^2-4x+7}+\log 16=4$

$\log 5^{x^2-4x+7}\cdot 16=4$

$10^4=5^{x^2-4x+7}\cdot 16$

$\cancel{2^4}\cdot 5^4=5^{x^2-4x+7}\cdot\cancel{2^4}$

$5^4=5^{x^2-4x+7}$

$4=x^2-4x+7$

$x^2-4x+3=0$

$x_1=3$

$x_2=1$

"CANDIDATAS":

$x=3$ $\quad\log 5^{3^2-4\cdot 3+7}+\log 16=4$ $\quad\underline{\underline{\text{Sí}}}$

$x=1$ $\quad\log 5^{1^2-4\cdot 1+7}+\log 16=4$ $\quad\underline{\underline{\text{Sí}}}$

9) $(x^2-x-3)\log 4=3\log\frac{1}{4}$

$\log 4^{x^2-x-3}=\log\left(\frac{1}{4}\right)^3$

$4^{x^2-x-3}=4^{-3}$

$x^2-x-\cancel{3}=-\cancel{3}$

$x^2-x=0$

$x^2 - x = 0$

$x(x-1) = 0$

1) $x = 0$

2) $x - 1 = 0 \Longrightarrow x = 1$

"CANDIDATAS"

$x = 0 \quad \log 4^{0^2 - 0 - 3} = 3 \log \frac{1}{4}$

$\log 4^{-3} = 3 \log \frac{1}{4} \Longrightarrow \underline{\underline{\text{SÍ}}}$

$x = 1 \quad \log 4^{1^2 - 1 - 3} = 3 \log \frac{1}{4} \Longrightarrow \underline{\underline{\text{SÍ}}}$

10) $\dfrac{\log(16 - x^2)}{\log(3x-4)} = 2$

$\log(16 - x^2) = 2 \log(3x - 4)$

$\log(16 - x^2) = \log(3x-4)^2$

$\cancel{16} - x^2 = 9x^2 + \cancel{16} - 24x$

$9x^2 + x^2 - 24x = 0$

$10x^2 - 24x = 0$

$2x(5x - 12) = 0$

1) $2x = 0 \Longrightarrow x = 0$

2) $5x - 12 = 0 \Longrightarrow x = \frac{12}{5}$

"CANDIDATAS".

$x=0 \qquad \dfrac{\log 16}{\cancel{\log(-4)}} = 2 \qquad$ NO VALE

$x = \dfrac{12}{5} \qquad \dfrac{\log\left(16-\left(\frac{12}{5}\right)^2\right)}{\log\left(3\cdot\frac{12}{5}-4\right)} = 2 \qquad$ SÍ

Única solución:

$$x = \frac{12}{5}$$

11) $\log x^2 - \log \dfrac{10x+11}{10} = 1$

$$\log \frac{x^2}{\frac{10x+11}{10}} = 1$$

$$10 = \frac{x^2}{\frac{10x+11}{10}}$$

$$\cancel{10}\cdot\frac{10x+11}{\cancel{10}} = x^2$$

$$10x+11-x^2=0$$

$$-x^2 + 10x + 11 = 0$$

$$x = \frac{-10 \pm \sqrt{100+44}}{-2} = \frac{-10 \pm 12}{-2} \begin{cases} x = -1 \\ x = 11 \end{cases}$$

"CANDIDATAS":

$x = -1$ $\quad \log (-1)^2 - \log \frac{10 \cdot (-1) + 11}{10} = 1$

$\log 1 - \log \frac{1}{10} = 1$ $\quad$ SÍ

$x = 11$ $\quad \log 11^2 - \log \frac{10 \cdot 11 + 11}{10} = 1$ $\quad$ SÍ

Las dos soluciones son válidas.

13) $\log x + \log 50 = \log 1000$

$$\log 50x = \log 1000$$

$$50x = 1000$$

$$x = 20$$

"CANDIDATA"

$x = 20$ $\quad \log 20 + \log 50 = \log 1000$

SÍ ES SOLUCIÓN

15) $(x^2-5x+9)\log 2+\log 125=3$

$$\log 2^{x^2-5x+9}+\log 125=3$$

$$\log 2^{x^2-5x+9}\cdot 125=3$$

$$10^3=2^{x^2-5x+9}\cdot 125$$

$$\cancel{5^3}\cdot 2^3=2^{x^2-5x+9}\cdot\cancel{5^3}$$

$$2^3=2^{x^2-5x+9}$$

$$3=x^2-5x+9$$

$$x^2-5x+6=0$$

$$x_1=3$$
$$x_2=2$$

"CANDIDATAS":

$x=3$ $\quad\log 2^{3^2-5\cdot 3+9}+\log 125=3$ $\quad\underline{\underline{\text{SÍ}}}$

$x=2$ $\quad\log 2^{2^2-5\cdot 2+9}+\log 125=3$ $\quad\underline{\underline{\text{SÍ}}}$

Las dos soluciones son válidas

16) $\log (2^{2-x})^{2+x}+\log 1250=4$

$$\log 2^{(2-x)\cdot(2+x)}\cdot 1250=4$$

$$10^4=2^{4-x^2}\cdot 1250$$

$$2^4\cdot 5^4=2^{4-x^2}\cdot 2\cdot 5^4$$

$$2^4=2^{4-x^2+1}$$

$\cancel{4} = \cancel{4} - x^2 + 1$

$x^2 = 1$

$x = \pm 1$

"CANDIDATAS":

$x = 1 \quad \log 2^3 + \log 1250 = 4$ Sí

$x = -1 \quad \log 2^3 + \log 1250 = 4$ Sí

Las dos soluciones son válidas.

7) $\log (7x-9)^2 + \log (3x-4)^2 = 2$

$\log (7x-9)^2 (3x-4)^2 = 2$

$(7x-9)^2(3x-4)^2 = 10^2$

$[(7x-9)(3x-4)]^2 = 10^2$

$x^2 = 9$

$x^2 = 3^2 \begin{cases} x = 3 \\ x = -3 \end{cases}$

1) $(7x-9)(3x-4) = 10$

$21x^2 - 28x - 27x + 36 = 10$

$21x^2 - 55x + 26 = 0$

$x = \dfrac{55 \pm \sqrt{55^2 - 4 \cdot 21 \cdot 26}}{2 \cdot 21} = \dfrac{55 \pm 29}{42} \begin{cases} x_1 = 2 \\ x_2 = \dfrac{13}{21} \end{cases}$

2) $(7x-9)(3x-4) = -10$

$21x^2 - 28x - 27x + 36 = -10$

$21x^2 - 28x - 27x + 46$

$$21x^2 - 55x + 46 = 0$$

$$x = \frac{+55 \pm \sqrt{55^2 - 4 \cdot 21 \cdot 46}}{2 \cdot 21} = \frac{55 \pm \sqrt{-839}}{42} \not\in \mathbb{R}$$

"CANDIDATAS":

$x = 2 \qquad \log(7 \cdot 2 - 9)^2 + \log(3 \cdot 2 - 4)^2 = 2$

$x = \frac{13}{21} \qquad \log\left(7 \cdot \frac{13}{21} - 9\right)^2 + \log\left(7 \cdot \frac{13}{21} - 4\right)^2 = 2$

Las dos soluciones son válidas.

12) $\log(5x+4) - \log 2 = \frac{1}{2}\log(x+4)$

$$\log \frac{5x+4}{2} = \log (x+4)^{1/2}$$

$$\frac{5x+4}{2} = \sqrt{x+4}$$

$$5x+4 = 2\sqrt{x+4}$$

$$(5x+4)^2 = (2\sqrt{x+4})^2$$

$$25x^2 + 16 + 40x = 4(x+4)$$

$$25x^2 + \cancel{16} + 40x = 4x + \cancel{16}$$

$$25x^2 + 36x = 0$$

$$x(25 + 36) = 0$$

1) $x = 0$

2) $25x + 36 = 0$

$$25x = -36$$

$$x = -\frac{36}{25}$$

"CANDIDATAS":

$x=0$ $\quad \log 4 - \log 2 = \frac{1}{2}\log 4$ **SÍ**

$x = -\frac{36}{25}$ $\quad \log\left(\cancel{-\frac{36}{5}}+4\right) - \log 2 = \frac{1}{2}\log\left(-\frac{36}{25}+4\right)$

NO VALE $\Uparrow$

La única solución válida es $x = 0$.