

Ecuaciones trigonométricas

Resuelve las siguientes ecuaciones trigonométricas:

1) $\cos 2\alpha + \sin^2 \alpha = 1$

$$\cos^2 \alpha - \sin^2 \alpha + \sin^2 \alpha = 1 \rightarrow \cos^2 \alpha = 1 \rightarrow \cos \alpha = \pm 1 \rightarrow \begin{cases} \cos \alpha = 1 \rightarrow \alpha = 0^\circ + 360^\circ k \\ \cos \alpha = -1 \rightarrow \alpha = 180^\circ + 360^\circ k \end{cases}$$

2) $\cos 2\alpha - \sin \alpha = 1$

$$\cos^2 \alpha - \sin^2 \alpha - \sin \alpha = 1 \rightarrow 1 - \sin^2 \alpha - \sin^2 \alpha - \sin \alpha = 1 \rightarrow \sin \alpha (2 \sin \alpha + 1) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \sin \alpha = 0 \rightarrow \alpha = \begin{cases} 0^\circ + 360^\circ k \\ 180^\circ + 360^\circ k \end{cases} \\ 2 \sin \alpha + 1 = 0 \rightarrow \sin \alpha = -\frac{1}{2} \rightarrow \alpha = \begin{cases} 210^\circ + 360^\circ k \\ 330^\circ + 360^\circ k \end{cases} \end{cases}$$

3) $\cos 2\alpha = \sin \alpha$

$$\cos^2 \alpha - \sin^2 \alpha = \sin \alpha \rightarrow 1 - \sin^2 \alpha - \sin^2 \alpha = \sin \alpha \rightarrow -2 \sin^2 \alpha - \sin \alpha + 1 = 0 \xrightarrow{x = \sin \alpha} \rightarrow$$

$$\rightarrow -2x^2 - x + 1 = 0 \rightarrow x = \begin{cases} \frac{1}{2} \\ -1 \end{cases} \rightarrow \sin \alpha = \begin{cases} \frac{1}{2} \rightarrow \alpha = \begin{cases} 30^\circ + 360^\circ k \\ 150^\circ + 360^\circ k \end{cases} \\ -1 \rightarrow \alpha = 270^\circ + 360^\circ k \end{cases}$$

4) $\sin \alpha + \cos 2\alpha = 1$

$$\sin \alpha + \cos^2 \alpha - \sin^2 \alpha = 1 \rightarrow \sin \alpha + 1 - \sin^2 \alpha - \sin^2 \alpha = 1 \rightarrow \sin \alpha - 2 \sin^2 \alpha = 0 \rightarrow$$

$$\rightarrow \sin \alpha (1 - 2 \sin \alpha) = 0 \rightarrow \begin{cases} \sin \alpha = 0 \rightarrow \begin{cases} 0^\circ + 360^\circ k \\ 180^\circ + 360^\circ k \end{cases} \\ 1 - 2 \sin \alpha = 0 \rightarrow \sin \alpha = \frac{1}{2} \rightarrow \alpha = \begin{cases} 30^\circ + 360^\circ k \\ 150^\circ + 360^\circ k \end{cases} \end{cases}$$

5) $\sin^2 \alpha + \cos \alpha = \frac{5}{4}$

$$1 - \cos^2 \alpha + \cos \alpha = \frac{5}{4} \rightarrow 4 \cos^2 \alpha - 4 \cos \alpha + 1 = 0 \rightarrow \cos \alpha = \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{1}{2} \rightarrow$$

$$\rightarrow \alpha = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases}$$

6) $\sin 2\alpha = 2 \cos \alpha$

$$2 \sin \alpha \cos \alpha = 2 \cos \alpha \rightarrow 2 \sin \alpha \cos \alpha - 2 \cos \alpha = 0 \rightarrow \cos \alpha (2 \sin \alpha - 2) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \cos \alpha = 0 \rightarrow \alpha = \begin{cases} 90^\circ + 360^\circ k \\ 270^\circ + 360^\circ k \end{cases} \\ 2 \sin \alpha - 2 = 0 \rightarrow \sin \alpha = 1 \rightarrow \alpha = 90^\circ + 360^\circ k \end{cases}$$

7) $\sin 4\alpha = \sin 2\alpha$

$$2 \sin 2\alpha \cos 2\alpha = \sin 2\alpha \rightarrow 2 \sin 2\alpha \cos 2\alpha - \sin 2\alpha = 0 \rightarrow \sin 2\alpha (2 \cos 2\alpha - 1) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \sin 2\alpha = 0 \rightarrow \alpha = \begin{cases} 0^\circ + 360^\circ k \\ 90^\circ + 360^\circ k \end{cases} \\ \cos 2\alpha = \frac{1}{2} \rightarrow 2\alpha = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \rightarrow \alpha = \begin{cases} 30^\circ + 180^\circ k \\ 150^\circ + 180^\circ k \end{cases} \end{cases}$$

8) $2 \sin \alpha = \operatorname{tg} \alpha$

$$2 \sin \alpha = \frac{\sin \alpha}{\cos \alpha} \rightarrow 2 \sin \alpha \cos \alpha = \sin \alpha \rightarrow 2 \sin \alpha \cos \alpha - \sin \alpha = 0 \rightarrow \sin \alpha (2 \cos \alpha - 1) = 0$$

$$\rightarrow \begin{cases} \sin \alpha = 0 \rightarrow \alpha = \begin{cases} 0^\circ + 360^\circ k \\ 180^\circ + 360^\circ k \end{cases} \\ 2 \cos \alpha - 1 = 0 \rightarrow \cos \alpha = \frac{1}{2} \rightarrow \alpha = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \end{cases}$$

9) $\sin \alpha = 1 + 2 \cos^2 \alpha$

$$\sin \alpha = 1 + 2(1 - \sin^2 \alpha) \rightarrow \sin \alpha = 1 + 2 - 2 \sin^2 \alpha \rightarrow 2 \sin^2 \alpha + \sin \alpha - 3 = 0 \rightarrow \sin \alpha = 1 \rightarrow \alpha = 90^\circ + 360^\circ k$$

10) $\sec \alpha + \operatorname{tg} \alpha = 0$

$$\frac{1}{\cos \alpha} + \frac{\sin \alpha}{\cos \alpha} = 0 \rightarrow 1 + \sin \alpha = 0 \rightarrow \sin \alpha = -1 \rightarrow \alpha = 270^\circ + 360^\circ k$$

11) $6 \cos^2 \frac{\alpha}{2} + \cos \alpha = 1$

$$6 \left(\sqrt{\frac{1 + \cos \alpha}{2}} \right)^2 + \cos \alpha = 1 \rightarrow 6 \left(\frac{1 + \cos \alpha}{2} \right) + \cos \alpha = 1 \rightarrow 3 + 3 \cos \alpha + \cos \alpha = 1 \rightarrow \cos \alpha = -\frac{1}{2} \rightarrow \alpha = \begin{cases} 120^\circ + 360^\circ k \\ 240^\circ + 360^\circ k \end{cases}$$

12) $6 \cos^2 \alpha + 6 \sin^2 \alpha = 5 + \sin \alpha$

$$6(\cos^2 \alpha + \sin^2 \alpha) = 5 + \sin \alpha \rightarrow 6 = 5 + \sin \alpha \rightarrow \sin \alpha = 1 \rightarrow \alpha = 90^\circ + 360^\circ k$$

13) $\operatorname{tg} 2\alpha \operatorname{tg} \alpha = 1$

$$\frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha} \operatorname{tg} \alpha = 1 \rightarrow \frac{2 \operatorname{tg}^2 \alpha}{1 - \operatorname{tg}^2 \alpha} = 1 \rightarrow 2 \operatorname{tg}^2 \alpha = 1 - \operatorname{tg}^2 \alpha \rightarrow \operatorname{tg} \alpha = \pm \frac{1}{\sqrt{3}} \rightarrow \alpha = \begin{cases} 30^\circ + 180^\circ k \\ 150^\circ + 180^\circ k \end{cases}$$

14) $\sin 2\alpha \cos \alpha = 6 \sin^3 \alpha$

$$2 \sin \alpha \cos \alpha \cos \alpha - 6 \sin^3 \alpha = 0 \rightarrow 2 \sin \alpha (\cos^2 \alpha - 3 \sin^2 \alpha) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \sin \alpha = 0 \rightarrow \alpha = \begin{cases} 0^\circ + 360^\circ k \\ 180^\circ + 360^\circ k \end{cases} \\ \cos^2 \alpha - 3 \sin^2 \alpha = 0 \rightarrow \cos^2 \alpha - 3(1 - \cos^2 \alpha) = 0 \rightarrow \cos \alpha = \pm \frac{\sqrt{3}}{2} \rightarrow \alpha = \begin{cases} 30^\circ + 180^\circ k \\ 150^\circ + 360^\circ k \end{cases} \end{cases}$$

15) $\sin \alpha + \cos \alpha = \sqrt{2}$

$$\frac{1}{2} \sin \alpha + \frac{\sqrt{3}}{2} \cos \alpha = 1 \rightarrow \sin(\alpha + 60^\circ) = 1 \rightarrow \alpha + 60^\circ = 90^\circ + 360^\circ k \rightarrow \alpha = 30^\circ + 360^\circ k$$

16) $\sin \alpha + \cos \alpha = \frac{5}{2}$

Esta ecuación no tiene solución ya que $\sin \alpha + \cos \alpha > 2$, y esto es imposible ya que $\sin \alpha, \cos \alpha \in [-1, 1]$, y por tanto, $-2 \leq \sin \alpha + \cos \alpha \leq 2$.

17) $2 \cos^2 \alpha + \cos \alpha - 1 = 0$

$$\cos \alpha = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} = \begin{cases} \frac{1}{2} \rightarrow \alpha = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \\ -1 \rightarrow \alpha = 180^\circ + 360^\circ k \end{cases}$$

18) $2 \sin^2 \alpha - 1 = 0$

$$\sin^2 \alpha = \frac{1}{2} \rightarrow \sin \alpha = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$\text{Si } \sin \alpha = \frac{\sqrt{2}}{2} \rightarrow \alpha = \begin{cases} 45^\circ + 360^\circ k \\ 135^\circ + 360^\circ k \end{cases}$$

$$\text{Si } \sin \alpha = -\frac{\sqrt{2}}{2} \rightarrow \alpha = \begin{cases} -45^\circ + 360^\circ k \\ 225^\circ + 360^\circ k \end{cases} = \begin{cases} 315^\circ + 360^\circ k \\ 225^\circ + 360^\circ k \end{cases}$$

19) $\tan^2 \alpha - \tan \alpha = 0$

$$\tan \alpha (\tan \alpha - 1) = 0 \rightarrow \begin{cases} \tan \alpha = 0 \rightarrow \alpha = 0^\circ + 180^\circ k \\ \tan \alpha - 1 = 0 \rightarrow \tan \alpha = 1 \rightarrow \alpha = 45^\circ + 180^\circ k \end{cases}$$

20) $2 \sin^2 \alpha + 3 \cos \alpha = 3$

$$2(1 - \cos^2 \alpha) + 3 \cos \alpha = 3 \rightarrow 2 - 2 \cos^2 \alpha + 3 \cos \alpha = 3 \rightarrow 2 \cos^2 \alpha - 3 \cos \alpha + 1 = 0 \rightarrow$$

$$\rightarrow \cos \alpha = \begin{cases} 1 \rightarrow \alpha = 0^\circ + 360^\circ k \\ \frac{1}{2} \rightarrow \alpha = \begin{cases} 60^\circ + 360^\circ k \\ 300^\circ + 360^\circ k \end{cases} \end{cases}$$

21) $\tan 2\alpha + 2 \cos \alpha = 0$

$$\frac{2 \tan \alpha}{1 - \tan^2 \alpha} + 2 \cos \alpha = 0 \rightarrow \frac{2 \frac{\sin \alpha}{\cos \alpha}}{1 - \frac{\sin^2 \alpha}{\cos^2 \alpha}} + 2 \cos \alpha = 0 \rightarrow \frac{\sin \alpha \cos \alpha}{\cos^2 \alpha - \sin^2 \alpha} + \cos \alpha = 0 \rightarrow$$

$$\rightarrow \sin \alpha \cos \alpha + \cos \alpha (\cos^2 \alpha - \sin^2 \alpha) = 0 \rightarrow \cos \alpha (\sin \alpha + \cos^2 \alpha - \sin^2 \alpha) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \cos \alpha = 0 \rightarrow \alpha = \begin{cases} 90^\circ + 360^\circ k \\ 270^\circ + 360^\circ k \end{cases} \\ \sin \alpha + \cos^2 \alpha - \sin^2 \alpha = 0 \rightarrow \sin \alpha + 1 - \sin^2 \alpha - \sin^2 \alpha = 0 \rightarrow -2\sin^2 \alpha + \sin \alpha + 1 = 0 \rightarrow \\ \rightarrow \sin \alpha = \begin{cases} -\frac{1}{2} \rightarrow \alpha = \begin{cases} 330^\circ + 360^\circ k \\ 210^\circ + 360^\circ k \end{cases} \\ 1 \rightarrow \alpha = 90^\circ + 360^\circ k \end{cases} \end{cases}$$

22) $\sqrt{2} \cos \frac{\alpha}{2} - \cos \alpha = 1$

$$\sqrt{2} \sqrt{\frac{1 + \cos \alpha}{2}} - \cos \alpha = 1 \rightarrow \sqrt{1 + \cos \alpha} - \cos \alpha = 1 \rightarrow \sqrt{1 + \cos \alpha} = 1 + \cos \alpha \rightarrow$$

$$\rightarrow 1 + \cos \alpha = 1 + \cos^2 \alpha + 2 \cos \alpha \rightarrow \cos^2 \alpha + \cos \alpha = 0 \rightarrow \cos \alpha (\cos \alpha + 1) = 0 \rightarrow$$

$$\rightarrow \begin{cases} \cos \alpha = 0 \rightarrow \alpha = \begin{cases} 90^\circ + 360^\circ k \\ \cancel{270^\circ + 360^\circ k} \end{cases} \\ \cos \alpha = -1 \rightarrow \alpha = 180^\circ + 360^\circ k \end{cases}$$

23) $2 \sin \alpha \cos^2 \alpha - 6 \sin^3 \alpha = 0$

$$2 \sin \alpha (\cos^2 \alpha - 3 \sin^2 \alpha) = 0 \rightarrow \begin{cases} \sin \alpha = 0 \\ \cos^2 \alpha - 3 \sin^2 \alpha = 0 \rightarrow \cos^2 \alpha + \sin^2 \alpha - 4 \sin^2 \alpha \rightarrow \end{cases}$$

$$\rightarrow 1 - 4 \sin^2 \alpha = 0 \rightarrow \sin^2 \alpha = \frac{1}{4} \rightarrow \sin \alpha = \pm \frac{1}{2} \rightarrow \begin{cases} \sin \alpha = \frac{1}{2} \rightarrow \alpha = \begin{cases} 30^\circ + 360^\circ k \\ 150^\circ + 360^\circ k \end{cases} \\ \sin \alpha = -\frac{1}{2} \rightarrow \alpha = \begin{cases} 210^\circ + 360^\circ k \\ 330^\circ + 360^\circ k \end{cases} \end{cases}$$