**1.** Dado el triángulo de vértices A(1,1), B(5,4) y C(-5,9), se pide:

**a)** Dibujarlo.

**b)** Demostrar que los vectores $\overrightarrow{AB}$ y $\overrightarrow{AC}$ son $\perp$

**c)** Hallar $|\overrightarrow{AB}|$ y $|\overrightarrow{AC}|$

**d)** Calcular su área. *(2 puntos)*

**2.** **a)** Hallar, en todas las formas conocidas, la ecuación de la recta **s** que tiene la misma pendiente que **r**: y=3x-1 y pasa por P(-1,2)

**b)** Hallar la distancia entre las dos rectas **r** y **s** anteriores.

**c)** Hallar el ángulo que forma **r** con la recta **t**: x-2y+4=0 *(2 puntos)*

**3.** Dadas las rectas $\left.\begin{array}{l} r: x+2y-3=0 \\ s: x-ky+4=0 \end{array}\right\}$ se pide:

**a)** Hallar **k** para que sean //

**b)** Hallar **k** para que sean $\perp$

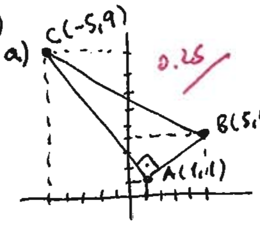
**c)** Hallar la ecuación general de la recta $\perp$ a r que pasa por el origen. *(2 puntos)*

**4.** **a)** Operar $\dfrac{(3-2i)(3+i)-(2i-3)^2}{i^{23}-i^{-13}}$ en forma binómica.

**b)** Calcular $\dfrac{(-2\sqrt{3}-2i)^5}{(-4+4\sqrt{3}\,i)^3\,2i}$ en forma polar, y pasar el resultado a binómica. *(2 puntos)*

**5.** **a)** Calcular $\sqrt[4]{\dfrac{8\sqrt{3}+8i}{-\sqrt{3}+i}}$, dando el resultado en binómica.

**b)** Dibujar los afijos de las raíces anteriores. *(2 puntos)*

**1** a) 


0.25

b) $\vec{AB} = B - A = (5,4) - (1,1) = (4,3)$
$\vec{AC} = C - A = (-5,9) - (1,1) = (-6,8)$
$\vec{AB}\cdot\vec{AC} = (4,3)\cdot(-6,8) = -24+24 = 0 \Rightarrow \vec{AB} \perp \vec{AC}$ 0.75

c) $|\vec{AB}| = \sqrt{16+9} = \boxed{5}$ 0.25
$|\vec{AC}| = \sqrt{36+64} = \boxed{10}$

d) $A = \frac{1}{2}$ base x altura $= \frac{1}{2}|\vec{AB}|\cdot|\vec{AC}| = \frac{1}{2}\cdot 5\cdot 10 = \boxed{25\ u^2}$ 0.75

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**2** a) $m = 3 \Rightarrow \vec{u}_r = (1,3)$ ; pasa por $P(-1,2)$

$\boxed{\begin{cases} x = -1+\lambda \\ y = 2+3\lambda \end{cases}}$ PARAMÉTRICAS $\Rightarrow \boxed{\frac{x+1}{1} = \frac{y-2}{3}}$ CONTINUA $\Rightarrow 3x+3 = y-2$ ; $\boxed{3x - y + 5 = 0}$ GRAL. O IMPLÍCITA $\Rightarrow \boxed{y = 3x+5}$ EXPLÍCITA ; $\boxed{y-2 = 3(x+1)}$ PTO-PDTE.

0.1 cada uno

b) $r: 3x - y - 1 = 0$ ; $\boxed{d(r,s) = d(P,r) = \frac{|-3-2-1|}{\sqrt{9+1}} = \frac{6}{\sqrt{10}} = \frac{6\sqrt{10}}{10} = \frac{3\sqrt{10}}{5}u.}$ 0.75

c) $\begin{cases} \vec{u}_r = (1,3) \\ \vec{u}_t = (2,1) \end{cases}$ $\cos\alpha = \frac{|\vec{u}_r\cdot\vec{u}_t|}{\|\vec{u}_r\|\cdot\|\vec{u}_t\|} = \frac{|2+3|}{\sqrt{1+9}\cdot\sqrt{4+1}} = \frac{5}{\sqrt{10}\cdot\sqrt{5}} = \frac{5}{\sqrt{50}} = \frac{5\sqrt{50}}{50} = \frac{\sqrt{50}}{10} = \frac{5\sqrt{2}}{10} = \frac{\sqrt{2}}{2} \Rightarrow \boxed{\alpha = \arccos\frac{\sqrt{2}}{2} = 45^\circ}$ 0.75

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**3** $\begin{cases} r: x+2y-3=0 \\ s: x - Ky + 4 = 0 \end{cases}$

a) $\frac{1}{1} = \frac{2}{-K}$ ; $-K = 2$ ; $\boxed{K=-2}$ 0.5

b) $r \perp s \Rightarrow \vec{u}_r\cdot\vec{u}_s = 0 \Rightarrow (-2,1)\cdot(K,1) = -2K+1 = 0 \Rightarrow \boxed{K = 1/2}$ 0.5

c) $\vec{u}_r = (-2,1) \Rightarrow \vec{n} = (1,2) \Rightarrow \frac{x}{1} = \frac{y}{2} \rightarrow \boxed{2x - y = 0}$ 1

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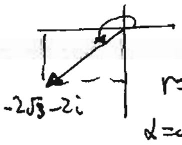
**4** a) $\frac{(3-2i)(3+i)-(2i-3)^2}{i^{23}-i^{-13}} = \frac{9+3i-6i-2i^2-(4i^2-12i+9)}{i^{23}-\frac{1}{i^{13}}} = \frac{9-3i+2-(-4-12i+9)}{i^3-\frac{1}{i}} = \frac{11-3i-5+12i}{-i-\frac{i}{-1}} = \frac{6+9i}{0} = \nexists$ 0.5 0.5

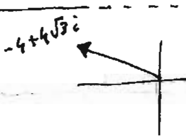
b) $\frac{(-2\sqrt{3}-2i)^5}{(-4+4\sqrt{3}i)^3\cdot 2i} = \frac{(4_{210^\circ})^5}{(8_{120^\circ})^3\cdot 2_{90^\circ}} = \frac{(4^5)_{405 0^\circ}}{(8^3)_{360^\circ}\cdot 2_{90^\circ}} = \frac{(2^{10})_{1050^\circ}}{(2^9)_{360^\circ}\cdot 2_{90^\circ}} = \frac{(2^{10})_{1050^\circ}}{(2^{10})_{450^\circ}} = 1_{600^\circ} = \boxed{1_{240^\circ}} = \cos 240^\circ + i\,\text{sen}\,240^\circ =$ 0.5

$\cos(180^\circ+60) = -\cos 60^\circ$ ; $\text{sen}(180+60) = -\text{sen}\,60^\circ$

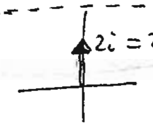
$= \boxed{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}$ 0.5




$r = \sqrt{12+4} = 4$
$\alpha = \text{arctg}\frac{-2}{-2\sqrt{3}} = \text{arctg}\frac{1}{\sqrt{3}} = 210^\circ$




$r = \sqrt{16+48} = 8$
$\alpha = \text{arctg}\frac{4\sqrt{3}}{-4} = \text{arctg}(-\sqrt{3}) = 120^\circ$

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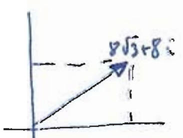
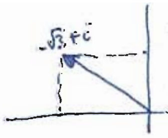
**5** a) $\sqrt[4]{\frac{8\sqrt{3}+8i}{-\sqrt{3}+i}} = \sqrt[4]{\frac{16_{30^\circ}}{2_{150^\circ}}} = \sqrt[4]{8_{-120^\circ}} = \sqrt[4]{8_{240^\circ}} = R_\beta$ siendo $\begin{cases} R = \sqrt[4]{8} \\ \beta = \frac{240^\circ + K\cdot 360^\circ}{4} = 60^\circ + K\cdot 90^\circ \end{cases}$ donde $K = 0,1,2,3$

0,25




$r = \sqrt{192+64} = 16$
$\alpha = \text{arctg}\frac{8}{8\sqrt{3}} = 30^\circ$




$r = \sqrt{3+1} = 2$
$\alpha = \text{arctg}\frac{1}{-\sqrt{3}} = \text{arctg}\left(-\frac{\sqrt{3}}{3}\right) = 150^\circ$

0,75

$K=0 \rightarrow z_1 = \boxed{(\sqrt[4]{8})_{60^\circ}} = \sqrt[4]{8}(\cos 60^\circ + i\,\text{sen}\,60^\circ) = \sqrt[4]{8}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = \frac{\sqrt[4]{8}}{2} + \frac{\sqrt[4]{\ }\cdot\sqrt{3}}{2}i = \frac{\sqrt[4]{8}}{2} + \frac{\sqrt[4]{\ }\cdot\sqrt[4]{3^2}}{2}i = \boxed{\frac{\sqrt[4]{8}}{2} + \frac{\sqrt[4]{72}}{2}i}$ 0,25

$K=1 \rightarrow z_2 = \boxed{(\sqrt[4]{8})_{150^\circ}} = \sqrt[4]{8}(\cos 150^\circ + i\,\text{sen}\,150^\circ) = \sqrt[4]{8}\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right) = \boxed{-\frac{\sqrt[4]{72}}{2} + \frac{\sqrt[4]{8}}{2}i}$ 0.25

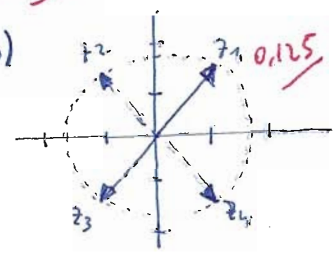
$\cos(180-30) = -\cos 30^\circ$ ; $\text{sen}(180-30) = \text{sen}\,30^\circ$

$K=2 \rightarrow z_3 = \boxed{(\sqrt[4]{8})_{240^\circ}} = \sqrt[4]{8}(\cos 240^\circ + i\,\text{sen}\,240^\circ) = \sqrt[4]{8}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) = \boxed{-\frac{\sqrt[4]{8}}{2} - \frac{\sqrt[4]{72}}{2}i}$ 0.25

$\cos(180+60) = -\cos 60^\circ$ ; $\text{sen}(180+60) = -\text{sen}\,60^\circ$

$K=3 \rightarrow z_4 = \boxed{(\sqrt[4]{8})_{330^\circ}} = \sqrt[4]{8}(\cos 330^\circ + i\,\text{sen}\,330^\circ) = \sqrt[4]{8}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right) = \boxed{\frac{\sqrt[4]{72}}{2} - \frac{\sqrt[4]{8}}{2}i}$ 0.25

$\cos(-30^\circ) = \cos 30^\circ$ ; $\text{sen}(-30^\circ) = -\text{sen}\,30^\circ$

b) 


0,125