

Problema 1 Sean $z_1 = 2 + i$ y $z_2 = -1 + 2i$. Calcular: $z_1 + z_2$, $z_1 \cdot z_2$ y $\frac{z_1}{z_2}$.

Solución:

- $z_1 + z_2 = 1 + 3i$
- $z_1 \cdot z_2 = -4 + 3i$
- $\frac{z_1}{z_2} = -i$

Problema 2 Resolver la ecuación $z^4 + 1 - i = 0$

Solución:

$$z^4 + 1 - i = 0 \implies z = \sqrt[4]{-1 + i} = \sqrt[8]{(2)_{135^\circ}} =$$

$$\left\{ \begin{array}{l} \sqrt[8]{2} \frac{135^\circ + 0^\circ}{4} = \sqrt[8]{2}(\cos 33^\circ 45' + i \sin 33^\circ 45') \\ \sqrt[8]{2} \frac{135^\circ + 360^\circ}{4} = \sqrt[8]{2}(\cos 123^\circ 45' + i \sin 123^\circ 45') \\ \sqrt[8]{2} \frac{135^\circ + 720^\circ}{4} = \sqrt[8]{2}(\cos 213^\circ 45' + i \sin 213^\circ 45') \\ \sqrt[8]{2} \frac{135^\circ + 1080^\circ}{4} = \sqrt[8]{2}(\cos 303^\circ 45' + i \sin 303^\circ 45') \end{array} \right.$$

Problema 3 Resolver por el método de Gauss los siguientes sistemas:

1.

$$\left\{ \begin{array}{lll} x+ & y+ & z = 2 \\ & y- & z = 1 \\ 2x+ & y & = 3 \end{array} \right.$$

2.

$$\left\{ \begin{array}{lll} x+ & 2y+ & z = 4 \\ 2x+ & y- & z = 2 \\ x- & y- & 2z = -2 \end{array} \right.$$

Solución:

1.

$$\left\{ \begin{array}{lll} x+ & y+ & z = 2 \\ & y- & z = 1 \\ 2x+ & y & = 3 \end{array} \right. \implies \left\{ \begin{array}{l} x = 1 \\ y = 1 \\ z = 0 \end{array} \right. , \text{ (S.C.D)}$$

2.

$$\left\{ \begin{array}{lll} x+ & 2y+ & z = 4 \\ 2x+ & y- & z = 2 \\ x- & y- & 2z = -2 \end{array} \right. \implies \left\{ \begin{array}{l} x = \lambda \\ y = 2 - \lambda \\ z = \lambda \end{array} \right. , \text{ (S.C.I)}$$

Problema 4 Resolver las siguientes ecuaciones:

1. $\log(x^2 - x) - \log(x + 1) = 1$
2. $3^{2x} - 3^{x+1} + 2 = 0$

Solución:

1. $\log(x^2 - x) - \log(x + 1) = 1 \implies x^2 - 11x - 10 = 0 \implies x = 11,844$ y $x = -0,844$. Las dos soluciones son válidas
2. $3^{2x} - 3^{x+1} + 2 = 0 \implies t^2 - 3t + 2 = 0 \implies t = 2$ y $t = 1 \implies x = 0,6309$ y $x = 0$.

Problema 5 Calcular los siguientes límites

1. $\lim_{x \rightarrow 2} \frac{2x^3 - 2x^2 - 8}{x^2 - x - 2}$
2. $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 5} - \sqrt{x + 1}}{x - 3}$
3. $\lim_{x \rightarrow \infty} (\sqrt{x^2 - x + 1} - \sqrt{x^2 + 1})$
4. $\lim_{x \rightarrow \infty} \frac{x^3 - 2\sqrt{x} + 1}{3x^3 + x - 1}$
5. $\lim_{x \rightarrow \infty} \left(\frac{x + 1}{x} \right)^{2x-1}$
6. $\lim_{x \rightarrow \infty} \left(\frac{2x^2 + 3}{3x^2} \right)^{\frac{x+1}{2}}$

Solución:

1. $\lim_{x \rightarrow 2} \frac{2x^3 - 2x^2 - 8}{x^2 - x - 2} = \frac{16}{3}$
2. $\lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 5} - \sqrt{x + 1}}{x - 3} = \frac{5}{4}$

$$3. \lim_{x \rightarrow \infty} (\sqrt{x^2 - x + 1} - \sqrt{x^2 + 1}) = -\frac{1}{2}$$

$$4. \lim_{x \rightarrow \infty} \frac{x^3 - 2\sqrt{x} + 1}{3x^3 + x - 1} = \frac{1}{3}$$

$$5. \lim_{x \rightarrow \infty} \left(\frac{x+1}{x} \right)^{2x-1} = e^2$$

$$6. \lim_{x \rightarrow \infty} \left(\frac{2x^2 + 3}{3x^2} \right)^{\frac{x+1}{2}} = 0$$

Problema 6 Calcular la derivada de las siguientes funciones

$$1. y = (3x^2 - x + 1)^8$$

$$2. y = \sin(2x - 1) \cdot \ln(2x - 1)$$

$$3. y = e^{\cos 2x}$$

$$4. y = \ln \left(\frac{\cos x}{x^2 + 1} \right)$$

$$5. y = \frac{\sin(x^2 + 1)}{e^x}$$

$$6. y = \tan(x^2 + 1)$$

$$7. y = 7^{x \sin x}$$

$$8. y = \log_5 \left(\frac{x^2 - 1}{x + 8} \right)$$

$$9. y = (x^2 + 2)^{x+1}$$

Solución:

$$1. y' = 8(3x^2 - x + 1)^7(6x - 1)$$

$$2. y' = 2 \cos(2x - 1) \cdot \ln(2x - 1) + \sin(2x - 1) \cdot \frac{2}{2x - 1}$$

$$3. y' = -2 \sin 2x e^{\cos 2x}$$

$$4. y' = \frac{-\sin x}{\cos x} - \frac{2x}{\cos x^2 + 1}$$

$$5. y' = \frac{2x \cos(x^2 + 1) \cdot e^x - \sin(x^2 + 1) \cdot e^x}{e^{2x}}$$

$$6. y' = \frac{2x}{\cos^2(x^2 + 1)}$$

$$7. y' = (\sin x + x \cos x) 7^{x \sin x} \ln 7$$

$$8. y' = \frac{2x}{(x^2 - 1) \ln 5} - \frac{1}{(x + 8) \ln 5}$$

$$9. y' = (x^2 + 2)^{x+1} \left(\ln(x^2 + 2) + \frac{2x(x+1)}{x^2 + 2} \right)$$

Problema 7 Calcular las rectas tangente y normal de la siguientes funciones en $x = 2$:

$$1. f(x) = e^{x^2-1}$$

$$2. f(x) = x^2 - x + 1$$

$$3. f(x) = \frac{x^2 + 1}{x^2 - 1}$$

Solución:

$$1. f(x) = e^{x^2-1} \implies f(2) = e^3, (2, e^3) \text{ y } f'(x) = 2xe^{x^2-1} \implies m = f'(2) = 4e^3$$

$$\text{Recta Tangente: } y - e^3 = 4e^3(x - 2)$$

$$\text{Recta Normal: } y - e^3 = -\frac{1}{4e^3}(x - 2)$$

$$2. f(x) = x^2 - x + 1 \implies f(2) = 3, (2, 3) \text{ y } f'(x) = 2x - 1 \implies m = f'(2) = 3$$

$$\text{Recta Tangente: } y - 3 = 3(x - 2)$$

$$\text{Recta Normal: } y - 3 = -\frac{1}{3}(x - 2)$$

$$3. f(x) = \frac{x^2 + 1}{x^2 - 1} \implies f(2) = \frac{5}{3}, \left(2, \frac{5}{3}\right) \text{ y } f'(x) = \frac{-4x}{(x^2 - 1)^2} \implies m = f'(2) = -\frac{8}{9}$$

$$\text{Recta Tangente: } y - \frac{5}{3} = -\frac{8}{9}(x - 2)$$

$$\text{Recta Normal: } y - \frac{5}{3} = \frac{9}{8}(x - 2)$$