

Problema 1 Sean $z_1 = 3 - 2i$ y $z_2 = -1 + 4i$. Calcular: $z_1 + z_2$, $z_1 \cdot z_2$ y $\frac{z_1}{z_2}$.

Solución:

$$\blacksquare z_1 + z_2 = 2 + 2i$$

$$\blacksquare z_1 \cdot z_2 = 5 + 14i$$

$$\blacksquare \frac{z_1}{z_2} = -\frac{11}{17} - \frac{10}{17}i$$

Problema 2 Resolver la ecuación $z^3 - 2i = 0$

Solución:

$$z^3 - 2i = 0 \implies z = \sqrt[3]{2i} = \sqrt[3]{(2)_{\pi/2}} = \begin{cases} \sqrt[3]{2}_{30^\circ} = \sqrt[3]{2}(\cos 30^\circ + i \sin 30^\circ) \\ \sqrt[3]{2}_{150^\circ} = \sqrt[3]{2}(\cos 150^\circ + i \sin 150^\circ) \\ \sqrt[3]{2}_{270^\circ} = \sqrt[3]{2}(\cos 270^\circ + i \sin 270^\circ) \end{cases}$$

Problema 3 Calcular los siguientes límites

$$1. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - \sqrt{x^2 + x + 1})$$

$$2. \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^3 - 1}$$

$$3. \lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 5} - \sqrt{x + 1}}{x - 3}$$

$$4. \lim_{x \rightarrow \infty} \frac{3x^2 - \sqrt{2x^5 + 1} + x - 1}{x^3 - x + 1}$$

Solución:

$$1. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - \sqrt{x^2 + x + 1}) = -\frac{1}{2}$$

$$2. \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^3 - 1} = 0$$

$$3. \lim_{x \rightarrow 3} \frac{\sqrt{x^2 - 5} - \sqrt{x + 1}}{x - 3} = \frac{5}{4}$$

$$4. \lim_{x \rightarrow \infty} \frac{3x^2 - \sqrt{2x^5 + 1} + x - 1}{x^3 - x + 1} = 0$$

Problema 4 Calcular la derivada de las siguientes funciones

$$1. y = (x^3 - 2x^2 + 1)^8$$

$$2. y = e^{x^2 - 3x + 1}$$

$$3. y = e^{2x} \cdot (x^2 + x - 1)$$

$$4. y = \frac{\sin(x^2)}{e^{2x}}$$

$$5. y = \ln\left(\frac{\sin x}{x}\right)$$

Solución:

$$1. y = (x^3 - 2x^2 + 1)^8 \implies y' = 8(x^3 - 2x^2 + 1)^7(3x^2 - 4x)$$

$$2. y = e^{x^2-3x+1} \implies y' = (2x - 3)e^{x^2-3x+1}$$

$$3. y = e^{2x} \cdot (x^2 + x - 1) \implies y' = 2e^{2x} \cdot (x^2 + x - 1) + e^{2x} \cdot (2x + 1)$$

$$4. y = \frac{\sin(x^2)}{e^{2x}} \implies y' = \frac{2x \cos(x^2)e^{2x} - 2e^{2x} \sin(x^2)}{(e^{2x})^2}$$

$$5. y = \ln\left(\frac{\sin x}{x}\right) \implies y' = \frac{\cos x}{\sin x} - \frac{1}{x}$$

Problema 5 Calcular las rectas tangente y normal de la siguiente función

$$f(x) = \frac{x^2 + 2}{x^2 - 1}$$

en $x = 2$

Solución:

$$f'(x) = \frac{-6x}{(x^2 - 1)^2} \implies f'(2) = -\frac{4}{3} \text{ y } f(2) = 2$$

$$\text{Recta Tangente: } y - 2 = -\frac{4}{3}(x - 2)$$

$$\text{Recta Normal: } y - 2 = \frac{3}{4}(x - 2)$$