

Problema 1 (2 puntos) Descompón cada polinomio como producto de factores de grado uno:

1. $P(x) = x^3 - 4x^2 + 3x + 2$

Solución:

$$P(x) = (x - 2)(x - 1 - \sqrt{2})(x - 1 + \sqrt{2})$$

2. $Q(x) = 2x^3 - x^2 - 2x + 1$

Solución:

$$Q(x) = (x - 1)(x + 1)(x - \frac{1}{2})$$

Problema 2 (3 puntos) Calcular las soluciones reales de:

1.

$$\frac{2x - 3}{x - 1} = \frac{3x - 7}{2x - 5}$$

Solución:

$$\begin{aligned} (2x - 3) \cdot (2x - 5) &= (x - 1) \cdot (3x - 7) \\ 4x^2 - 10x - 6x + 15 &= 3x^2 - 7x - 3x + 7 \\ x^2 - 6x + 8 &= 0 \\ x &= \frac{6 \pm \sqrt{36 - 32}}{2} = \frac{6 \pm 2}{2} \implies x = 4, x = 2 \end{aligned}$$

2.

$$\frac{2 + x}{x^2 + x} = \frac{2 - x}{x^2 - x}$$

Solución:

$$\begin{aligned} (2 + x) \cdot (x^2 - x) &= (2 - x) \cdot (x^2 + x) \\ 2x^2 - 2x + x^3 - x^2 &= 2x^2 + 2x - x^3 - x^2 \\ 2x^3 - 4x &= 0 \implies x \cdot (2x^2 - 4) = 0 \implies \\ x &= 0 \text{ (No sería una solución lógica)} \\ 2x^2 - 4 &= 0 \implies x^2 = 2 \implies x = \pm\sqrt{2} \end{aligned}$$

Problema 3 (2 puntos) Halla las soluciones reales de:

$$\sqrt{x+6} + \sqrt{2-x} = 4$$

Solución:

$$\begin{aligned}(\sqrt{x+6})^2 &= (4 - \sqrt{2-x})^2 \\ x+6 &= 16 + (\sqrt{2-x})^2 - 8\sqrt{2-x}\end{aligned}$$

$$2x - 12 = -8\sqrt{2-x}$$

$$x - 6 = -4\sqrt{2-x}$$

$$(x-6)^2 = (-4\sqrt{2-x})^2$$

$$x^2 + 36 - 12x = 16(2-x)$$

$$x^2 + 4x + 4 = 0 \implies x = \frac{-4 \pm \sqrt{16-16}}{2} = -2 \text{ doble}$$

Problema 4 (3 puntos) Halla las soluciones de:

1.

$$3^{x^2+5x-4} \cdot 9^{2x+3} = 27^{x-1}$$

Solución:

$$3^{x^2+5x-4} \cdot 3^{2(2x+3)} = 3^{3(x-1)}$$

$$3^{x^2+5x-4+2(2x+3)} = 3^{3(x-1)}$$

$$x^2 + 5x - 4 + 4x + 6 = 3x - 3$$

$$x^2 + 6x + 5 = 0 \implies x = \frac{-6 \pm \sqrt{36-20}}{4}$$

$$x = \frac{-6 \pm 4}{2} \implies x = -1, \quad x = -5$$

2.

$$\log_{10}(3x^2 - 2) = 1 + \log_{10}(x-1)$$

Solución:

$$\log_{10}(3x^2 - 2) = \log_{10}10 + \log_{10}(x-1)$$

$$\log_{10}(3x^2 - 2) = \log_{10}10(x-1)$$

$$3x^2 - 2 = 10(x-1)$$

$$3x^2 - 10x + 8 = 0 \implies x = \frac{10 \pm \sqrt{100-96}}{6}$$

$$x = \frac{10 \pm 2}{6} \implies x = 2, \quad x = \frac{4}{3}$$