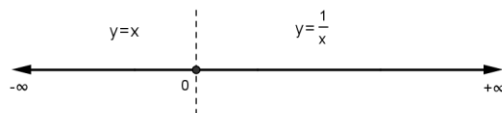


CALCULA LOS SIGUIENTES LÍMITES:

$$1) \quad f(x) = \begin{cases} x & \text{si } x \leq 0 \\ \frac{1}{x} & \text{si } x > 0 \end{cases} \quad \text{Dom}(f) = \mathbb{R}$$



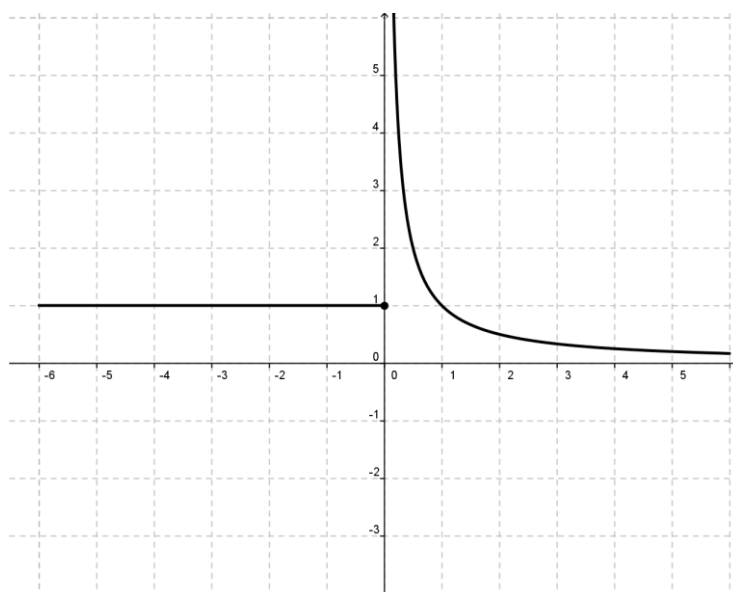
$$\Rightarrow \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} x = -\infty$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}$$

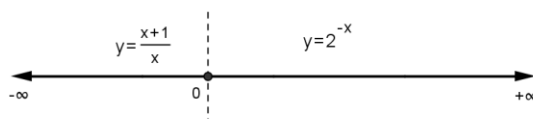
$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{1}{x} = \frac{1}{+\infty} = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = \begin{cases} \lim_{x \rightarrow 0^-} x = 0 \\ \lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty \end{cases} \Rightarrow \nexists \lim_{x \rightarrow 0} f(x)$$

$$\Rightarrow \lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} x = -3$$



$$2) \quad f(x) = \begin{cases} \frac{x+1}{x} & \text{si } x < 0 \\ 2^{-x} & \text{si } x \geq 0 \end{cases} \quad \text{Dom}(f) = \mathbb{R}$$



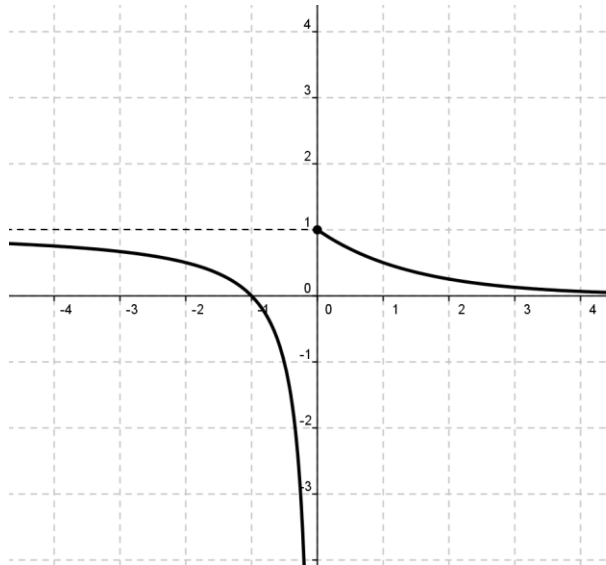
$$\Rightarrow \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x+1}{x} = \frac{-\infty}{-\infty} \text{ (I)} = \lim_{x \rightarrow -\infty} \frac{\frac{x}{x} + \frac{1}{x}}{\frac{x}{x}} = \lim_{x \rightarrow -\infty} \frac{1 + \frac{1}{x}}{1} = \frac{1}{1} = 1 \Rightarrow y = 1 \text{ es A.H. por la izquierda}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} 2^{-x} = 2^{-(+\infty)} = 2^{-\infty} = 0^+ \Rightarrow y = 0 \text{ es A.H. por la derecha}$$

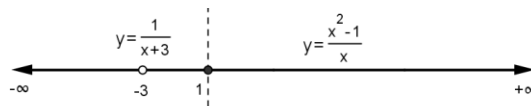
$$\Rightarrow \lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} \frac{x+1}{x} = \frac{-3+1}{-3} = \frac{-2}{-3} = \frac{2}{3}$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} 2^{-x} = 2^{-2} = \frac{1}{4}$$

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = \begin{cases} \lim_{x \rightarrow 0^-} \frac{x+1}{x} = \frac{1}{0^-} = -\infty \\ \lim_{x \rightarrow 0^+} 2^{-x} = 2^0 = 1 \end{cases} \Rightarrow \nexists \lim_{x \rightarrow 0} f(x) \quad x=0 \text{ es A.V. por la izquierda}$$



$$3) \quad f(x) = \begin{cases} \frac{1}{x+3} & \text{si } x < 1 \\ \frac{x^2-1}{x} & \text{si } x \geq 1 \end{cases} \quad \text{Dom}(f) = \mathbb{R} - \{-3\}$$



$$\Rightarrow \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{1}{x+3} = \frac{1}{-\infty} = 0^- \Rightarrow y=0 \text{ es A.H. por la izquierda}$$

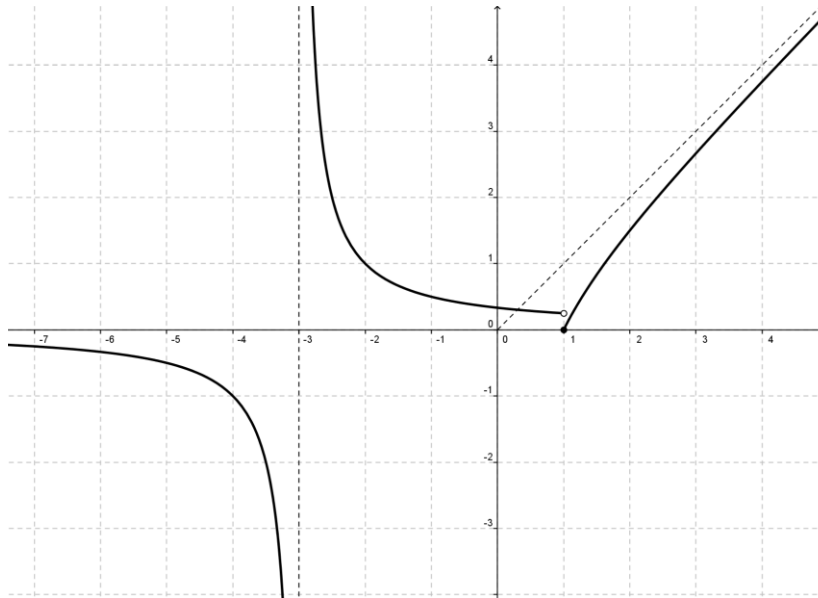
$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{x^2-1}{x} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{x^2}{x} = \lim_{x \rightarrow +\infty} x = +\infty$$

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{1}{x+3} = \frac{1}{3}$$

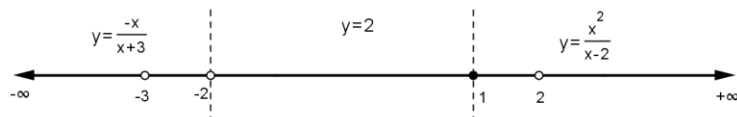
$$\Rightarrow \lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} \frac{1}{x+3} = \frac{1}{0} = \begin{cases} \lim_{x \rightarrow -3^-} \frac{1}{x+3} = \frac{1}{0^-} = -\infty \\ \lim_{x \rightarrow -3^+} \frac{1}{x+3} = \frac{1}{0^+} = +\infty \end{cases} \Rightarrow x = -3 \text{ es A.V.}$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x^2-1}{x} = \frac{2^2-1}{2} = \frac{3}{2}$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = \begin{cases} \lim_{x \rightarrow 1^-} \frac{1}{x+3} = \frac{1}{4} \\ \lim_{x \rightarrow 1^+} \frac{x^2-1}{x} = \frac{0}{1} = 0 \end{cases} \Rightarrow \nexists \lim_{x \rightarrow 1} f(x)$$



$$4) \quad f(x) = \begin{cases} \frac{-x}{x+3} & \text{si } x < -2 \\ 2 & \text{si } -2 < x < 1 \\ \frac{x^2}{x-2} & \text{si } x \geq 1 \end{cases} \Rightarrow \text{Dom}(f) = \mathbb{R} - \{-3, -2, 2\}$$



$$\Rightarrow \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{-x}{x+3} = \frac{+\infty}{-\infty} \text{ (I)} = \lim_{x \rightarrow -\infty} \frac{-x}{x} = \lim_{x \rightarrow -\infty} -1 = -1 \Rightarrow y = -1 \text{ es A.H. por la izquierda}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{x^2}{x-2} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{x^2}{x} = \lim_{x \rightarrow +\infty} x = +\infty$$

$$\Rightarrow \lim_{x \rightarrow -5} f(x) = \lim_{x \rightarrow -5} \frac{-x}{x+3} = \frac{5}{2}$$

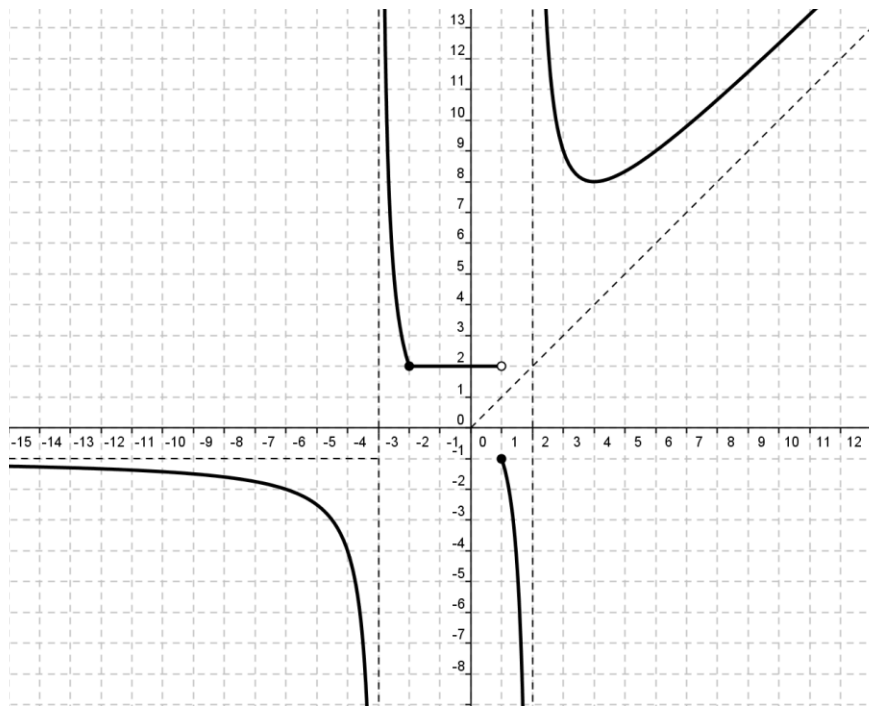
$$\Rightarrow \lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} \frac{-x}{x+3} = \frac{3}{0} = \begin{cases} \lim_{x \rightarrow -3^-} \frac{-x}{x+3} = \frac{3}{0^-} = -\infty \\ \lim_{x \rightarrow -3^+} \frac{-x}{x+3} = \frac{3}{0^+} = +\infty \end{cases} \Rightarrow x = -3 \text{ es A.V.}$$

$$\Rightarrow \lim_{x \rightarrow -2} f(x) = \begin{cases} \lim_{x \rightarrow -2^-} \frac{-x}{x+3} = \frac{2}{1} = 2 \\ \lim_{x \rightarrow -2^+} 2 = 2 \end{cases} \Rightarrow \lim_{x \rightarrow -2} f(x) = 2$$

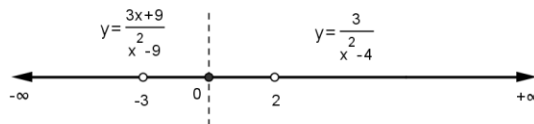
$$\Rightarrow \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} 2 = 2$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = \begin{cases} \lim_{x \rightarrow 1^-} 2 = 2 \\ \lim_{x \rightarrow 1^+} \frac{x^2}{x-2} = \frac{1}{-1} = -1 \end{cases} \Rightarrow \nexists \lim_{x \rightarrow 1} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x^2}{x-2} = \frac{4}{0} = \begin{cases} \lim_{x \rightarrow 2^-} \frac{x^2}{x-2} = \frac{4}{0^-} = -\infty \\ \lim_{x \rightarrow 2^+} \frac{x^2}{x-2} = \frac{4}{0^+} = +\infty \end{cases} \Rightarrow x=2 \text{ es A.V.}$$



$$5) f(x) = \begin{cases} \frac{3x+9}{x^2-9} & \text{si } x \leq 0 \\ \frac{3}{x^2-4} & \text{si } x > 0 \end{cases} \Rightarrow \text{Dom}(f) = \mathbb{R} - \{-3, 2\}$$



$$\Rightarrow \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{3x+9}{x^2-9} = \frac{-\infty}{+\infty} \text{ (I) } \lim_{x \rightarrow -\infty} \frac{3x}{x^2} = \lim_{x \rightarrow -\infty} \frac{3}{x} = \frac{3}{-\infty} = 0^- \Rightarrow y=0 \text{ es A.H. por la izquierda}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{3}{x^2-4} = \frac{3}{+\infty} = 0^+ \Rightarrow y=0 \text{ es A.H. por la derecha}$$

$$\Rightarrow \lim_{x \rightarrow -5} f(x) = \lim_{x \rightarrow -5} \frac{3x+9}{x^2-9} = \frac{-15+9}{25-9} = -\frac{6}{16} = -\frac{3}{8}$$

$$\Rightarrow \lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} \frac{3x+9}{x^2-9} = \frac{0}{0} \text{ (I) } = \lim_{x \rightarrow -3} \frac{3(x+3)}{(x-3)(x+3)} = \lim_{x \rightarrow -3} \frac{3}{x-3} = \frac{3}{-6} = -\frac{1}{2} \Rightarrow \lim_{x \rightarrow -3} f(x) = -\frac{1}{2}$$

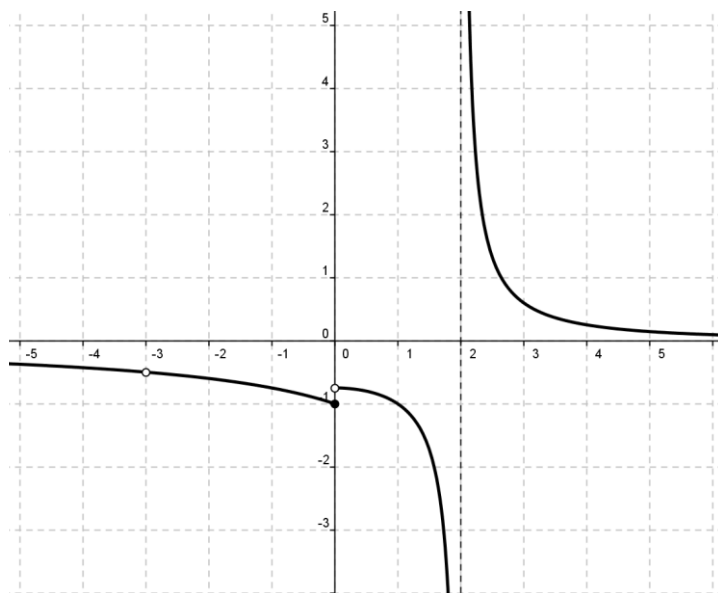
Observación

$$\left. \begin{aligned} \exists \lim_{x \rightarrow -3} f(x) &= -\frac{1}{2} \\ \nexists f(-3) \end{aligned} \right\} \Rightarrow x = -3 \text{ discontinuidad evitable ("punto en blanco")}$$

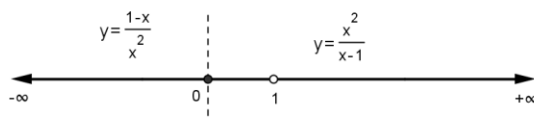
$$\Rightarrow \lim_{x \rightarrow 0} f(x) = \begin{cases} \lim_{x \rightarrow 0^-} \frac{3x+9}{x^2-9} = \frac{9}{-9} = -1 \\ \lim_{x \rightarrow 0^+} \frac{3}{x^2-4} = -\frac{3}{4} \end{cases} \Rightarrow \nexists \lim_{x \rightarrow 0} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{3}{x^2-4} = \frac{3}{(1)^2-4} = \frac{3}{-3} = -1 \Rightarrow \lim_{x \rightarrow 1} f(x) = -1$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{3}{x^2-4} = \frac{3}{0} = \begin{cases} \lim_{x \rightarrow 2^-} \frac{3}{x^2-4} = \frac{3}{0^-} = -\infty \\ \lim_{x \rightarrow 2^+} \frac{3}{x^2-4} = \frac{3}{0^+} = +\infty \end{cases} \Rightarrow x=2 \text{ es A.V.}$$



$$6) f(x) = \begin{cases} \frac{1-x}{x^2} & \text{si } x < 0 \\ \frac{x^2}{x-1} & \text{si } x \geq 0 \end{cases} \Rightarrow \text{Dom}(f) = \mathbb{R} - \{1\}$$



$$\Rightarrow \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{1-x}{x^2} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow -\infty} \frac{-x}{x^2} = \lim_{x \rightarrow -\infty} \frac{-1}{x} = \frac{-1}{-\infty} = 0^+ \Rightarrow y=0 \text{ es A.H. por la izquierda}$$

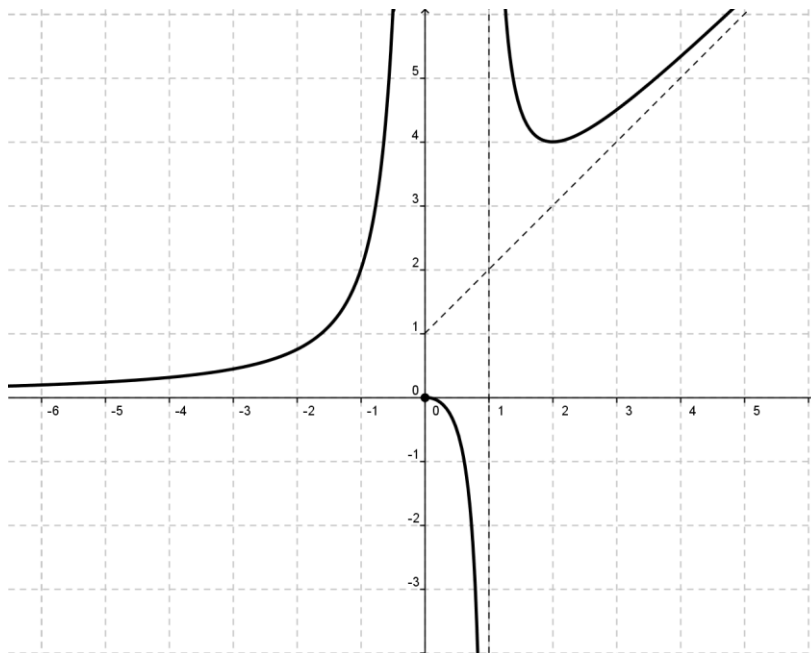
$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{x^2}{x-1} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{x^2}{x} = \lim_{x \rightarrow +\infty} x = +\infty$$

$$\Rightarrow \lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} \frac{1-x}{x^2} = \frac{1-(-1)}{(-1)^2} = \frac{2}{1} = 2 \Rightarrow \lim_{x \rightarrow -1} f(x) = 2$$

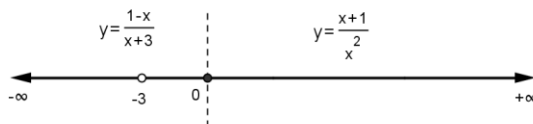
$$\Rightarrow \lim_{x \rightarrow 0} f(x) = \begin{cases} \lim_{x \rightarrow 0^-} \frac{1-x}{x^2} = \frac{1}{0^+} = +\infty \\ \lim_{x \rightarrow 0^+} \frac{x^2}{x-1} = \frac{0}{-1} = 0 \end{cases} \Rightarrow \nexists \lim_{x \rightarrow 0} f(x) \quad x=0 \text{ es A.V. por la izquierda}$$

$$\Rightarrow \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x^2}{x-1} = \frac{4}{1} = 4 \Rightarrow \lim_{x \rightarrow 2} f(x) = 4$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2}{x-1} = \frac{1}{0} = \begin{cases} \lim_{x \rightarrow 1^-} \frac{x^2}{x-1} = \frac{1}{0^-} = -\infty \\ \lim_{x \rightarrow 1^+} \frac{x^2}{x-1} = \frac{1}{0^+} = +\infty \end{cases} \Rightarrow x=1 \text{ es A.V.}$$



$$7) \quad h(x) = \begin{cases} \frac{1-x}{x+3} & \text{si } x \leq 0 \\ \frac{x+1}{x^2} & \text{si } x > 0 \end{cases} \Rightarrow \text{Dom}(f) = \mathbb{R} - \{-3\}$$



$$\Rightarrow \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{1-x}{x+3} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow -\infty} \frac{-x}{x} = \lim_{x \rightarrow -\infty} -1 \Rightarrow y = -1 \text{ es A.H. por la izquierda}$$

$$\Rightarrow \lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{x+1}{x^2} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{x}{x^2} = \lim_{x \rightarrow +\infty} \frac{1}{x} = \frac{1}{+\infty} = 0^+ \Rightarrow y = 0 \text{ es A.H. por la derecha}$$

$$\Rightarrow \lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} \frac{1-x}{x+3} = \frac{1-(-1)}{-1+3} = \frac{2}{2} = 1 \Rightarrow \lim_{x \rightarrow -1} f(x) = 1$$

$$\Rightarrow \lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} \frac{1-x}{x+3} = \frac{4}{0} = \begin{cases} \lim_{x \rightarrow -3^-} \frac{1-x}{x+3} = \frac{4}{0^-} = -\infty \\ \lim_{x \rightarrow -3^+} \frac{1-x}{x+3} = \frac{4}{0^+} = +\infty \end{cases} \Rightarrow \nexists \lim_{x \rightarrow -3} f(x) \quad x = -3 \text{ es A.V.}$$

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = \begin{cases} \lim_{x \rightarrow 0^-} \frac{1-x}{x+3} = \frac{1}{3} \\ \lim_{x \rightarrow 0^+} \frac{x+1}{x^2} = \frac{1}{0^+} = +\infty \end{cases} \Rightarrow \nexists \lim_{x \rightarrow 0} f(x) \quad x = 0 \text{ es A.V. por la derecha}$$

$$\triangleright \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x+1}{x^2} = \frac{3}{4} \Rightarrow \lim_{x \rightarrow 2} f(x) = \frac{3}{4}$$

