COCIENTE DE POLINOMIOS

Sean $a_n = a_0 n^p + a_1 n^{p-1} + \ldots + a_p$ y $b_n = b_0 n^q + b_1 n^{q-1} + \ldots + b_q$, con $p, q \in N$.

Estudiemos el

$$\boxed{\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\infty}{\infty}}$$

Método: *se divide el numerador y el denominador por la mayor potencia de la n del denominador.*

Casos:

Caso I: grado Numerador = grado Denominador

$$\lim_{n\to\infty} \frac{3n^2+5n-7}{-2\boxed{n^2}+n-3} = \left(\frac{+\infty}{-\infty}\right) = \lim_{n\to\infty} \frac{\dfrac{3n^2+5n-7}{n^2}}{\dfrac{-2n^2+n-3}{n^2}} = \lim_{n\to\infty} \frac{3+\dfrac{5}{n}-\dfrac{7}{n^2}}{-2+\dfrac{1}{n}-\dfrac{3}{n^2}} = \frac{3+0-0}{-2+0-0} = \boxed{-\frac{3}{2}}$$

Caso II: grado Numerador < grado Denominador

$$\lim_{n\to\infty} \frac{-n^2-3n+4}{\boxed{n^4}+n^3+2n^2+1} = \left(\frac{-\infty}{+\infty}\right) = \lim_{n\to\infty} \frac{\dfrac{-n^2-3n+4}{n^4}}{\dfrac{n^4+n^3+2n^2+1}{n^4}} = \lim_{n\to\infty} \frac{-\dfrac{1}{n^2}-\dfrac{3}{n^3}+\dfrac{4}{n^4}}{1+\dfrac{1}{n}+\dfrac{2}{n^2}+\dfrac{1}{n^3}} = \frac{-0-0+4}{1+0+0+0} = \boxed{0}$$

Caso III: grado Numerador > grado Denominador

$$\lim_{n\to\infty} \frac{2n^3-4n^2+5n-1}{-3\boxed{n^2}+8n-1} = \left(\frac{+\infty}{-\infty}\right) = \lim_{n\to\infty} \frac{\dfrac{2n^3-4n^2+5n-1}{n^2}}{\dfrac{-3n^2+8n-1}{n^2}} = \lim_{n\to\infty} \frac{2n-4+\dfrac{5}{n}-\dfrac{1}{n^2}}{-3+\dfrac{8}{n}-\dfrac{1}{n^2}} = \frac{\infty-4+0-0}{-3+0-0} = \boxed{-\infty}$$

Resumen:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{a_0 n^p + a_1 n^{p-1} + \ldots + a_p}{b_0 n^q + b_1 n^{q-1} + \ldots + b_q} = \begin{cases} \dfrac{a_0}{b_0} & \text{si} \quad p = q \\ 0 & \text{si} \quad p < q \\ \pm\infty & \text{si} \quad p > q \quad (*) \end{cases}$$

(*) El signo depende de los signos de a_0 y b_0.

Observación:
Algunos límites se reducen a este tipo:

$$\boxed{\lim_{n\to\infty}\left(\frac{3n^2+1}{n-4}-\frac{n^2+2}{3n-1}\right)=(\infty-\infty)}$$

Realizando la diferencia:

$$\frac{3n^2+1}{n-4}-\frac{n^2+2}{3n-1}=\frac{\left(3n^2+1\right)\cdot(3n-1)-\left(n^2+2\right)\cdot(n-4)}{(n-4)\cdot(3n-1)}=$$

$$\frac{9n^3-3n^2+3n-1-\left(n+2n-4n^2-8\right)}{3n^2-n-12n+1}=\frac{9n^3+n^2+7}{3n^2-13n+1}\Rightarrow$$

$$\lim_{n\to\infty}\left(\frac{3n^2+1}{n-4}-\frac{n^2+2}{3n-1}\right)=\lim_{n\to\infty}\frac{9n^3+n^2+7}{3n^2-13n+1}=\left(\frac{\infty}{\infty}\right)=\boxed{+\infty}$$