

Nombre: _____ Curso: _____

1. Opera con radicales dejando el resultado de la forma más reducida posible:

a) $\frac{\sqrt[4]{\frac{b^6}{b^3}}}{\sqrt[1]{b}} \div \sqrt[6]{a}$ (1 punto)

b) $\sqrt[3]{81a^3} + 2a\sqrt[3]{24}$ (0,5 punto)

c) $\sqrt{x^3}\sqrt[4]{x^3}$ (0,5 punto)

d) $\sqrt[3]{\sqrt{2^3}\sqrt{4}}$ (0,5 punto)

2. Opera y simplifica: (1 punto)

$$\frac{3}{3 - \frac{1}{2}} - \frac{2}{\frac{1}{3} + \frac{1}{2}}$$

3. Expresa mediante intervalo los siguientes conjuntos de números reales y represéntalos en la recta. (1 punto)

a) $|2x + 6| < 1$

b) $|x - \frac{1}{2}| > \frac{1}{4}$

4. Reduce a un solo logaritmo, aplicando las propiedades de los logaritmos. (1 punto)

$$3 \log(x + 10) - \log \frac{2x + 20}{3} + \log \frac{3}{2}$$

5. Resuelve las siguientes ecuaciones. (3 puntos)

a) $4^{x^2+1} = 2^{5x+5}$

b) $\frac{x+1}{x+1} + \frac{2x+3}{2x+3} = 5$

c) $6x^3 - 7x^2 - 14x + 15 = 0$

6. Resuelve el siguiente sistema. (1,5 puntos)

$$\begin{cases} 2x + 4y = 10 \\ x^2 + 3xy = -8 \end{cases}$$

$$1) a) \frac{\sqrt[12]{(a^3 b)^3} \cdot \sqrt[12]{(a^4 b^3)^2}}{\sqrt[12]{\frac{1}{b^6}}} : \sqrt{a^2} = \left(\sqrt[12]{\frac{a^9 b^3 \cdot a^8 b^6}{1} : \frac{1}{b^6}} \right) : \sqrt{a^2} =$$

$$\sqrt[12]{a^9 b^3 a^8 b^6 \cdot b^6 : a^2} = \sqrt[12]{a^{9+8-2} b^{3+6+6}} = \sqrt[12]{a^{15} \cdot b^{15}} = \sqrt[4]{a^5 \cdot b^5} = ab^4 ab.$$

$$b) \sqrt[3]{81a^3} + 2a\sqrt[3]{24} = \sqrt[3]{3^4 a^3} + 2a\sqrt[3]{2^3 \cdot 3} = 3a\sqrt[3]{3} + 2a \cdot 2\sqrt[3]{3} =$$

$$= \boxed{7a\sqrt[3]{3}}$$

$$c) \sqrt{x} \sqrt[4]{x} \sqrt[3]{x^3} = \sqrt[12]{x^6 \cdot x^4 \cdot (x^3)^3} = \sqrt[12]{x^{19}} = \boxed{x \sqrt[12]{x^7}}$$

$$d) \sqrt[3]{\sqrt{2}} \sqrt[4]{4} = \sqrt[3]{\sqrt{2^3} \cdot 2^2} = \sqrt[18]{2^3 \cdot 2^4} = \boxed{\sqrt[18]{2^7}}$$

$$2) \frac{3}{\sqrt{3}-\sqrt{2}} + \frac{2}{\sqrt{3}+\sqrt{2}} = 3\sqrt{3}+3\sqrt{2}-2\sqrt{3}+2\sqrt{2} = \boxed{\sqrt{3}+5\sqrt{2}}$$

$$\left(\frac{3}{\sqrt{3}-\sqrt{2}} \right) \cdot \left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} \right) = \frac{3\sqrt{3}+3\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{3\sqrt{3}+3\sqrt{2}}{3-2} = 3\sqrt{3}+3\sqrt{2}$$

$$\left(\frac{2}{\sqrt{3}+\sqrt{2}} \right) \cdot \left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \right) = \frac{2\sqrt{3}-2\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{2\sqrt{3}-2\sqrt{2}}{3-2} = 2\sqrt{3}-2\sqrt{2}$$

$$3) a) |2x+6| < 1 \begin{cases} 2x+6 < 1 \rightarrow x < \frac{1-6}{2}; x < -\frac{5}{2} \\ 2x+6 > -1 \rightarrow x > -\frac{7}{2}; x > -\frac{7}{2} \end{cases}$$

$$-\frac{7}{2} < x < -\frac{5}{2} \rightarrow \boxed{\left(-\frac{7}{2}, -\frac{5}{2}\right)}$$

$$b) \left| x - \frac{1}{2} \right| > \frac{1}{4} \Rightarrow \begin{cases} x - \frac{1}{2} > \frac{1}{4}; x > \frac{1}{4} + \frac{1}{2}; \boxed{x > \frac{3}{4}} \\ x - \frac{1}{2} < -\frac{1}{4}; x < -\frac{1}{4} + \frac{1}{2}; x < \frac{1}{4}; \boxed{x < \frac{1}{4}} \end{cases}$$

$$\boxed{\left(-\infty, \frac{1}{4}\right) \cup \left(\frac{3}{4}, \infty\right)}$$

$$4) \log(x+10)^3 - \log \frac{2x+20}{3} + \log \frac{3}{2} = \log \left(\frac{(x+10)^3}{\frac{2x+20}{3}} \cdot \frac{3}{2} \right)$$

$$= \log \left[\frac{3 \cdot (x+10)^3}{2(2x+10)} \cdot \frac{3}{2} \right] = \boxed{\log \left(\frac{9(x+10)^2}{4} \right)}$$

$$a) 4^{x^2+1} = 2^{5x+5}, \quad (2^2)^{x^2+1} = 2^{5x+5}; \quad 2^{2(x^2+1)} = 2^{5x+5}$$

$$2x^2+2=5x+5; \quad 2x^2-5x-3=0; \quad x = \frac{5 \pm \sqrt{25-4 \cdot 2 \cdot (-3)}}{2 \cdot 2} = \frac{5 \pm 7}{4}$$

$x = \frac{5+7}{4} = \frac{12}{4} = 3$	<u>Soluciones</u>
$x = \frac{5-7}{4} = \frac{-2}{4} = -\frac{1}{2}$	

$$b) \sqrt{x+1} + \sqrt{2x+3} = 5$$

$$(\sqrt{x+1})^2 = (5 - \sqrt{2x+3})^2; \quad x+1 = 25 + (\sqrt{2x+3})^2 - 2 \cdot 5 \cdot \sqrt{2x+3};$$

$$x+1 = 25 + 2x+3 - 10\sqrt{2x+3}; \quad 10\sqrt{2x+3} = 25 + 2x+3 - x - 1;$$

$$10\sqrt{2x+3} = (27+x); \quad \frac{10^2 \cdot (2x+3)}{100} = 27^2 + x^2 + 2 \cdot 27 \cdot x;$$

$$200x+300 = 729 + x^2 + 54x; \quad x^2 - 146x + 429 = 0$$

$$x = \frac{146 \pm \sqrt{146^2 - 4 \cdot 1 \cdot 429}}{2} = \frac{146 \pm \sqrt{19600}}{2} = \frac{146 \pm 140}{2}$$

$$x = \frac{146+140}{2} = 143 \text{ No}$$

$$\sqrt{143+1} + \sqrt{2 \cdot 143+3} = 12 + \dots \neq 5$$

$$x = \frac{146-140}{2} = 3 \text{ Si}$$

$$\sqrt{3+1} + \sqrt{2 \cdot 3+3} = \sqrt{4} + \sqrt{9} = 2+3=5$$

$$c) 6x^3 - 7x^2 - 14x + 15 = 0$$

	6	-7	-14	15
1	6	-1	-15	
	6	-1	-15	0

$$6x^2 - x - 15 = 0$$

$$x = \frac{1 \pm \sqrt{1 - 4 \cdot 6 \cdot (-15)}}{2 \cdot 6} = \frac{1 \pm \sqrt{361}}{12}$$

$$x = \frac{1+19}{12} = \frac{20}{12} = \frac{5}{3} = x$$

$$x = \frac{-18}{12} = -\frac{3}{2} = x$$

$$x = 1$$

$$\textcircled{6} \quad \begin{cases} 2x + 4y = 10 \\ x^2 + 3xy = -8 \end{cases} \rightarrow x + 2y = 5 \quad ; \quad x = 5 - 2y$$

$$(5 - 2y)^2 + 3(5 - 2y)y = -8 \quad ; \quad 25 + 4y^2 - 20y + 15y - 6y^2 = -8$$

$$-2y^2 - 5y + 33 = 0$$

$$y = \frac{5 \pm \sqrt{5^2 - 4(-2) \cdot 33}}{2(-2)} = \frac{5 \pm \sqrt{289}}{-4} = \frac{5 \pm 17}{-4} \quad \begin{cases} \frac{22}{-4} = \textcircled{-\frac{11}{2}} \\ \frac{-12}{-4} = \textcircled{3} \end{cases}$$