Polinomios

Ejercicios resueltos

2.1-1 Realiza la suma de los siguientes polinomios:

a) $p(x)=x^5+x^4-4x^3+6x^2+x-7$

$q(x)=x^6+2x^4+x^2+5$

b) $p(x)=9x^5-2x^4+12x^3+x^2-x+10$

$q(x)=-x^5+5x^4-12x^3-2x^2+x-15$

c) $p(x)=-5x^4+6x^3-2x^2+3x+8$

$q(x)=2x^4-3x^3+2x^2-4$

d) $p(x)=3x^4+x^3-2x^2+x-14$

$q(x)=6x^4-8x^3+2x^2-3x$

$r(x)=2x+14$

e) $p(x)=-x^6+4x^5-2x^4-7x^3+6x^2+x-2$

$q(x)=3x^6+2x^5-x^3+2x^2-2x+5$

$r(x)=-2x^6-6x^5+2x^4+8x^3-8x^2+x-3$

f) $p(x)=x^4-3x^3+x^2-7x+11$

$q(x)=2x^5-3x^4+x^3-x^2-7$

$r(x)=-3x^5+2x^4-5x^3+8x^2+3x-4$

Solución

a) $\left.\begin{array}{l} p(x)=x^5+x^4-4x^3+6x^2+x-7 \\ q(x)=x^6+2x^4+x^2+5 \end{array}\right\}$

$p(x)+q(x)=x^6+x^5+3x^4-4x^3+7x^2+x-2$

b) $\left.\begin{array}{l} p(x)=9x^5-2x^4+12x^3+x^2-x+10 \\ q(x)=-x^5+5x^4-12x^3-2x^2+x-15 \end{array}\right\}$

$p(x)+q(x)=8x^5+3x^4-x^2-5$

c) $$\left.\begin{array}{l} p(x) = -5x^4 + 6x^3 - 2x^2 + 3x + 8 \\ q(x) = 2x^4 - 3x^3 + 2x^2 - 4 \end{array}\right\}$$

$$p(x) + q(x) = -3x^4 + 3x^3 + 3x + 4$$

d) $$\left.\begin{array}{l} p(x) = 3x^4 + x^3 - 2x^2 + x - 14 \\ q(x) = 6x^4 - 8x^3 + 2x^2 - 3x \\ r(x) = 2x + 14 \end{array}\right\}$$

$$p(x) + q(x) + r(x) = \left[p(x) + q(x)\right] + r(x) =$$
$$= \left[9x^4 - 7x^3 - 2x - 14\right] + 2x + 14 = 9x^4 - 7x^3$$

e) $$\left.\begin{array}{l} p(x) = -x^6 + 4x^5 - 2x^4 - 7x^3 + 6x^2 + x - 2 \\ q(x) = 3x^6 + 2x^5 - x^3 + 2x^2 - 2x + 5 \\ r(x) = -2x^6 - 6x^5 + 2x^4 + 8x^3 - 8x^2 + x - 3 \end{array}\right\}$$

$$p(x) + q(x) + r(x) = \left[p(x) + q(x)\right] + r(x) =$$
$$= \left[2x^6 + 6x^5 - 2x^4 - 8x^3 + 8x^2 - x + 3\right] +$$
$$+ \left[-2x^6 - 6x^5 + 2x^4 + 8x^3 - 8x^2 + x - 3\right] = 0$$

f) $$\left.\begin{array}{l} p(x) = x^4 - 3x^3 + x^2 - 7x + 11 \\ q(x) = 2x^5 - 3x^4 + x^3 - x^2 - 7 \\ r(x) = -3x^5 + 2x^4 - 5x^3 + 8x^2 + 3x - 4 \end{array}\right\}$$

$$p(x) + q(x) + r(x) = \left[p(x) + q(x)\right] + r(x) =$$
$$= \left[2x^5 - 2x^4 - 2x^3 - 7x + 4\right] + \left[-3x^5 + 2x^4 - 5x^3 + 8x^2 + 3x - 4\right] =$$
$$= -x^5 - 7x^3 + 8x^2 - 4x$$

2.1-2 Realiza la resta de los siguientes polinomios:

$$a)\quad p(x)=x^6+2x^5-3x^4+x^3+4x^2+4x-4$$
$$q(x)=-x^6+2x^5-5x^4+x^3+2x^2+3x-8$$

$$b)\quad p(x)=-3x^3+7x^2-3x-2$$
$$q(x)=5x^3+5x^2+5x+5$$

$$c)\quad p(x)=x^4+4x^3-2x^2+7x+10$$
$$q(x)=-2x^4+5x^3-8x^2+3x+11$$

$$d)\quad p(x)=-x^5+5x^3+4x^2-x+1$$
$$q(x)=x^4+9x^3-3x^2+x-1$$

$$e)\quad p(x)=-7x^3+x^2-12x-2$$
$$q(x)=-6x^3+3x^2-13x+15$$

$$f)\quad p(x)=x^4+3x^3-3x^2+2x+14$$
$$q(x)=-x^5-2x^4+3x^3-3x+14$$

Solución

$$a)\quad \left.\begin{array}{l} p(x)=x^6+2x^5-3x^4+x^3+4x^2+4x-4 \\ q(x)=-x^6+2x^5-5x^4+x^3+2x^2+3x-8 \end{array}\right\}$$

$$p(x)-q(x)=p(x)+\left[-q(x)\right]=$$
$$=x^6+2x^5-3x^4+x^3+4x^2+4x-4-$$
$$-\left[-x^6+2x^5-5x^4+x^3+2x^2+3x-8\right]$$

$$p(x)-q(x)=2x^6+2x^4+2x^2+x+4$$

$$b)\quad \left.\begin{array}{l} p(x)=-3x^3+7x^2-3x-2 \\ q(x)=5x^3+5x^2+5x+5 \end{array}\right\}$$

$$p(x)-q(x)=p(x)+\left[-q(x)\right]=$$
$$=-3x^3+7x^2-3x-2-\left[5x^3+5x^2+5x+5\right]$$

$$p(x)-q(x)=-8x^3+2x^2-8x-7$$

c) $\left.\begin{array}{l} p(x) = x^4 + 4x^3 - 2x^2 + 7x + 10 \\ q(x) = -2x^4 + 5x^3 - 8x^2 + 3x + 11 \end{array}\right\}$

$$p(x) - q(x) = p(x) + [-q(x)] =$$
$$= x^4 + 4x^3 - 2x^2 + 7x + 10 - [-2x^4 + 5x^3 - 8x^2 + 3x + 11]$$

$$p(x) - q(x) = 3x^4 - x^3 + 6x^2 + 4x - 1$$

d) $\left.\begin{array}{l} p(x) = -x^5 + 5x^3 + 4x^2 - x + 1 \\ q(x) = x^4 + 9x^3 - 3x^2 + x - 1 \end{array}\right\}$

$$p(x) - q(x) = p(x) + [-q(x)] =$$
$$= -x^5 + 5x^3 + 4x^2 - x + 1 - [x^4 + 9x^3 - 3x^2 + x - 1]$$

$$p(x) - q(x) = -x^5 - x^4 - 4x^3 + 7x^2 - 2x + 2$$

e) $\left.\begin{array}{l} p(x) = -7x^3 + x^2 - 12x - 2 \\ q(x) = -6x^3 + 3x^2 - 13x + 15 \end{array}\right\}$

$$p(x) - q(x) = p(x) + [-q(x)] =$$
$$= -7x^3 + x^2 - 12x - 2 - [-6x^3 + 3x^2 - 13x + 15]$$

$$p(x) - q(x) = -x^3 - 2x^2 + x - 17$$

f) $p(x) = x^4 + 3x^3 - 3x^2 + 2x + 14$

$q(x) = -x^5 - 2x^4 + 3x^3 - 3x + 14$

$$p(x) - q(x) = p(x) + [-q(x)] =$$
$$= x^4 + 3x^3 - 3x^2 + 2x + 14 - [-x^5 - 2x^4 + 3x^3 - 3x + 14]$$

$$p(x) - q(x) = x^5 + 3x^4 - 3x^2 + 5x$$

2.1-3 Realiza el producto de los siguientes polinomios:

a) $p(x) = x^4 + 2x^3 - x^2 + 3x + 1$

$q(x) = 2x$

b) $p(x) = -x^5 + x^4 - x^3 + x^2 - x + 1$

$q(x) = -5x^4$

c) $p(x) = 2x^6 + 3x^4 + x^2 - 6$

$q(x) = x^3 + x$

d) $p(x) = x^2 + 2x + 3$

$q(x) = -x^2 + x + 4$

e) $p(x) = x^5 + 3x^3 + 6x$

$q(x) = -x^2 + 2x - 2$

$r(x) = 2x^3 + 5x^2 - 2x + 3$

f) $p(x) = -x^2 + 2x + 3$

$q(x) = x^2 + x + 1$

$r(x) = x^4 + x^3 + x^2 + x + 4$

Solución

a) $\left.\begin{array}{l} p(x) = x^4 + 2x^3 - x^2 + 3x + 1 \\ q(x) = 2x \end{array}\right\}$

$$p(x)\cdot q(x) = \left[x^4 + 2x^3 - x^2 + 3x + 1\right]\cdot 2x =$$
$$= x^4\cdot 2x + 2x^3\cdot 2x - x^2\cdot 2x + 3x\cdot 2x + 1\cdot 2x =$$
$$= 2x^5 + 4x^4 - 2x^3 + 6x^2 + 2x$$

b) $\left.\begin{array}{l} p(x) = -x^5 + x^4 - x^3 + x^2 - x + 1 \\ q(x) = -5x^4 \end{array}\right\}$

$$p(x)\cdot q(x) = \left[-x^5 + x^4 - x^3 + x^2 - x + 1\right]\cdot\left(-5x^4\right) =$$
$$= 5x^9 - 5x^8 + 5x^7 - 5x^6 + 5x^5 - 5x^4$$

c) $\left.\begin{array}{l} p(x) = 2x^6 + 3x^4 + x^2 - 6 \\ q(x) = x^3 + x \end{array}\right\}$

$$\begin{aligned} p(x)\cdot q(x) &= \left[2x^6 + 3x^4 + x^2 - 6\right]\cdot\left[x^3 + x\right] = \\ &= \left[2x^6 + 3x^4 + x^2 - 6\right]\cdot x^3 + \left[2x^6 + 3x^4 + x^2 - 6\right]\cdot x = \\ &= \left[2x^9 + 3x^7 + x^5 - 6x^3\right] + \left[2x^7 + 3x^5 + x^3 - 6x\right] = \\ &= 2x^9 + 5x^7 + 4x^5 - 5x^3 - 6x \end{aligned}$$

d) $\left.\begin{array}{l} p(x) = x^2 + 2x + 3 \\ q(x) = -x^2 + x + 4 \end{array}\right\}$

$$\begin{aligned} p(x)\cdot q(x) &= \left[x^2 + 2x + 3\right]\cdot\left[-x^2 + x + 4\right] = \\ &= \left[x^2 + 2x + 3\right]\cdot\left(-x^2\right) + \left[x^2 + 2x + 3\right]\cdot x + \left[x^2 + 2x + 3\right]\cdot 4 = \\ &= -x^4 - 2x^3 - 3x^2 + x^3 + 2x^2 + 3x + 4x^2 + 8x + 12 = \\ &= -x^4 - x^3 + 3x^2 + 11x + 12 \end{aligned}$$

e) $\left.\begin{array}{l} p(x) = x^5 + 3x^3 + 6x \\ q(x) = -x^2 + 2x - 2 \\ r(x) = 2x^3 + 5x^2 - 2x + 3 \end{array}\right\}$

$$\begin{aligned} p(x)\cdot q(x)\cdot r(x) &= \left[p(x)\cdot q(x)\right]\cdot r(x) = \\ &= \left[\left(x^5 + 3x^3 + 6x\right)\cdot\left(-x^2 + 2x - 2\right)\right]\cdot\left(2x^3 + 5x^2 - 2x + 3\right) = \\ &= \left(-x^7 - 3x^5 - 6x^3 + 2x^6 + 6x^4 + 12x^2 - 2x^5 - 6x^3 - 12x\right)\cdot \\ &\cdot\left(2x^3 + 5x^2 - 2x + 3\right) \end{aligned}$$

$$\begin{aligned} p(x)\cdot q(x)\cdot r(x) &= \left(-x^7 + 2x^6 - 5x^5 + 6x^4 - 12x^3 + 12x^2 - 12x\right)\cdot \\ &\cdot\left(2x^3 + 5x^2 - 2x + 3\right) \end{aligned}$$

$$\begin{aligned} p(x)\cdot q(x)\cdot r(x) &= \\ &= -2x^{10} + 4x^9 - 10x^8 + 12x^7 - 24x^6 + 24x^5 - 24x^4 - \\ &- 5x^9 + 10x^8 - 25x^7 + 30x^6 - 60x^5 + 60x^4 - 60x^3 + \\ &+ 2x^8 - 4x^7 + 10x^6 - 12x^5 + 24x^4 - 24x^3 + 24x^2 - \\ &- 3x^7 + 6x^6 - 15x^5 + 18x^4 - 36x^3 + 36x^2 - 36x \end{aligned}$$

$$\begin{aligned} &p(x)\cdot q(x)\cdot r(x) = \\ &= -2x^{10} - x^9 + 2x^8 - 20x^7 + 22x^6 - 63x^5 + 78x^4 - 120x^3 + 60x^2 - 36x \end{aligned}$$

$f)$ $$\left.\begin{aligned} p(x)&=-x^2+2x+3\\ q(x)&=x^2+x+1\\ r(x)&=x^4+x^3+x^2+x+4\end{aligned}\right\}$$

$$p(x)\cdot q(x)\cdot r(x)=\left[p(x)\cdot q(x)\right]\cdot r(x)=$$
$$=\left[\left(-x^2+2x+3\right)\cdot\left(x^2+x+1\right)\right]\cdot\left(x^4+x^3+x^2+x+4\right)$$

$$p(x)\cdot q(x)\cdot r(x)=\left[p(x)\cdot q(x)\right]\cdot r(x)=$$
$$=\left(-x^4+2x^3+3x^2-x^3+2x^2+3x-x^2+2x+3\right)\cdot$$
$$\cdot\left(x^4+x^3+x^2+x+4\right)$$

$$p(x)\cdot q(x)\cdot r(x)=\left[p(x)\cdot q(x)\right]\cdot r(x)=$$
$$=\left(-x^4+x^3+4x^2+5x+3\right)\cdot\left(x^4+x^3+x^2+x+4\right)$$

$$p(x)\cdot q(x)\cdot r(x)=\left[p(x)\cdot q(x)\right]\cdot r(x)=$$
$$=-x^8+x^7+4x^6+5x^5+3x^4-$$
$$-x^7+x^6+4x^5+5x^4+3x^3-$$
$$-x^6+x^5+4x^4+5x^3+3x^2-$$
$$-x^5+x^4+4x^3+5x^2+3x-$$
$$-4x^4+4x^3+16x^2+20x+12$$

$$p(x)\cdot q(x)\cdot r(x)=$$
$$=-x^8+4x^6+9x^5+9x^4+16x^3+24x^2+23x+12$$

2.1-4 Dados los siguientes polinomios, realiza la operación que se indica:

a) $\left.\begin{array}{l} p(x)=x^3+6x^2-x+2 \\ q(x)=2x^2+4x-3 \end{array}\right\}$ $\quad (p(x)+q(x))\cdot(p(x)-q(x))$

b) $\left.\begin{array}{l} p(x)=x^2-5x+2 \\ q(x)=-x^2+3x-4 \end{array}\right\}$ $\quad (3\cdot p(x)+q(x))\cdot(p(x)-2\cdot q(x))$

Solución

a) $\left.\begin{array}{l} p(x)=x^3+6x^2-x+2 \\ q(x)=2x^2+4x-3 \end{array}\right\}$ $\quad (p(x)+q(x))\cdot(p(x)-q(x))$

$$\left.\begin{array}{l} p(x)+q(x)=x^3+8x^2+3x-1 \\ p(x)-q(x)=x^3+4x^2-5x+5 \end{array}\right\}$$

$$(p(x)+q(x))\cdot(p(x)-q(x))=\left(x^3+8x^2+3x-1\right)\cdot\left(x^3+4x^2-5x+5\right)$$

$$\begin{aligned}(p(x)+q(x))\cdot(p(x)-q(x))=\,&x^6+8x^5+3x^4-x^3+\\ &+4x^5+32x^4+12x^3-4x^2-\\ &-5x^4-40x^3-15x^2+5x+\\ &+5x^3+40x^2+15x-5\end{aligned}$$

$$(p(x)+q(x))\cdot(p(x)-q(x))=x^6+12x^5+30x^4-24x^3+21x^2+20x-5$$

b) $\left.\begin{array}{l} p(x)=x^2-5x+2 \\ q(x)=-x^2+3x-4 \end{array}\right\}$ $\quad (3\cdot p(x)+q(x))\cdot(p(x)-2\cdot q(x))$

$$\left.\begin{array}{l} 3\cdot p(x)+q(x)=3x^2-15x+6-x^2+3x-4=2x^2-12x+2 \\ p(x)-2\cdot q(x)=x^2-5x+2+2x^2-6x+8=3x^2-11x+10 \end{array}\right\}$$

$$(3\cdot p(x)+q(x))\cdot(p(x)-2\cdot q(x))=\left(2x^2-12x+2\right)\cdot\left(3x^2-11x+10\right)$$

$$\begin{aligned}(3\cdot p(x)+q(x))\cdot(p(x)-2\cdot q(x))=\,&6x^4-36x^3+6x^2-\\ &-22x^3+132x^2-22x+\\ &+20x^2-120x+20\end{aligned}$$

$$(3\cdot p(x)+q(x))\cdot(p(x)-2\cdot q(x))=6x^4-58x^3+158x^2-142x+20$$

2.1-5 Calcula las siguientes potencias de binomios utilizando el desarrollo del binomio de Newton:

$a)\ (x+1)^4$

$b)\ (x-4)^3$

$c)\ (2x+3)^5$

$d)\ (x^2+2)^3$

$e)\ (x^3+1)^5$

$f)\ (3x^2-1)^4$

Solución

$$a)\ (x+1)^4 = \binom{4}{0}x^4 + \binom{4}{1}x^3\cdot 1^1 + \binom{4}{2}x^2\cdot 1^2 + \binom{4}{3}x\cdot 1^3 + \binom{4}{4}1^4 =$$

$$= \frac{4!}{4!\cdot 0!}x^4 + \frac{4!}{3!\cdot 1!}x^3 + \frac{4!}{2!\cdot 2!}x^2 + \frac{4!}{1!\cdot 3!}x + \frac{4!}{0!\cdot 4!} =$$

$$= x^4 + 4x^3 + 6x^2 + 4x + 1$$

$$b)\ (x-4)^3 = \binom{3}{0}x^3 + \binom{3}{1}x^2\cdot(-4)^1 + \binom{3}{2}x\cdot(-4)^2 + \binom{3}{3}(-4)^3 =$$

$$= \frac{3!}{3!\cdot 0!}x^3 - 4\frac{3!}{2!\cdot 1!}x^2 + 16\frac{3!}{1!\cdot 2!}x - 64\frac{3!}{0!\cdot 3!} =$$

$$= x^3 - 12x^2 + 48x - 64$$

$$c)\ (2x+3)^5 =$$

$$= \binom{5}{0}(2x)^5 + \binom{5}{1}(2x)^4\cdot(3)^1 + \binom{5}{2}(2x)^3\cdot(3)^2 +$$

$$+ \binom{5}{3}(2x)^2\cdot(3)^3 + \binom{5}{4}(2x)^1\cdot(3)^4 + \binom{5}{5}(3)^5 =$$

$$= 32\frac{5!}{5!\cdot 0!}x^5 + 48\frac{5!}{4!\cdot 1!}x^4 + 72\frac{5!}{3!\cdot 2!}x^3 +$$

$$+ 108\frac{5!}{2!\cdot 3!}x^2 + 162\frac{5!}{1!\cdot 4!}x + 243\frac{5!}{0!\cdot 5!} =$$

$$= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$$

d) $\left(x^2+2\right)^3=\binom{3}{0}\left(x^2\right)^3+\binom{3}{1}\left(x^2\right)^2\cdot(2)^1+\binom{3}{2}\left(x^2\right)^1\cdot(2)^2+\binom{3}{3}(2)^3=$

$$=\frac{3!}{3!\cdot 0!}x^6+2\frac{3!}{2!\cdot 1!}x^4+4\frac{3!}{1!\cdot 2!}x^2+8\frac{3!}{0!\cdot 3!}=$$

$$=x^6+6x^4+12x^2+8$$

e) $\left(x^3+1\right)^5=$

$$=\binom{5}{0}\left(x^3\right)^5+\binom{5}{1}\left(x^3\right)^4\cdot(1)^1+\binom{5}{2}\left(x^3\right)^3\cdot(1)^2+$$

$$+\binom{5}{3}\left(x^3\right)^2\cdot(1)^3+\binom{5}{4}\left(x^3\right)^1\cdot(1)^4+\binom{5}{5}(1)^5=$$

$$=\frac{5!}{5!\cdot 0!}x^{15}+\frac{5!}{4!\cdot 1!}x^{12}+\frac{5!}{3!\cdot 2!}x^9+\frac{5!}{2!\cdot 3!}x^6+\frac{5!}{1!\cdot 4!}x^3+\frac{5!}{0!\cdot 5!}=$$

$$=x^{15}+5x^{12}+10x^9+10x^6+5x^3+1$$

f) $\left(3x^2-1\right)^4=$

$$=\binom{4}{0}\left(3x^2\right)^4+\binom{4}{1}\left(3x^2\right)^3\cdot(-1)^1+$$

$$+\binom{4}{2}\left(3x^2\right)^2\cdot(-1)^2+\binom{4}{3}\left(3x^2\right)^1\cdot(-1)^3+\binom{4}{4}(-1)^4=$$

$$=\frac{4!}{4!\cdot 0!}81x^8-\frac{4!}{3!\cdot 1!}27x^6+\frac{4!}{2!\cdot 2!}9x^4-\frac{4!}{1!\cdot 3!}3x^2+\frac{4!}{0!\cdot 4!}=$$

$$=81x^8-108x^6+54x^4-12x^2+1$$

2.1-6 Realiza la división entera de los siguientes polinomios:

$$a)\ \begin{array}{l} p(x)=x^2-6x+4 \\ q(x)=x^2-2 \end{array}$$

$$b)\ \begin{array}{l} p(x)=x^5+3x^3+6x-2 \\ q(x)=x^3+x \end{array}$$

$$c)\ \begin{array}{l} p(x)=x^6+3 \\ q(x)=x^2+2x-4 \end{array}$$

$$d)\ \begin{array}{l} p(x)=x^6+2x^5-x^4+2x^3+6x^2-x+3 \\ q(x)=x^3+10x^2-2x+3 \end{array}$$

Solución

$$a)\ \left.\begin{array}{l} p(x)=x^2-6x+4 \\ q(x)=x^2-2 \end{array}\right\} \Rightarrow p(x)=c(x)\cdot q(x)+r(x)$$

$$\begin{array}{r|l} x^2-6x+4 & x^2-2 \\ \cline{2-2} -x^2+2 & 1 \\ \hline -6x+6 & \end{array}$$

$$\left.\begin{array}{l} c(x)=1 \\ r(x)=-6x+6 \end{array}\right\} \Rightarrow x^2-6x+4=1\cdot\left(x^2-2\right)+\left(-6x+6\right)$$

$$b)\ \left.\begin{array}{l} p(x)=x^5+3x^3+6x-2 \\ q(x)=x^3+x \end{array}\right\} \Rightarrow p(x)=c(x)\cdot q(x)+r(x)$$

$$\begin{array}{r|l} x^5+3x^3+6x-2 & x^3+x \\ \cline{2-2} -x^5-x^3 & x^2+2 \\ \hline 2x^3+6x-2 & \\ -2x^3-2x & \\ \hline 4x-2 & \end{array}$$

$$\left.\begin{array}{l} c(x)=x^2+2 \\ r(x)=4x-2 \end{array}\right\} \Rightarrow x^5+3x^3+6x-2=\left(x^2+2\right)\cdot\left(x^3+x\right)+\left(4x-2\right)$$

c) $\left.\begin{array}{l} p(x)=x^6+3 \\ q(x)=x^2+2x-4 \end{array}\right\} \Rightarrow p(x)=c(x)\cdot q(x)+r(x)$

$$\begin{array}{c|l} x^6+3 & x^2+2x-4 \\ \cline{2-2} -x^6-2x^5+4x^4 & x^4-2x^3+8x^2-24x+80 \\ \hline -2x^5+4x^4+3 & \\ 2x^5+4x^4-8x^3 & \\ \hline 8x^4-8x^3+3 & \\ -8x^4-16x^3+32x^2 & \\ \hline -24x^3+32x^2+3 & \\ 24x^3+48x^2-96x & \\ \hline 80x^2-96x+3 & \\ -80x^2-160x+320 & \\ \hline -256x+323 & \end{array}$$

$\left.\begin{array}{l} c(x)=x^4-2x^3+8x^2-24x+80 \\ r(x)=-256x+323 \end{array}\right\}$

$$\Rightarrow x^6+3=\left(x^4-2x^3+8x^2-24x+80\right)\cdot\left(x^2+2x-4\right)+\left(-256x+323\right)$$

d) $\left.\begin{array}{l} p(x)=x^6+2x^5-x^4+2x^3+6x^2-x+3 \\ q(x)=x^3+10x^2-2x+3 \end{array}\right\} \Rightarrow p(x)=c(x)\cdot q(x)+r(x)$

$$\begin{array}{c|l} x^6+2x^5-x^4+2x^3+6x^2-x+3 & x^3+10x^2-2x+3 \\ \cline{2-2} -x^6-10x^5+2x^4-3x^3 & x^3-8x^2+81x-827 \\ \hline -8x^5+x^4-x^3+6x^2-x+3 & \\ 8x^5+80x^4-16x^3+24x^2 & \\ \hline 81x^4-17x^3+30x^2-x+3 & \\ -81x^4-810x^3+162x^2-243x & \\ \hline -827x^3+192x^2-244x+3 & \\ 827x^3+8270x^2-1654x+2481 & \\ \hline 8462x^2-1898x+2484 & \end{array}$$

$\left.\begin{array}{l} c(x)=x^3-8x^2+81x-827 \\ r(x)=8462x^2-1898x+2484 \end{array}\right\}$

$$\Rightarrow x^6+2x^5-x^4+2x^3+6x^2-x+3=$$

$$=\left(x^3-8x^2+81x-827\right)\cdot\left(x^3+10x^2-2x+3\right)+\left(8462x^2-1898x+2484\right)$$

2.1-7 Factoriza según sus raíces reales los siguientes polinomios:

$a)\ p(x)=x^2-4x-5$

$b)\ p(x)=x^3-5x^2+6x$

$c)\ p(x)=x^4-13x^2+36$

$d)\ p(x)=x^4-16$

Solución

$a)\ p(x)=x^2-4x-5$

$$x^2-4x-5=0 \Rightarrow x=\frac{4\pm\sqrt{16+20}}{2}=\frac{4\pm 6}{2}=5,-1$$

$$p(x)=x^2-4x-5=(x-5)\cdot(x+1)$$

$b)\ p(x)=x^3-5x^2+6x$

$$x^3-5x^2+6x=x\cdot\left(x^2-5x+6\right)=0 \Rightarrow \begin{cases} x=0 \\ x^2-5x+6=0 \end{cases}$$

$$x^2-5x+6=0 \Rightarrow x=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm 1}{2}=3,2$$

$$p(x)=x^3-5x^2+6x=x\cdot(x-3)\cdot(x-2)$$

$c)\ p(x)=x^4-13x^2+36$

$$x^2=t \Rightarrow x^4-13x^2+36=0 \Rightarrow t^2-13t+36=0$$

$$t^2-13t+36=0 \Rightarrow x^2=t=\frac{13\pm\sqrt{169-144}}{2}=\frac{13\pm 5}{2}=9,4$$

$$x=\pm\sqrt{9},\pm\sqrt{4}=3,-3,2,-2$$

$$p(x)=x^4-13x^2+36=(x-3)\cdot(x+3)\cdot(x-2)\cdot(x+2)$$

$d)\ p(x)=x^4-16=\left(x^2-4\right)\cdot\left(x^2+4\right)$

$$\left(x^2-4\right)\cdot\left(x^2+4\right)=0 \Rightarrow \begin{cases} x^2-4=0 \\ x^2+4=0 \end{cases} \Rightarrow \begin{cases} x=2,-2 \\ x^2=\pm\sqrt{-4}\notin\mathbb{R} \end{cases}$$

$$p(x)=x^4-16=\left(x^2-4\right)\cdot\left(x^2+4\right)=(x-2)\cdot(x+2)\cdot\left(x^2+4\right)$$

2.1-8 Factoriza según sus raíces reales los siguientes polinomios utilizando la regla de Ruffini:

a) $p(x)=x^3-3x^2-6x+8$

b) $p(x)=x^3+3x^2-13x-15$

c) $p(x)=x^3-7x^2+x-7$

d) $p(x)=x^4-2x^3-x+2$

e) $p(x)=x^5-x^4-10x^3+9x^2+9x$

f) $p(x)=x^4-\frac{5}{6}x^3-\frac{7}{3}x^2-\frac{1}{6}x+\frac{1}{3}$

Solución

a) $p(x)=x^3-3x^2-6x+8$

Divisores enteros del término independiente: 1, –1, 2, –2, 4, –4, 8, –8

	1	–3	–6	8
1		1	–2	–8
	1	–2	–8	0

Resto cero

$$p(x)=x^3-3x^2-6x+8=(x^2-2x-8)\cdot(x-1)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$x^2-2x-8=0\Rightarrow x=\frac{2\pm\sqrt{4+32}}{2}=\frac{2\pm 6}{2}=4,-2$$

Por tanto:

$$p(x)=x^3-3x^2-6x+8=(x-4)\cdot(x+2)\cdot(x-1)$$

b) $p(x)=x^3+3x^2-13x-15$

Divisores enteros del término independiente: 1, –1, 3, –3, 5, –5, 15, –15

	1	3	–13	–15
–1		–1	–2	15
	1	2	–15	0

Resto cero

$$p(x) = x^3 + 3x^2 - 13x - 15 = (x^2 + 2x - 15) \cdot (x + 1)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$x^2 + 2x - 15 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4 + 60}}{2} = \frac{-2 \pm 8}{2} = 3, -5$$

Por tanto:

$$p(x) = x^3 + 3x^2 - 13x - 15 = (x - 3) \cdot (x + 5) \cdot (x + 1)$$

c) $p(x) = x^3 - 7x^2 + x - 7$

Divisores enteros del término independiente: 1, –1, 7, –7

	1	–7	1	–7
7		7	0	7
	1	0	1	0

Resto cero

$$p(x) = x^3 - 7x^2 + x - 7 = (x^2 + 1) \cdot (x - 7)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$x^2 + 1 = 0 \Rightarrow x^2 = -1 \Rightarrow x = \pm\sqrt{-1}$, que no son raíces reales.

Por tanto:

$$p(x) = x^3 - 7x^2 + x - 7 = (x^2 + 1) \cdot (x - 7)$$

d) $p(x) = x^4 - 2x^3 - x + 2$

Divisores enteros del término independiente: 1, –1, 2, –2

	1	–2	0	–1	2
1		1	–1	–1	–2
	1	–1	–1	–2	0

Resto cero

$$p(x) = x^4 - 2x^3 - x + 2 = (x^3 - x^2 - x - 2) \cdot (x - 1)$$

Para el polinomio de grado 3 que resulta utilizamos otra vez Ruffini con los divisores enteros del término independiente: 1, –1, 2, –2

	1	–1	–1	–2
2		2	2	2
	1	1	1	0

Resto cero

Por tanto:

$$p(x) = x^4 - 2x^3 - x + 2 = (x^2 + x + 1) \cdot (x - 2) \cdot (x - 1)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$x^2 + x + 1 = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{1-4}}{2}$, que no son raíces reales.

Por tanto:

$$p(x) = x^4 - 2x^3 - x + 2 = (x^2 + x + 1) \cdot (x - 2) \cdot (x - 1)$$

e) $p(x) = x^5 - x^4 - 10x^3 + 9x^2 + 9x$

Podemos factorizar fácilmente el polinomio sacando *x* factor común:

$$p(x) = x^5 - x^4 - 10x^3 + 9x^2 + 9x = (x^4 - x^3 - 10x^2 + 9x + 9) \cdot x$$

Para el polinomio de grado 4 que resulta utilizamos Ruffini con los divisores enteros del término independiente: 1, –1, 3, –3, 9, –9

	1	–1	–10	9	9
3		3	6	–12	–9
	1	2	–4	–3	0

Resto cero

$$p(x) = x^5 - x^4 - 10x^3 + 9x^2 + 9x = (x^3 + 2x^2 - 4x - 3) \cdot (x - 3) \cdot x$$

Para el polinomio de grado 3 que resulta utilizamos otra vez Ruffini con los divisores enteros del término independiente: 1, –1, 3, –3

	1	2	–4	–3
–3		–3	3	3
	1	–1	–1	0

Resto cero

Por tanto:

$$p(x) = x^5 - x^4 - 10x^3 + 9x^2 + 9x = (x^2 - x - 1) \cdot (x + 3) \cdot (x - 3) \cdot x$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$x^2 - x - 1 = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{1+4}}{2} = \dfrac{1+\sqrt{5}}{2}, \dfrac{1-\sqrt{5}}{2}$, que son raíces reales.

Por tanto:

$$p(x)=\left(x-\frac{1+\sqrt{5}}{2}\right)\cdot\left(x-\frac{1-\sqrt{5}}{2}\right)\cdot(x+3)\cdot(x-3)\cdot x$$

f) $p(x)=x^4-\frac{5}{6}x^3-\frac{7}{3}x^2-\frac{1}{6}x+\frac{1}{3}$

Como el término independiente no es entero y aparecen términos racionales, multiplicamos todo el polinomio por 6:

$$6p(x)=6x^4-5x^3-14x^2-x+2$$

Para este polinomio de grado 4 que resulta utilizamos Ruffini con los divisores enteros del término independiente: 1, –1, 2, –2

$$\begin{array}{c|ccccc} & 6 & -5 & -14 & -1 & 2 \\ 2 & & 12 & 14 & 0 & -2 \\ \hline & 6 & 7 & 0 & -1 & \lfloor 0 \end{array}$$

Resto cero

$$6p(x)=6x^4-5x^3-14x^2-x+2=\left(6x^3+7x^2-1\right)\cdot(x-2)$$

Para el polinomio de grado 3 que resulta utilizamos otra vez Ruffini con los divisores enteros del término independiente: 1, –1

$$\begin{array}{c|cccc} & 6 & 7 & 0 & -1 \\ -1 & & -6 & -1 & 1 \\ \hline & 6 & 1 & -1 & \lfloor 0 \end{array}$$

Resto cero

Por tanto:

$$6p(x)=6x^4-5x^3-14x^2-x+2=\left(6x^2+x-1\right)\cdot(x+1)\cdot(x-2)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$6x^2+x-1=0\Rightarrow x=\frac{-1\pm\sqrt{1+24}}{12}=\frac{-1\pm 5}{12}=\frac{1}{3},-\frac{1}{2}$$, son raíces reales.

Por tanto:

$$6p(x)=6x^4-5x^3-14x^2-x+2=6\cdot\left(x-\frac{1}{3}\right)\cdot\left(x+\frac{1}{2}\right)\cdot(x+1)\cdot(x-2)$$

Finalmente:

$$p(x)=x^4-\frac{5}{6}x^3-\frac{7}{3}x^2-\frac{1}{6}x+\frac{1}{3}=\left(x-\frac{1}{3}\right)\cdot\left(x+\frac{1}{2}\right)\cdot(x+1)\cdot(x-2)$$