

PROBLEMAS RESUELTOS DE CÁLCULO DE DERIVADAS

1) Calcular las derivadas de:

a) $f(x) = -\frac{2x^5}{\cos x}$

$$f'(x) = -\frac{10x^4 \cos x + 2x^5 \operatorname{sen} x}{\cos^2 x}$$

b) $g(x) = -\frac{3}{2} \ln \sqrt{7x}$

Simplificamos antes de derivar, aplicando propiedades de logaritmos neperianos: $g(x) = -\frac{3}{2} \frac{1}{2} \ln 7x = -\frac{3}{4} \ln 7x \Rightarrow g'(x) = -\frac{3}{4} \frac{1}{7x} = -\frac{3}{4x}$

c) $h(x) = \frac{e^{3x-5}}{2}$

Como $h(x) = \frac{1}{2} e^{3x-5} \Rightarrow h'(x) = \frac{1}{2} 3e^{3x-5} = \frac{3}{2} e^{3x-5}$

2) Halle $f'(2)$, $g'(4)$ y $h'(0)$ para las funciones definidas de la siguiente forma (L designa *logaritmo neperiano*):

$$f(x) = x^2 - \frac{16}{x^2}; \quad g(x) = (x+9)^3; \quad h(x) = L(x^2+1).$$

Simplemente, aplicando las reglas de derivación, se obtiene:

$$f'(x) = 2x - \frac{32x}{x^4} = 2x - \frac{32}{x^3} \Rightarrow f'(2) = 4 - \frac{32}{8} = 4 - 4 = 0$$

$$g'(x) = 3(x+9)^2 \Rightarrow g'(4) = 3(16+9) = 75$$

$$h'(x) = \frac{2x}{x^2+1} \Rightarrow h'(0) = \frac{0}{1} = 0$$

3) Derivar y simplificar:

a) $f(x) = \frac{3x-1}{x} - (5x-x^2)^2$

$$\begin{aligned} f'(x) &= \frac{3x-(3x-1)}{x^2} - 2(5x-x^2)(5-2x) = \frac{1}{x^2} - 2(25x-10x^2-5x^2+2x^3) = \\ &= \frac{1}{x^2} - 2(2x^3-15x^2+25x) = \frac{1}{x^2} - 4x^3 + 30x^2 - 50x = \\ &= \boxed{\frac{-4x^5 + 30x^4 - 50x^3 + 1}{x^2}} \end{aligned}$$

b) $g(x) = (x^2-1) \cdot \ln x$

$$g'(x) = 2x \ln x + \frac{x^2-1}{x} = \boxed{\frac{2x^2 \ln x + x^2 - 1}{x}}$$

c) $h(x) = 2^{5x}$.

$$h'(x) = \boxed{5 \cdot 2^{5x} \ln 2}$$

d) $i(x) = (x^3 - 6x) \cdot (x^2 + 1)^3$.

$$\begin{aligned} i'(x) &= (3x^2 - 6)(x^2 + 1)^3 + (x^3 - 6x) 3 \cdot 2x (x^2 + 1)^2 = \\ &= (x^2 + 1)^2 ((3x^2 - 6)(x^2 + 1) + 6x(x^3 - 6x)) = \\ &= (x^2 + 1)^2 (3x^4 + 3x^2 - 6x^2 - 6 + 6x^4 - 36x^2) = \boxed{(x^2 + 1)^2 (9x^4 - 39x^2 - 6)} \end{aligned}$$

e) $j(x) = (x+1) \cdot e^{2x+1}$.

$$j'(x) = e^{2x+1} + (x+1) 2 e^{2x+1} = e^{2x+1} (1 + 2x + 2) = \boxed{e^{2x+1} (2x + 3)}$$

f) $k(x) = 3x \cos 3x^2$.

$$k'(x) = 3 \cos 3x^2 - 3x \cdot 6x \sin 3x^2 = \boxed{3 \cos 3x^2 - 18x^2 \sin 3x^2}$$

Nota: La expresión simplificada final siempre puede resultar subjetiva, y debe entenderse como una expresión cómoda para operar y para volver a derivar si es preciso. Por ejemplo, en el d y el f se podría extraer 3 factor común.

4) Calcule las derivadas de las siguientes funciones:

a) $f(x) = \left(\frac{2-5x}{3} \right)^2 + \frac{1-2x}{x^2}$

$$\begin{aligned} f'(x) &= 2 \left(\frac{2-5x}{3} \right) \frac{-5}{3} + \frac{-2x^2 - (1-2x)2x}{x^4} = \frac{-10(2-5x)}{9} + \frac{-2x^2 - 2x + 4x^2}{x^4} = \\ &= \frac{50x - 20}{9} + \frac{2x^2 - 2x}{x^4} = \frac{50x - 20}{9} + \frac{x(2x - 2)}{x^4} = \frac{50x - 20}{9} + \frac{2x - 2}{x^3} = \\ &= \frac{50x^4 - 20x^3}{9x^3} + \frac{18x - 18}{9x^3} = \boxed{\frac{50x^2 - 20x^3 + 18x - 18}{9x^3}} \end{aligned}$$

b) $g(x) = (3x + 2)^2 \ln(1 + x^2)$

$$\begin{aligned} g'(x) &= 2(3x + 2)3\ln(1 + x^2) + (3x + 2)^2 \frac{2x}{1 + x^2} = \\ &= \boxed{(18x + 12)\ln(1 + x^2) + \frac{2x(3x + 2)^2}{1 + x^2}} \end{aligned}$$

c) $h(x) = 2^{5x} + \frac{1}{x^2}$

$$h'(x) = \boxed{5 \cdot 2^{5x} \ln 2 - \frac{2}{x^3}}$$

5) Calcule las derivadas de las siguientes funciones:

a) $f(x) = \frac{3x-1}{x} - (5x-x^2)^2$.

$$f'(x) = \frac{3x - (3x-1)}{x^2} - 2(5x-x^2)(5-2x) = \frac{3x-3x+1}{x^2} - (5x-x^2)(10-4x) =$$

$$= \frac{1}{x^2} - (50x - 20x^2 - 10x^2 + 4x^3) = \frac{1 - 50x^2 + 30x^4 - 4x^5}{x^2} =$$

$$= \frac{-4x^5 + 30x^4 - 50x^2 + 1}{x^2}$$

b) $g(x) = (x^2 - 1) \cdot \ln x.$

$$g'(x) = 2x \ln x + \frac{x^2 - 1}{x}$$

c) $h(x) = 2^{3x}.$

$$h'(x) = 3 \cdot 2^{3x} \ln 2$$

d) $i(x) = (x^3 - 6x) \cdot (x^2 + 1)^3.$

$$\begin{aligned} i'(x) &= (3x^2 - 6)(x^2 + 1)^3 + (x^3 - 6x)3(x^2 + 1)^2 2x = \\ &= (3x^2 - 6)(x^2 + 1)^3 + (6x^4 - 36x^2)(x^2 + 1)^2 = \\ &= (x^2 + 1)^2 [(3x^2 - 6)(x^2 + 1) + 6x^4 - 36x^2] = \\ &= (x^2 + 1)^2 (3x^4 + 3x^2 - 6x^2 - 6 + 6x^4 - 36x^2) = \\ &= (x^2 + 1)^2 (9x^4 - 39x^2 - 6) \end{aligned}$$

6) Calcular las derivadas de:

a) $y = \frac{\sin x}{1 + \cos x} \Rightarrow y' = \frac{\cos x(1 + \cos x) - \sin x(-\sin x)}{(1 + \cos x)^2} =$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2} = \frac{\cos x + 1}{(1 + \cos x)^2} = \frac{(1 + \cos x)}{(1 + \cos x)^2} = \frac{1}{1 + \cos x}$$

b) $y = \arctg(e^{-2x}) \Rightarrow y' = \frac{-2e^{-2x}}{1 + (e^{-2x})^2} = -\frac{2e^{-2x}}{1 + e^{-4x}}$

c) $y = \sin^3 3x \Rightarrow y' = 3(\sin^2 3x)(3 \cos 3x) = 9 \sin^2 3x \cos 3x$

d) $y = \ln \frac{(x-2)^3}{\sqrt{2x-1}} = \ln(x-2)^3 - \ln \sqrt{2x-1} = 3 \ln(x-2) - \frac{1}{2} \ln(2x-1) \Rightarrow$

$$y' = 3 \frac{1}{x-2} - \frac{1}{2} \frac{2}{2x-1} = \frac{3}{x-2} - \frac{1}{2x-1} = \frac{3}{x-2} - \frac{1}{2x-1}$$

e) $y = x^3 e^{-3x} \Rightarrow y' = 3x^2 e^{-3x} + x^3 (-3) e^{-3x} = 3x^2 e^{-3x} - 3x^3 e^{-3x} =$

$$= 3x^2 e^{-3x} (1 - x)$$

7) Derivar y simplificar: $y = \arctg 3x^2$; $y = \frac{x^2 e^{1-x}}{3}$; $y = \ln \sqrt[3]{\frac{(x-2)^2}{x-3}}$; $y = 2 \cos^2 4x$

• $y = \arctg 3x^2 \Rightarrow y' = \frac{6x}{1 + (3x^2)^2} = \frac{6x}{1 + 9x^4}$

• $y = \frac{x^2 e^{1-x}}{3} = \frac{1}{3} x^2 e^{1-x} \Rightarrow y' = \frac{1}{3} (2x e^{1-x} + x^2 (-1) e^{1-x}) = \frac{x e^{1-x} (2 - x)}{3}$

• $y = \ln \sqrt[3]{\frac{(x-2)^2}{x-3}} = \frac{1}{3} \ln \frac{(x-2)^2}{x-3} = \frac{1}{3} [\ln(x-2)^2 - \ln(x-3)] =$

$$= \frac{1}{3} [2 \ln(x-2) - \ln(x-3)] \Rightarrow y' = \frac{1}{3} \left(2 \frac{1}{x-2} - \frac{1}{x-3} \right) = \frac{2}{3(x-2)} - \frac{1}{3(x-3)}$$

• $y = 2 \cos^2 4x \Rightarrow y' = 2 \cdot 2 (\cos 4x) (-\sin 4x) 4 = -16 \sin 4x \cos 4x =$

$$= -8 \cdot 2 \operatorname{sen} 4x \cos 4x = -8 \operatorname{sen} (2 \cdot 4x) = \boxed{-8 \operatorname{sen} 8x}$$

8) Derivar y simplificar:

(2 puntos)

$$a) y = e^{2x} \operatorname{tg} x \Rightarrow y' = 2e^{2x} \operatorname{tg} x + e^{2x} (1 + \operatorname{tg}^2 x) = \boxed{e^{2x} (\operatorname{tg}^2 x + 2 \operatorname{tg} x + 1)}$$

$$b) y = \ln \sqrt[3]{\frac{x^2}{x^2-4}} = \frac{1}{3} (2 \ln x - \ln(x^2-4)) \Rightarrow y' = \boxed{\frac{1}{3} \left(\frac{2}{x} - \frac{2x}{x^2-4} \right)}$$

$$c) y = 2 \cos^3 3x \Rightarrow y' = 2 \cdot 3 \cos^2 3x (-\operatorname{sen} 3x) \cdot 3 = \boxed{-18 \operatorname{sen} 3x \cos^2 3x}$$

$$d) y = \operatorname{arcsen} x^3 \Rightarrow y' = \frac{3x^2}{\sqrt{1-(x^3)^2}} = \boxed{\frac{3x^2}{\sqrt{1-x^6}}}$$

9) Derivar las siguientes funciones, simplificando los resultados:

$$a) y = 2e^{\cos 3x} \Rightarrow y' = 2e^{\cos 3x} (-\operatorname{sen} 3x) \cdot 3 = \boxed{-6 e^{\cos 3x} \operatorname{sen} 3x}$$

$$b) y = \operatorname{arctg} \sqrt{2x} \Rightarrow$$

$$y' = \frac{\frac{2}{2\sqrt{2x}}}{1 + (\sqrt{2x})^2} = \frac{\frac{1}{\sqrt{2x}}}{1 + 2x} = \frac{\frac{1}{\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}}}{1 + 2x} = \frac{\frac{\sqrt{2x}}{2x}}{1 + 2x} = \frac{\sqrt{2x}}{2x(1 + 2x)} = \boxed{\frac{\sqrt{2x}}{4x^2 + 2x}}$$

$$c) y = \ln \sqrt[3]{\frac{(2x-3)^2}{x-3}} = \frac{1}{3} \ln \frac{(2x-3)^2}{x-3} = \frac{1}{3} [\ln(2x-3)^2 - \ln(x-3)] =$$

$$= \frac{1}{3} [2 \ln(2x-3) - \ln(x-3)]. \text{ Derivando:}$$

$$y' = \frac{1}{3} \left[2 \frac{2}{2x-3} - \frac{1}{x-3} \right] = \boxed{\frac{1}{3} \left[\frac{4}{2x-3} - \frac{1}{x-3} \right]}$$

$$d) y = 3x \operatorname{tg} 4x \Rightarrow y' = 3 \operatorname{tg} 4x + 3x \frac{4}{\cos^2 4x} = \boxed{3 \operatorname{tg} 4x + \frac{12x}{\cos^2 4x}}$$

$$e) y = 2 \operatorname{sen}^2 3x \Rightarrow$$

$$y' = 4 (\operatorname{sen} 3x \cos 3x) \cdot 3 = 6 \cdot 2 \operatorname{sen} 3x \cos 3x = 6 \operatorname{sen} 2 \cdot 3x = \boxed{6 \operatorname{sen} 6x}$$

10) Derivar las siguientes funciones, simplificando los resultados:

$$a) y = 2xe^{\operatorname{sen} 3x}$$

$$y' = 2e^{\operatorname{sen} 3x} + 2xe^{\operatorname{sen} 3x} (\cos 3x) \cdot 3 = \boxed{2e^{\operatorname{sen} 3x} (1 + 3x \cos 3x)}$$

$$b) y = \ln \sqrt[3]{\frac{(4x-3)^2}{x-1}}$$

Simplificando la expresión antes de derivar, aplicando propiedades de logaritmos:

$$y = \frac{1}{3} [\ln(4x-3)^2 - \ln(x-1)] = \frac{1}{3} [2 \ln(4x-3) - \ln(x-1)] \Rightarrow$$

$$\Rightarrow y' = \frac{1}{3} \left(2 \frac{4}{4x-3} - \frac{1}{x-1} \right) = \boxed{\frac{1}{3} \left(\frac{8}{4x-3} - \frac{1}{x-1} \right)}$$

$$c) y = 3x^{\cos 2x}$$

Tomamos $\ln$ antes de derivar: $\ln y = \ln 3x^{\cos 2x} = \ln 3 + \ln x^{\cos 2x} = \ln 3 + \cos 2x \ln x$

Derivando miembro a miembro:

$$\frac{y'}{y} = 0 + -2\operatorname{sen} 2x \ln x + (\cos 2x) \frac{1}{x} = -2\operatorname{sen} 2x \ln x + \frac{\cos 2x}{x} = \frac{-2x\operatorname{sen} 2x \ln x + \cos 2x}{x}$$

$$\Rightarrow y' = 3x^{\cos 2x} \frac{-2x\operatorname{sen} 2x \ln x + \cos 2x}{x} = \boxed{3x^{\cos 2x-1} (\cos 2x - 2x\operatorname{sen} 2x \ln x)}$$

d) $y = 2 \operatorname{sen}^2 3x$

$$y' = 2 \cdot 2 \operatorname{sen} 3x (\cos 3x) 3 = 6 \cdot 2 \operatorname{sen} 3x \cos 3x = 6 \operatorname{sen} 2 \cdot 3x = \boxed{6 \operatorname{sen} 6x}$$

11) Derivar las siguientes funciones, simplificando los resultados:

a) $y = 3xe^{\cos x^2}$

$$y' = 3e^{\cos x^2} + 3xe^{\cos x^2} 2x(-\operatorname{sen} x^2) = \boxed{3e^{\cos x^2} (1 - 2x^2 \operatorname{sen} x^2)}$$

b) $y = \ln \sqrt[5]{\frac{4x^2 - 3}{(x-1)^2}}$

Simplificando la expresión antes de derivar, aplicando propiedades de logaritmos:

$$y = \frac{1}{5} [\ln(4x^2 - 3) - \ln(x-1)^2] = \frac{1}{5} [\ln(4x^2 - 3) - 2\ln(x-1)] \Rightarrow$$

$$\Rightarrow y' = \frac{1}{5} \left(\frac{8x}{4x^2 - 3} - 2 \frac{1}{x-1} \right) = \boxed{\frac{1}{5} \left(\frac{8x}{4x^2 - 3} - \frac{2}{x-1} \right)}$$

c) $y = \operatorname{arctg} \sqrt{3x}$

$$y' = \frac{\frac{3}{2\sqrt{x}}}{1 + (\sqrt{3x})^2} = \frac{\frac{3}{2\sqrt{x}}}{1 + 3x} = \boxed{\frac{3}{2\sqrt{3x}(1+3x)}}$$

d) $y = 3 \cos^2 5x$

$$y' = 3 \cdot 2 \cos 5x (-\operatorname{sen} 5x) = -15 \cdot 2 \operatorname{sen} 5x \cos 5x = -15 \operatorname{sen} 2 \cdot 5x = \boxed{-15 \operatorname{sen} 10x}$$

12) Calcule las derivadas de las siguientes funciones:

$$f(x) = \frac{2^x + x^2}{x}; \quad g(x) = (x^2 + 1)^2 \cdot \ln(e^{3x} + 4)$$

$$f'(x) = \frac{(2^x \ln 2 + 2x)x - (2^x + x^2) \cdot 1}{x^2} = \frac{2^x x \ln 2 + 2x^2 - 2^x - x^2}{x^2} = \boxed{\frac{2^x x \ln 2 + x^2 - 2^x}{x^2}}$$

$$g'(x) = 2(x^2 + 1)2x \ln(e^{3x} + 4) + (x^2 + 1)^2 \frac{3e^{3x}}{e^{3x} + 4} =$$

$$= \boxed{(4x^3 + 4x) \ln(e^{3x} + 4) + \frac{3e^{3x}(x^2 + 1)^2}{e^{3x} + 4}}$$

13) Derivar y simplificar:

a) $y = 2(7x^3 - 3x)^6$

$$y' = 2 \cdot 6(7x^3 - 3x)^5 (21x^2 - 3) = 12(21x^2 - 3) (7x^3 - 3x)^5 =$$

$$= \boxed{(252x^2 - 36) (7x^3 - 3x)^5}$$

$$\text{b) } y = \frac{3x^2 - 12}{x - 1}$$

$$y' = \frac{6x(x-1) - (3x^2 - 12)1}{(x-1)^2} = \frac{6x^2 - 6x - 3x^2 + 12}{(x-1)^2} = \boxed{\frac{3x^2 - 6x + 12}{(x-1)^2}}$$

$$\text{c) } y = \sqrt{2x^2 + 1}$$

$$y' = \frac{4x}{2\sqrt{2x^2 + 1}} = \boxed{\frac{2x}{\sqrt{2x^2 + 1}}}$$

$$\text{d) } y = (x + 1)e^{2x+1}$$

$$y' = 1 \cdot e^{2x+1} + (x+1)2e^{2x+1} = e^{2x+1}[1 + 2(x+1)] = e^{2x+1}(1 + 2x + 2) = \boxed{e^{2x+1}(2x+3)}$$

14) Derivar y simplificar:

$$\text{a) } y = 2(7x^2 - 3x)^5$$

$$y' = 2 \cdot 5(7x^2 - 3x)^4(14x - 3) = \boxed{10(14x - 3)(7x^2 - 3x)^4}$$

$$\text{b) } y = \frac{x-1}{3x^4 - 2}$$

$$y' = \frac{3x^4 - 2 - 12x^3(x-1)}{(3x^4 - 2)^2} = \frac{3x^4 - 2 - 12x^4 + 12x^3}{(3x^4 - 2)^2} = \boxed{\frac{-9x^4 + 12x^3 - 2}{(3x^4 - 2)^2}}$$

$$\text{c) } y = \sin \sqrt{2x}$$

$$y' = \frac{2}{2\sqrt{2x}} \cos \sqrt{2x} = \boxed{\frac{\cos \sqrt{2x}}{\sqrt{2x}}}$$

$$\text{d) } y = e^{2x+1} \ln 3x$$

$$y' = 2e^{2x+1} \ln 3x + e^{2x+1} \frac{3}{3x} = e^{2x+1} \left(2 \ln 3x + \frac{1}{x} \right) = \boxed{e^{2x+1} \frac{1 + 2x \ln 3x}{x}}$$