

1. Expresa, utilizando logaritmos, las siguientes igualdades:

1. $4^3 = 64$

2. $7^{-2} = \frac{1}{49}$

3. $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

4. $5^0 = 1$

5. $2^{x+1} = 12$

2. Expresa, usando potencias, las siguientes igualdades:

1. $\log_3 81 = 4$

2. $\log_5 0'04 = -2$

3. $x = \log_2 5$

4. $x = \log_3 2 + 1$

5. $y = \log_2 x + \log_2 3$

3. Halla, sin usar calculadora, los siguientes logaritmos:

1. $\log_2 32$

2. $\log_3 \frac{1}{27}$

3. $\log_5 0'04$

4. $\log_{\frac{2}{3}} 2'25$

5. $\log \sqrt{0'1}$

4. Calcula el valor de x para que la igualdad sea cierta:

1. $\log_x 64 = 6$

2. $\log_x 9 = -2$

3. $\log_x 4 = 4$

4. $\log_2 x = 4$

5. $\log_x 4 = 0'5$

5. Calcula el logaritmo de M y desarrolla por logaritmos:

1. $M = 2x^2y$

2. $M = \frac{2x^2}{3y}$

3. $M = \sqrt[3]{\frac{3x}{y^2}}$

4. $M = \frac{2\sqrt{x+1}}{(y+1)^2}$

5. $M = 2\left(\frac{2x^2y}{3z}\right)^3$

6. Halla el valor de M, siendo:

1. $\log M = 2 \cdot \log x - 3 \cdot \log y$

2. $\log M = \log x - \frac{\log y}{2} - 1$

3. $\log M = \log x - \frac{\log y}{2} - \frac{2 \cdot \log z}{3}$

7. Comprueba, sin calculadora, si son ciertas las siguientes igualdades:

1. $\log_a b \cdot \log_b a = 1$

2. $\log_2 3 \cdot \log_3 4 = 2$

3. $\log_4 x - \log_8 x = \log_2 \sqrt[6]{x}$

8. Reduce al máximo las siguientes expresiones:

1. $2^{3 \cdot \log_4 x}$

2. $\log_2 \sqrt{x} + \log_4 x$

3. $\log_{\sqrt{x}} 4 - \log_x 4 - \log_{x^2} 4$

9. Halla, razonadamente, los valores de x que cumplen la igualdad:

1. $4^{\log_2 x} = 3$

2. $2^{\log_4 x} = 3$

3. $4^{\log_2 x} = 2^{\log_2 8x-2}$

4. $4^{\log_2 x} = 2^{1-\log_4 x}$

10. Sabiendo que $\log 2 = 0'301$ y $\log 3 = 0'477$, halla, sin usar calculadora:

1. $\log 72$

2. $\log 56'25$

3. $\log \frac{0'125}{\sqrt{6}}$

4. $\log \frac{20\sqrt{6}}{\sqrt[3]{30}}$

11. Sabiendo que $\log x = 2'3$ y $\log y = 3'2$, calcula:

1. $\log x^3 \sqrt{y}$

2. $\log_{xy} y - \log_y x$

3. $\log_x 10 + \log_y 100$

4. $\log_{x^2y} \sqrt{10}$

12. Calcula, aplicando logaritmos:

1. $\frac{0'5\sqrt{3}}{1'2}$

2. $\sqrt{\frac{1'22}{0'3^3}}$

3. $\frac{2100}{20\sqrt[3]{2'21}}$

4. $\left(2'1\sqrt[3]{2'2\sqrt{2'3}}\right)^3$

13. Despeja x en la siguiente igualdad, usando logaritmos decimales si es necesario:

1. $M = (y+1)^{x+1}$

2. $M = \frac{2y^{2x-1}}{3} + 1$

3. $M = \sqrt{(y^2-1)^{x-1}}$

4. $\log x - \log y = \log (x-y)$

14. Halla el valor de x (aproximando a las centésimas), para los valores que se indican:

1. $y = 2^{x+1}$; $y = 0'5$

2. $c = 2(a+2)^{2x}$; $c = 5, a = 0'3$

3. $y = \frac{2 \cdot \sqrt[k]{k}}{k-1}$; $y = -2'3, k = 0'5$

15. Resuelve las siguientes ecuaciones exponenciales:

1. $2^{2x-3} = 8$

2. $3^{\frac{4-x}{3}} = 9$

3. $3^{x-1} - 9^{1-x} = 0$

4. $\frac{4^{2x+1}}{8^{2-x}} = 1$

5. $9^{x^2+x} = 3^{x+1}$

6. $5^{x^2-x-2} - 1 = 0$

7. $\sqrt{2^{2x-3}} = 4^{x-1}$

8. $\sqrt{3^{x+2}} = \sqrt[3]{9^{2x-1}}$

9. $\sqrt{2^x} \cdot \sqrt{2^{x+2}} \cdot \sqrt{2^{x-2}} = 4$

10. $\sqrt[3]{3^{x+1}} \cdot \sqrt[3]{3^x} \cdot \sqrt[3]{3^{x+2}} = 3$

11. $\sqrt[3]{2} = \sqrt{x+1}$

12. $\sqrt[4]{4^{x+2}} = \sqrt[2]{8^{x+2}}$

13. $\sqrt{2} \cdot \sqrt{x+1} = \sqrt[3]{8^{x-2}}$

14. $\sqrt[3]{2} \cdot \sqrt{x+3} = \sqrt[4]{2^{3-x}}$

15. $2^{x+1} = 5$

16. $2^{2x-1} = 3^{1-x}$

17. $2^{x^2-2} = 3^{x-1}$

18. $\sqrt{2^{x+2}} = \sqrt[3]{3^x}$

19. $2^{x+2} - 2^{x+1} - 2^x = 2^3$

20. $3^x - 3^{x-1} - 3^{x-2} = 5$

21. $\frac{11}{3} - \frac{1}{3^{1-x}} + \frac{1}{3^{2-x}} = 3^x$

22. $\frac{1}{2^{1-x}} + \frac{1}{2^{2-x}} + \frac{5}{8} = 2^{x+1}$

23. $\frac{1}{5^{1-x}} - 4 \cdot 5^{x-2} = 5$

24. $5 \cdot 2^{x-1} - \frac{3}{2^{1-x}} - \frac{3}{2^{2-x}} = 2$

25. $2^{2x-1} + 2^{2x-3} - 2^{x+1} = 2$

26. $3 + 3^{2x+1} - 3^{x+2} = 3^x$

27. $5^{1+x} + 5^{1-x} = 26$

28. $2^{2-x} - 2^{1-x} = 4 + 2^{4+x}$

29. $\frac{1}{2^{x-2}} - \frac{1}{2^{x-3}} = 2^x - 5$

30. $\frac{1}{5^{x-1}} = 5^x - 4$

31. $2^{x+4} - 2^{x+3} - 2^{x+2} = 4^{x+2}$

32. $3^{2x+3} - 3^{x+2} = 1 - 9^{x+1}$

33. $2 \cdot 3^{2x-1} + 3^{x+1} - 9^x = 1 - 3^{x-2}$

34. $\frac{3}{2^{1-x}} - 4^{x-1} = 1 - 5 \cdot 2^{x-3}$

35. $3^{2x+1} - 3^{x+1} - 2 \cdot 9^x = 1 - \frac{1}{3^{x-1}}$

36. $8^{x-1} - \frac{5}{4^{1-x}} + \frac{21}{2^{3-x}} = 1 + 4^{x-2}$

37. $5^{2x+1} - 4 \cdot 25^x + 25^{1-x} = 26$

38. $2 \cdot 4^{x+1} + \frac{1}{2^{3-2x}} - \frac{1}{2^{1-4x}} = 2$

39. $x^2 2^x - x 2^{x+1} - 2^{x+3} = 0$

40. $\frac{x^2}{2^{1-x}} - 3x 2^x + 2^{x+2} = 0$

41. $2^{x+1} - 2^x - 2^{x-1} = 3$

42. $3^x - \frac{2}{3^{1-x}} - \frac{1}{3^{2-x}} = 4$

43. $2 \cdot 3^{x-2} + 3^x - 9^{x-1} = 2$

44. $3 \cdot 2^x + \frac{1}{3 \cdot 2^{1-x}} - 2^{2x-1} = 1$

16. Resuelve los siguientes sistemas exponenciales:

1. $\begin{cases} 2^{2x-y} = 8 \\ 3^{2x+y} = 3 \end{cases}$

2. $\begin{cases} 4^{2x-y} = 8 \\ 9^{x-2y} = 1 \end{cases}$

3. $\begin{cases} 8^{x+y+1} - 4^{y-x} = 0 \\ 9^{x+1} - 3^{x+y-1} = 0 \end{cases}$

4. $\begin{cases} 2^{x+y-1} = 4^{2-x} \\ 3^{x-y} = 9^{2-x} \end{cases}$

5. $\begin{cases} 3^{x+y+1} = 9^{x-y-3} \\ 4^{x+y} = \sqrt{2^{2x+3y}} \end{cases}$

6. $\begin{cases} 3^{x-y} = \sqrt[3]{3^{x+1}} \\ 2^{x-2} = \sqrt[3]{4^{1-y}} \end{cases}$

7. $\begin{cases} \sqrt[2]{2^{x-y}} = \sqrt[3]{9^{x+2}} \\ \sqrt[4]{4^{x+4}} = \sqrt[4]{8^{y+1}} \end{cases}$

8. $\begin{cases} \sqrt[4]{4^{x+1}} = \sqrt[2]{2^{2y-1}} \\ \sqrt[3]{3^{2x-1}} = \sqrt[4]{9^{y-2}} \end{cases}$

9. $\begin{cases} 4^{x-1} = \sqrt{2^{y+2}} \\ \sqrt[3]{3^y} = 9^{x-2} \end{cases}$

10. $\begin{cases} \sqrt[2]{2^{x-y}} = \sqrt[3]{9^{x+2}} \\ \sqrt[2]{2^{x+1}} = \sqrt[4]{2^y} \end{cases}$

11. $\begin{cases} 3^y - 2^x = 1 \\ 2^{x+2} - 3^{y+1} = 5 \end{cases}$

12. $\begin{cases} 2^{x+3} - 3^{y-1} = 1 \\ 2^{x+2} + 3^{y-2} = 3 \end{cases}$

13. $\begin{cases} 2^{x+3} + 2^{x+2} - 3^{y-1} = 0 \\ 3^y - 3^{y-2} - 2^{x+4} = 4 \end{cases}$

14. $\begin{cases} 2^{y+1} + 3 \cdot 2^y - 3^{x+3} = 1 \\ 3^{x+2} - 5 \cdot 3^{x+1} + 2^{y+1} = 2 \end{cases}$

15. $\begin{cases} \sqrt{2^x} - \sqrt{3^y} = 1 \\ \sqrt{2^{x-2}} - \sqrt{3^{y-2}} = 1 \end{cases}$

16. $\begin{cases} \sqrt{2^{x+1}} - \sqrt{3^{y-1}} = 1 \\ \sqrt{3^{y+1}} - \sqrt{2^{x+3}} = 1 \end{cases}$

17. $\begin{cases} 3^{y+1} - 2^{x+1} = 1 \\ 9^{y-1} - 4^{x-2} = 0 \end{cases}$

18. $\begin{cases} 3^{y-1} - 2^{x+2} = 1 \\ 4^{x+1} - 5 \cdot 3^{y-2} = 1 \end{cases}$

19. $\begin{cases} 2^y + 3^{x+1} - 9^x = 4 \\ 5 \cdot 3^x - 2^y - 9^x = 2 \end{cases}$

20. $\begin{cases} 2^{x+2} - 9^{y+1} = 7 \\ 2 \cdot 3^{y+2} - 4^x = 2 \end{cases}$

21. $\begin{cases} 2^x = 3^y \\ x + y = 2 \end{cases}$

22. $\begin{cases} 2^{x+1} = 5^{y-1} \\ x + y = 1 \end{cases}$

23. $\begin{cases} 2^x - 3^y = 1 \\ 2^{x+3} - 3^{y+2} = 6 \end{cases}$

24. $\begin{cases} 3^y + 3 \cdot 2^x = 3 \\ 2 \cdot 9^y - 9 \cdot 2^{x+1} = 7 \end{cases}$

17. Resuelve las siguientes ecuaciones logarítmicas:

1. $2 \cdot \log x - \log 4 = \log 9$

2. $\log 2x + \log 2 = 2 \cdot \log x$

3. $2 \cdot \log x - \log 8 = \log \frac{x}{2}$

4. $5 \cdot \log x = 3 \cdot \log x + 2 \cdot \log 3$

5. $3 \cdot \log \frac{x}{2} + 2 \cdot \log \frac{x}{3} = 3 \cdot \log x - \log 8$

6. $3 \cdot \log x - 2 \cdot \log \frac{x}{3} = 2 \cdot \log 3 + 2 \cdot \log 2x$

7. $\log \frac{x}{5} + 1 = 2 \cdot \log x$

8. $2 \cdot \log 4x + \log \frac{x}{2} = 3$

9. $\log_2 \sqrt{x} - \log_2 \sqrt{2} = \frac{1}{2}$

10. $\log \sqrt[3]{x} - \log 2 = \log \sqrt[3]{2} - \log \sqrt{2x}$ 11. $\log (2x+12) - \log (3x-2) = \log 2$ 12. $\log x - \log 2 = 2 \cdot \log (x-3)$
 13. $\log(x-1) + \log(x-2) - \log 3 = 2 \cdot \log(x-3)$ 14. $2 \cdot \log(x-1) - \log(x+2) = \log(x+1) - \log 2$ 15. $\log(5x+4) - \log 2 = \log \sqrt{x+4}$
 16. $\log(2x+14) - \log 2 = \log \sqrt{x+5} + \log 3$ 17. $\log \sqrt{3x-2} - \log \sqrt{2x-3} = 1 - \log 5$ 18. $\log \sqrt{3x+4} + \log \sqrt{5x+1} = 1 + \log 3$
 19. $\frac{\log(x^2-3)}{\log(x+3)} = 2$ 20. $\frac{\log(4-x^2)}{\log(2+x)} = 2$ 21. $\frac{\log 2 + \log(x+2)}{\log(x+1)} = 2$
 22. $\frac{1 + \log_2(x+6)}{\log_2(x+2)} = 2$ 23. $\frac{1 + 2 \cdot \log x}{\log x} - \log x = 2$ 24. $\log_2 \sqrt{x} - \log_2 \sqrt{2} = \frac{1}{\log_2 x}$
 25. $\log_2 x - \log_4 x = 1$ 26. $\log_2 x - \log_x 8 = 2$ 27. $\log_{2x} 16 - \log_x 2 = 0$
 28. $\log_x 4 + \log_{2x} 4 = 3$ 29. $\frac{3 \cdot \log x}{x} + \log \sqrt{x} = \log x^x$ 30. $\log_2 x^6 - 8 \cdot \log_{2x} x = \log_2 x^x$

18. Resuelve los siguientes sistemas logarítmicos:

1. $\begin{cases} x + y = 8 \\ \log x - \log y = \log 3 \end{cases}$ 2. $\begin{cases} x + y = 6 \\ \log 2x - \log y = 1 \end{cases}$ 3. $\begin{cases} 4^{x-1} - 2^{y+1} = 0 \\ \log 5x - \log \frac{y}{2} = 1 \end{cases}$ 4. $\begin{cases} 3^{1-x} - 9^{y-2} = 0 \\ \log_2 2x - \log_2 3y = 1 \end{cases}$
 5. $\begin{cases} x + y = 7 \\ \log x + \log y = 1 \end{cases}$ 6. $\begin{cases} x + 2y = 5 \\ \log 5x + \log y = 1 \end{cases}$ 7. $\begin{cases} 9^{x-6} - 3^{3-2y} = 0 \\ \log 2x + \log 4y = 2 \end{cases}$ 8. $\begin{cases} 8^{x-2} - 2^{1-3y} = 0 \\ \log_2 2x + \log_2 3y = 2 \end{cases}$
 9. $\begin{cases} 2(x^2+y) = 5(3x-4) \\ \log_2 y = 2(\log_2 x - 1) \end{cases}$ 10. $\begin{cases} 6x - x^2 - y = 3 \\ \log x + \log y = 1 \end{cases}$ 11. $\begin{cases} y - \log_2 x = 1 \\ \log_2 x^3 - y = 1 \end{cases}$ 12. $\begin{cases} \log_2 x + y = 3 \\ \log_2 x^3 + y^2 = 7 \end{cases}$
 13. $\begin{cases} \log 2x + \log 5x = 2 \\ \log 4x - \log y = 1 \end{cases}$ 14. $\begin{cases} \log_2 x + \log_2 y = 3 \\ \log_2 8x - \log_2 2y = 1 \end{cases}$ 15. $\begin{cases} \log x^2 + \log y^3 = 2 \\ \log x^3 - \log y^2 = 3 \end{cases}$ 16. $\begin{cases} \log_2 x + \log_2 y = 3 \\ \log_2 x^2 - \log_2 y^3 = 1 \end{cases}$
 17. $\begin{cases} \log_2 x + \log_4 y = 2 \\ \log_4 x - \log_2 y = 1 \end{cases}$ 18. $\begin{cases} \log_3 x^3 - \log_2 y^2 = 1 \\ \log_9 x + \log_4 y = 1 \end{cases}$ 19. $\begin{cases} \log_x 4 + \log_y 2 = 2 \\ \log_2 x - \log_2 y = 1 \end{cases}$ 20. $\begin{cases} \log_4 x + \log_9 y = 1 \\ \log_x 4 - \log_y 3 = 1 \end{cases}$

— Soluciones —

- 1.1. $\log_4 64 = 3$ 1.2. $\log_{7/49} \frac{1}{2} = -2$ 1.3. $\log_{\frac{1}{2}} \frac{1}{16} = 4$ 1.4. $\log_5 1 = 0$ 1.5. $\log_2 12 = x+1$ 2.1. $3^4 = 81$ 2.2. $5^{-2} = 0.04$ 2.3. $2^x = 5$ 2.4. $3^{x-1} = 2$ 2.5. $2^y = 3x$ 3.1. 5
 3.2. -3 3.3. -2 3.4. -2 3.5. $-\frac{1}{2}$ 4.1. 2 4.2. $\frac{1}{3}$ 4.3. $\sqrt{2}$ 4.4. 16 4.5. 2 5.1. $\log M = \log 2 + 2 \log x + \log y$ 5.2. $\log M = \log 2 + 2 \log x - \log 3 - \log y$ 5.3. $\log M = \frac{\log 3 + \log x - 2 \log y}{3}$
 5.4. $\log M = \log 2 + \frac{\log(x+1)}{2} - 2 \log(y+1)$ 5.5. $\log M = 4 \log 2 + 6 \log x + 3 \log y - 3 \log z$ 6.1. $\frac{x^2}{y^3}$ 6.2. $\frac{x}{10\sqrt{y}}$ 6.3. $\frac{x\sqrt[3]{z^2}}{\sqrt{y}}$ 8.1. $x\sqrt{x}$ 8.2. $\log_2 x$ 8.3. $\log_x 2$
 9.1. $\sqrt{3}$ 9.2. 9 9.3. 2 9.4. $\sqrt[5]{4}$ 10.1. 1'857 10.2. 1'75 10.3. -1'292 10.4. 1'198 11.1. 8'5 11.2. 0'673 11.3. -0'19 11.4. 0'064 12.1. 0'722 12.2. 6'722
 12.3. 80'611 12.4. 9'847 13.1. $\frac{\log M}{\log(y+1)} - 1$ 13.2. $\frac{\log^3(M+1)}{2 \log y}$ 13.3. $\frac{2 \log M}{\log(y^2-1)} + 1$ 13.4. $\frac{y^2}{y-1}$ 14.1. -2 14.2. 0'55 14.3. 3'25 15.1. 3 15.2. -2
 15.3. 1 15.4. 4 15.5. $-1, \frac{1}{2}$ 15.6. -1, 2 15.7. $\frac{1}{2}$ 15.8. 2 15.9. 2 15.10. 1 15.11. 1 15.12. -2, 5 15.13. 3 15.14. 1 15.15. 1'32 15.16. 0'72 15.17. -0'23, 1'81
 15.18. -0'70, 2'87 15.19. 3 15.20. 2 15.21. 1 15.22. -1 15.23. 3 15.24. 3 15.25. 2 15.26. -1, 1 15.27. -1, 1 15.28. -2 15.29. 0, 2 15.30. 1 15.31. -2 15.32. -1 15.33. -1, 2 15.34. -1, 3 15.35. 0, 1 15.36. -1, 1, 3 15.37. 0, 1 15.38. -1, 2 15.39. -2, 4 15.40. 2, 4 15.41. 2'58 15.42. 2'63
 15.43. 0'63 15.44. -1'58, 2'58 16.1. (1,-1) 16.2. $(1, \frac{1}{2})$ 16.3. (-1,2) 16.4. $(\frac{3}{2}, \frac{1}{2})$ 16.5. (1,-2) 16.6. (2,1) 16.7. (2,2) 16.8. (2,-1) 16.9. (1,-2), (3,6)
 16.10. (1,2), $(-2, \frac{1}{2})$ 16.11. (3,2) 16.12. (-1,2) 16.13. (-2,2) 16.14. (-1,1) 16.15. (4,2) 16.16. (3,3) 16.17. (2,1) 16.18. (1,3) 16.19. (0,1), (1,2)
 16.20. (1,-1), (2,0) 16.21. $(\log_6 9, \log_6 4)$ 16.22. $(-\log_2 9, \log_2 20)$ 16.23. $(\log_2 3, \log_3 2)$ 16.24. $(-\log_2 3, \log_3 2)$ 17.1. 6 17.2. 4 17.3. 4 17.4. 3 17.5. 3 17.6. $\frac{1}{4}$
 17.7. 2 17.8. 5 17.9. 4 17.10. 2 17.11. 4 17.12. $\frac{9}{2}$ 17.13. 5 17.14. 7 17.15. 0 17.16. -1, -4 17.17. 2 17.18. 7 17.19. $-\frac{5}{3}$ 17.20. 0
 17.21. $\sqrt{3}$ 17.22. 2 17.23. $\frac{1}{10}, 10$ 17.24. $\frac{1}{2}, 4$ 17.25. 4 17.26. $\frac{1}{2}, 8$ 17.27. $\sqrt[3]{2}$ 17.28. 2, $\frac{\sqrt[3]{2}}{2}$ 17.29. 1, 2 17.30. 1, 2, 4 18.1. (6,2) 18.2. (5,1) 18.3. (3,3)
 18.4. (3,1) 18.5. (5,2), (2,5) 18.6. (1,2), $(4, \frac{1}{2})$ 18.7. $(\frac{5}{2}, 5)$, $(5, \frac{5}{2})$ 18.8. $(\frac{1}{3}, 2)$, $(2, \frac{1}{3})$ 18.9. (2,1), (4,4) 18.10. (2,5), (5,2) 18.11. (2,2) 18.12. (2,2), (4,1)
 18.13. (5,2) 18.14. (2,4) 18.15. (10,1) 18.16. (4,2) 18.17. (4,1) 18.18. (3,2) 18.19. $(\sqrt{2}, \frac{\sqrt{2}}{2})$ 18.20. (2,3), $(16, \frac{1}{9})$