

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \rightarrow \infty} \frac{5x^3 + 2x^2 - 3x + 1}{2x^5 + 3x^2 - 2}$
2. $\lim_{x \rightarrow \infty} \left(\frac{5x^2 - x + 1}{x^2 + x - 2} \right)^{x^2 - 1}$
3. $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 2}{x^2 - 1} \right)^{2x - 1}$
4. $\lim_{x \rightarrow \infty} \frac{\sqrt{5x^2 + 3}}{-x^2 + 4}$
5. $\lim_{x \rightarrow 1} \frac{6x^5 + 2x^4 - 7x^3 + x^2 - x - 1}{4x^5 + 7x^4 - 12x^3 + 2x - 1}$
6. $\lim_{x \rightarrow 2} \frac{2x^3 + x^2 - 12x + 4}{x^3 + 5x^2 - 15x + 2}$
7. $\lim_{x \rightarrow 3} \frac{\sqrt{4x^2 + 1} - \sqrt{10x + 7}}{x - 3}$
8. $\lim_{x \rightarrow 5} \frac{\sqrt{2x^2 - 3} - \sqrt{9x + 2}}{x - 5}$

Solución:

1. $\lim_{x \rightarrow \infty} \frac{5x^3 + 2x^2 - 3x + 1}{2x^5 + 3x^2 - 2} = 0$
2. $\lim_{x \rightarrow \infty} \left(\frac{5x^2 - x + 1}{x^2 + x - 2} \right)^{x^2 - 1} = \infty$
3. $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 2}{x^2 - 1} \right)^{2x - 1} = e^2$
4. $\lim_{x \rightarrow \infty} \frac{\sqrt{5x^2 + 3}}{-x^2 + 4} = 0$
5. $\lim_{x \rightarrow 1} \frac{6x^5 + 2x^4 - 7x^3 + x^2 - x - 1}{4x^5 + 7x^4 - 12x^3 + 2x - 1} = \frac{9}{7}$
6. $\lim_{x \rightarrow 2} \frac{2x^3 + x^2 - 12x + 4}{x^3 + 5x^2 - 15x + 2} = \frac{16}{17}$

$$7. \lim_{x \rightarrow 3} \frac{\sqrt{4x^2 + 1} - \sqrt{10x + 7}}{x - 3} = \frac{7\sqrt{37}}{37}$$

$$8. \lim_{x \rightarrow 5} \frac{\sqrt{2x^2 - 3} - \sqrt{9x + 2}}{x - 5} = \frac{11\sqrt{47}}{94}$$

Problema 2 Calcular las siguientes derivadas:

$$1. y = e^{2x^3 - 4x^2 + x - 1}$$

$$2. y = \ln(3x^4 + 1)$$

$$3. y = (5x^2 + x - 3)^{15}$$

$$4. y = (2x^2 - 3x + 1)(x^3 + 5x^2 - 2)$$

$$5. y = \frac{3x^2 - 2x + 1}{5x - 8}$$

$$6. y = \ln \frac{x^2 - 5}{x + 3}$$

Solución:

$$1. y = e^{2x^3 - 4x^2 + x - 1} \implies y' = (6x^2 - 8x + 1)e^{2x^3 - 4x^2 + x - 1}$$

$$2. y = \ln(3x^4 + 1) \implies y' = \frac{12x^3}{3x^4 + 1}$$

$$3. y = (5x^2 + x - 3)^{15} \implies y' = 15(5x^2 + x - 3)^{14}(10x + 1)$$

$$4. y = (2x^2 - 3x + 1)(x^3 + 5x^2 - 2) \implies y' = (4x - 3)(x^3 + 5x^2 - 2) + (2x^2 - 3x + 1)(3x^2 + 10x)$$

$$5. y = \frac{3x^2 - 2x + 1}{5x - 8} \implies y' = \frac{(6x - 2)(5x - 8) - (3x^2 - 2x + 1)5}{(5x - 8)^2}$$

$$6. y = \ln \frac{x^2 - 5}{x + 3} = \ln(x^2 - 5) - \ln(x + 3) \implies y' = \frac{2x}{x^2 - 5} - \frac{1}{x + 3}$$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

$$1. f(x) = \frac{5x^2 - 1}{x^2 + 2} \text{ en el punto } x = 1.$$

$$2. f(x) = \frac{x^2 + 3}{2x - 1} \text{ en el punto } x = 0.$$

Solución:

$$1. \ b = f(a) \implies b = f(1) = \frac{4}{3} \text{ e } y - b = m(x - a)$$

$$f'(x) = -\frac{22x}{(x^2 + 2)^2} \implies m = f'(1) = \frac{22}{9}$$

$$\text{Recta Tangente: } y - \frac{4}{3} = \frac{22}{9}(x - 1)$$

$$\text{Recta Normal: } y - \frac{4}{3} = -\frac{9}{22}(x - 1)$$

$$2. \ b = f(a) \implies b = f(0) = -3 \text{ e } y - b = m(x - a)$$

$$f'(x) = \frac{2(x^2 - x - 3)}{(2x - 1)^2} \implies m = f'(0) = -6$$

$$\text{Recta Tangente: } y + 3 = -6x$$

$$\text{Recta Normal: } y + 3 = \frac{1}{6}x$$