

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \rightarrow \infty} \frac{9x^3 - 5x^2 + x + 1}{3x^3 - 4x^2 + 5}$
2. $\lim_{x \rightarrow \infty} \left(\frac{7x^2 - 2x + 1}{3x^2 + 2x - 2} \right)^{x^2 - 9}$
3. $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 2x + 1}{x^2 - 5} \right)^{3x}$
4. $\lim_{x \rightarrow \infty} \frac{\sqrt{5x^3 + 3x + 1}}{x^2 + 5}$
5. $\lim_{x \rightarrow 1} \frac{6x^5 + 2x^4 - 9x^3 + x^2 - 2x + 2}{3x^5 + 5x^4 - 10x^3 + x + 1}$
6. $\lim_{x \rightarrow 2} \frac{3x^3 - 5x^2 - x - 2}{x^3 - x^2 - x - 2}$
7. $\lim_{x \rightarrow 3} \frac{\sqrt{5x^2 + 2} - \sqrt{15x + 2}}{x - 3}$
8. $\lim_{x \rightarrow 5} \frac{\sqrt{x^2 + 2} - \sqrt{4x + 7}}{x - 5}$

Solución:

1. $\lim_{x \rightarrow \infty} \frac{9x^3 - 5x^2 + x + 1}{3x^3 - 4x^2 + 5} = 3$
2. $\lim_{x \rightarrow \infty} \left(\frac{7x^2 - 2x + 1}{3x^2 + 2x - 2} \right)^{x^2 - 9} = \infty$
3. $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 2x + 1}{x^2 - 5} \right)^{3x} = e^{-6}$
4. $\lim_{x \rightarrow \infty} \frac{\sqrt{5x^3 + 3x + 1}}{x^2 + 5} = 0$
5. $\lim_{x \rightarrow 1} \frac{6x^5 + 2x^4 - 9x^3 + x^2 - 2x + 2}{3x^5 + 5x^4 - 10x^3 + x + 1} = \frac{11}{6}$
6. $\lim_{x \rightarrow 2} \frac{3x^3 - 5x^2 - x - 2}{x^3 - x^2 - x - 2} = \frac{15}{7}$

$$7. \lim_{x \rightarrow 3} \frac{\sqrt{5x^2 + 2} - \sqrt{15x + 2}}{x - 3} = \frac{15\sqrt{47}}{94}$$

$$8. \lim_{x \rightarrow 5} \frac{\sqrt{x^2 + 2} - \sqrt{4x + 7}}{x - 5} = \frac{\sqrt{3}}{3}$$

Problema 2 Calcular las siguientes derivadas:

$$1. y = e^{4x^3 - 3x^2 - 2x - 1}$$

$$2. y = \ln(5x^3 + 4)$$

$$3. y = (x^2 + 7x - 1)^{16}$$

$$4. y = (x^2 + x - 3)(2x^3 - x^2 + 5)$$

$$5. y = \frac{x^2 + 9}{5x + 3}$$

$$6. y = \ln \frac{x^2 - 1}{x^2 + 8}$$

Solución:

$$1. y = e^{4x^3 - 3x^2 - 2x - 1} \implies y' = (12x^2 - 6x - 2)e^{4x^3 - 3x^2 - 2x - 1}$$

$$2. y = \ln(5x^3 + 4) \implies y' = \frac{15x^2}{5x^3 + 4}$$

$$3. y = (x^2 + 7x - 1)^{16} \implies y' = 16(x^2 + 7x - 1)^{15}(2x + 7)$$

$$4. y = (x^2 + x - 3)(2x^3 - x^2 + 5) \implies y' = (2x + 1)(2x^3 - x^2 + 5) + (x^2 + x - 3)(6x^2 - 2x)$$

$$5. y = \frac{x^2 + 9}{5x + 3} \implies y' = \frac{(2x)(5x + 3) - (x^2 + 9)5}{(5x + 3)^2}$$

$$6. y = \ln \frac{x^2 - 1}{x^2 + 8} = \ln(x^2 - 1) - \ln(x^2 + 8) \implies y' = \frac{2x}{x^2 - 1} - \frac{2x}{x^2 + 8}$$

Problema 3 Calcular

1. las rectas tangente y normal a la siguiente función: $f(x) = \frac{x^2 - 3}{x^2 + 1}$ en el punto $x = 1$.

2. la recta tangente a la función $f(x) = 7x^2 - 3x - 1$ que sea paralela a la recta $y = 11x + 7$.

Solución:

$$1. \ b = f(a) \implies b = f(1) = -1 \text{ e } y - b = m(x - a)$$

$$f'(x) = -\frac{8x}{(x^2 + 1)^2} \implies m = f'(1) = 2$$

$$\text{Recta Tangente: } y + 1 = 2(x - 1)$$

$$\text{Recta Normal: } y + 1 = -\frac{1}{2}(x - 1)$$

$$2. \ m = 1, \ f'(x) = 14x - 3 \implies m = f'(a) = 14a - 3 = 11 = 1 \implies a = 1$$

$$b = f(a) \implies b = f(1) = 3 \text{ e } y - b = m(x - a) \text{ Recta Tangente:}$$

$$y - 3 = 11(x - 1)$$