

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \rightarrow \infty} \frac{3x^3 + 5x^2 - 5x - 1}{4x^3 - 7x^2 + 3}$
2. $\lim_{x \rightarrow \infty} \left(\frac{3x^2 - 6x + 2}{7x^2 - 3x - 2} \right)^{x^2 - 8}$
3. $\lim_{x \rightarrow \infty} \left(\frac{x^2 - x + 2}{x^2 + 5} \right)^{3x}$
4. $\lim_{x \rightarrow \infty} \frac{\sqrt{7x^2 - 5} + 3}{x^2 + 1}$
5. $\lim_{x \rightarrow 1} \frac{8x^5 + 2x^4 - 9x^3 + 2x^2 - 2x - 1}{3x^5 - 4x^4 + 10x^3 - 10x + 1}$
6. $\lim_{x \rightarrow 2} \frac{4x^3 - 2x^2 - 5x - 14}{2x^3 - 3x^2 - 5x + 6}$
7. $\lim_{x \rightarrow 3} \frac{\sqrt{7x^2 - 2} - \sqrt{20x + 1}}{x - 3}$
8. $\lim_{x \rightarrow 5} \frac{\sqrt{x^2 + 4} - \sqrt{4x + 9}}{x - 5}$

Solución:

1. $\lim_{x \rightarrow \infty} \frac{3x^3 + 5x^2 - 5x - 1}{4x^3 - 7x^2 + 3} = \frac{3}{4}$
2. $\lim_{x \rightarrow \infty} \left(\frac{3x^2 - 6x + 2}{7x^2 - 3x - 2} \right)^{x^2 - 8} = 0$
3. $\lim_{x \rightarrow \infty} \left(\frac{x^2 - x + 2}{x^2 + 5} \right)^{3x} = e^{-3}$
4. $\lim_{x \rightarrow \infty} \frac{\sqrt{7x^2 - 5} + 3}{x^2 + 1} = 0$
5. $\lim_{x \rightarrow 1} \frac{8x^5 + 2x^4 - 9x^3 + 2x^2 - 2x - 1}{3x^5 - 4x^4 + 10x^3 - 10x + 1} = \frac{23}{19}$
6. $\lim_{x \rightarrow 2} \frac{4x^3 - 2x^2 - 5x - 14}{2x^3 - 3x^2 - 5x + 6} = 5$

$$7. \lim_{x \rightarrow 3} \frac{\sqrt{7x^2 - 2} - \sqrt{20x + 1}}{x - 3} = \frac{11\sqrt{61}}{61}$$

$$8. \lim_{x \rightarrow 5} \frac{\sqrt{x^2 + 4} - \sqrt{4x + 9}}{x - 5} = \frac{3\sqrt{29}}{29}$$

Problema 2 Calcular las siguientes derivadas:

$$1. y = e^{3x^3 - x^2 + 5x - 1}$$

$$2. y = \ln(5x^2 + 1)$$

$$3. y = (3x^2 - 2x + 5)^{15}$$

$$4. y = (x^2 + 5x - 1)(2x^3 + 3x^2 - 1)$$

$$5. y = \frac{x^2 - x - 3}{7x + 2}$$

$$6. y = \ln \frac{x^2 + 5}{x^2 - 2}$$

Solución:

$$1. y = e^{3x^3 - x^2 + 5x - 1} \implies y' = (9x^2 - 2x + 5)e^{3x^3 - x^2 + 5x - 1}$$

$$2. y = \ln(5x^2 + 1) \implies y' = \frac{10x}{5x^2 + 1}$$

$$3. y = (3x^2 - 2x + 5)^{15} \implies y' = 15(3x^2 - 2x + 5)^{14}(6x - 2)$$

$$4. y = (x^2 + 5x - 1)(2x^3 + 3x^2 - 1) \implies y' = (2x + 5)(2x^3 + 3x^2 - 1) + (x^2 + 5x - 1)(6x^2 + 6x)$$

$$5. y = \frac{x^2 - x - 3}{7x + 2} \implies y' = \frac{(2x - 1)(7x + 2) - (x^2 - x - 3)7}{(7x + 2)^2}$$

$$6. y = \ln \frac{x^2 + 5}{x^2 - 2} = \ln(x^2 + 5) - \ln(x^2 - 2) \implies y' = \frac{2x}{x^2 + 5} - \frac{2x}{x^2 - 2}$$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

$$1. f(x) = \frac{5x^2 + 7}{x^2 - 3} \text{ en el punto } x = 1.$$

$$2. f(x) = \frac{x^2 + 3}{5x - 1} \text{ en el punto } x = 0.$$

Solución:

$$1. \ b = f(a) \implies b = f(1) = -6 \text{ e } y - b = m(x - a)$$

$$f'(x) = -\frac{44x}{(x^2 - 3)^2} \implies m = f'(1) = -11$$

$$\text{Recta Tangente: } y + 6 = -11(x - 1)$$

$$\text{Recta Normal: } y + 6 = \frac{1}{11}(x - 1)$$

$$2. \ b = f(a) \implies b = f(0) = -3 \text{ e } y - b = m(x - a)$$

$$f'(x) = \frac{5x^2 - 2x - 15}{(5x - 1)^2} \implies m = f'(0) = -15$$

$$\text{Recta Tangente: } y + 3 = -15x$$

$$\text{Recta Normal: } y + 3 = \frac{1}{15}x$$