Problema 1 Calcular los siguientes límites:

1. $\displaystyle \lim_{x\longrightarrow\infty} \frac{-3x^2-x+7}{5x^3+3}$

2. $\displaystyle \lim_{x\longrightarrow\infty} \left(\frac{5x^2-1}{3x^2+8}\right)^{x^2-9}$

3. $\displaystyle \lim_{x\longrightarrow\infty} \left(\frac{x^2+3x+1}{x^2-1}\right)^{3x}$

4. $\displaystyle \lim_{x\longrightarrow\infty} \frac{\sqrt{7x^2-6}}{-x+5}$

5. $\displaystyle \lim_{x\longrightarrow 1} \frac{8x^4+x^3-8x^2-3x+2}{5x^3+x^2-8x+2}$

6. $\displaystyle \lim_{x\longrightarrow 6} \frac{\sqrt{x^2-3}-\sqrt{5x+3}}{x-6}$

Solución:

1. $\displaystyle \lim_{x\longrightarrow\infty} \frac{-3x^2-x+7}{5x^3+3} = 0$

2. $\displaystyle \lim_{x\longrightarrow\infty} \left(\frac{5x^2-1}{3x^2+8}\right)^{x^2-9} = \infty$

3. $\displaystyle \lim_{x\longrightarrow\infty} \left(\frac{x^2+3x+1}{x^2-1}\right)^{3x} = e^9$

4. $\displaystyle \lim_{x\longrightarrow\infty} \frac{\sqrt{7x^2-6}}{-x+5} = -\sqrt{7}$

5. $\displaystyle \lim_{x\longrightarrow 1} \frac{8x^4+x^3-8x^2-3x+2}{5x^3+x^2-8x+2} = \frac{16}{9}$

6. $\displaystyle \lim_{x\longrightarrow 6} \frac{\sqrt{x^2-3}-\sqrt{5x+3}}{x-6} = \frac{7\sqrt{33}}{66}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{6x^3+7x^2-2x+3}$

2. $y = \ln(7x^5 + 2)$
3. $y = (2x^2 - 7)^{14}$
4. $y = (3x^2 + 5x - 3)(2x^3 - 3x^2 + 1)$
5. $y = \frac{5x^2 - x + 2}{7x - 1}$
6. $y = x^9 \ln x$

Solución:

1. $y = e^{6x^3+7x^2-2x+3} \Longrightarrow y' = (18x^2 + 14x - 2)e^{6x^3+7x^2-2x+3}$
2. $y = \ln(7x^5 + 2) \Longrightarrow y' = \dfrac{35x^4}{7x^5 + 2}$
3. $y = (2x^2 - 7)^{14} \Longrightarrow y' = 14(2x^2 - 7)^{13}(4x)$
4. $y = (3x^2 + 5x - 3)(2x^3 - 3x^2 + 1) \Longrightarrow y' = (6x + 5)(2x^3 - 3x^2 + 1) + (3x^2 + 5x - 3)(6x^2 - 6x)$
5. $y = \dfrac{5x^2 - x + 2}{7x - 1} \Longrightarrow y' = \dfrac{(10x - 1)(7x - 1) - (5x^2 - x + 2)7}{(2x - 3)^2}$
6. $y = x^9 \ln x \Longrightarrow y' = 9x^8 \ln x + x^9 \dfrac{1}{x}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{7x^2 - 1}{x^2 - 2}$ en el punto $x = 1$.
2. $f(x) = \dfrac{3x^2 - 2}{2x + 3}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = -6$ e $y - b = m(x - a)$

   $$f'(x) = -\frac{26x}{(x^2 - 2)^2} \Longrightarrow m = f'(1) = -26$$

   Recta Tangente: $y + 6 = -26(x - 1)$

   Recta Normal: $y + 6 = \frac{1}{26}(x - 1)$

2. $b = f(a) \Longrightarrow b = f(0) = -\dfrac{2}{3}$ e $y - b = m(x - a)$

$$f'(x) = \frac{2(3x^2 + 9x + 2)}{(2x+3)^2} \Longrightarrow m = f'(0) = \frac{4}{9}$$

Recta Tangente: $y + \dfrac{2}{3} = \dfrac{4}{9}x$

Recta Normal: $y + \dfrac{2}{3} = -\dfrac{9}{4}x$

Problema 4 Calcular las siguientes integrales:

1. $\displaystyle\int (5x^2 - 2x + 5)\, dx$
2. $\displaystyle\int \left(\frac{5x^2 - 6x + 3}{x} - 9e^x \right) dx$
3. $\displaystyle\int \left(\frac{3x^2 - 2\sqrt[5]{x^2} + 5}{x} \right) dx$

Solución:

1. $\displaystyle\int (5x^2 - 2x + 5)\, dx = \frac{5x^3}{3} - x^2 + 5x + C$
2. $\displaystyle\int \left(\frac{5x^2 - 6x + 3}{x} - 9e^x \right) dx = \frac{5x^2}{2} - 6x + 3\ln|x| - 9e^x + C$
3. $\displaystyle\int \left(\frac{3x^2 - 2\sqrt[5]{x^2} + 5}{x} \right) dx = \frac{3x^2}{2} - 5\sqrt[5]{x^2} + 5\ln|x| + C$