52623

1. $3^x = 6561 = 3^8 \rightarrow \boxed{x=8}$

$2^{3x+2} = 0'25 = 2^{-2} \rightarrow 3x+2=-2 \quad \boxed{x=-4/3}$

$\left(\frac{1}{3}\right)^x = 81 \;;\quad 3^{-x} = 3^4 \rightarrow \boxed{x=-4}$

$3^x = \frac{1}{\sqrt{27}} = 3^{-3/2} \rightarrow \boxed{x=-3/2}$

2. $3^x = 6000 \;;\quad x = \frac{\ln 6000}{\ln 3} = \boxed{7'919}$

$2^x = 0'3 \;;\quad x = \frac{\ln 0'3}{\ln 2} = \boxed{-1'737}$

$2^{3x+2} = 1000 \;;\quad 3x+\cancel{2} = \frac{\ln 1000}{\ln 2} = 9'966 \rightarrow x = \frac{7'966}{3} = \boxed{2'655}$

$\left(\frac{1}{3}\right)^x = 100 \;;\quad x = \frac{\ln 100}{\ln 1/3} = \boxed{-4'192}$

$x = \log_5 38 = \frac{\ln 38}{\ln 5} = \boxed{2'260}$

3. $\log_2 32 = \boxed{5} \qquad \log_{10} 0'1 = \boxed{-1} \qquad \log_3 \frac{27}{\sqrt{3}} = \log_3 \frac{3^3}{3^{1/2}} = \log_3 3^{5/2} = \boxed{\frac{5}{2}} \qquad \ln e^2 = \boxed{2}$

4. $10^{\log_{10} x} = \boxed{x} \qquad e^{\ln x} = \boxed{x}$

$4^{\log_2 x} = (2^2)^{\log_2 x} = 2^{2\log_2 x} = 2^{\log_2(x^2)} = \boxed{x^2}$

$2^{3\log_4 x} = (\sqrt{4})^{3\log_4 x} = 4^{\frac{3}{2}\log_4 x} = 4^{\log_4(x^{3/2})} = \boxed{x^{3/2}} = \boxed{\sqrt{x^3}}$

$3^{\log_9 x} = (9^{1/2})^{\log_9 x} = 9^{\frac{1}{2}\log_9 x} = 9^{\log_9(x^{1/2})} = \boxed{x^{1/2}} = \boxed{\sqrt{x}}$

Otra forma:

$3^{\log_9 x} = 3^{\frac{\log_3 x}{\log_3 9}} = 3^{\frac{\log_3 x}{2}} = 3^{\frac{1}{2}\log_3 x} = 3^{\log_3(x^{1/2})} = \boxed{x^{1/2}} = \boxed{\sqrt{x}}$

5. $\log_x 9 = 2 \rightarrow x^2 = 9 \rightarrow x = \boxed{3}$, $\cancel{-3}$ No puede ser negativa la base de un logarit.

$\log_x\left(\frac{1}{3}\right) = -2 \rightarrow x^{-2} = \frac{1}{3} \;;\quad \frac{1}{x^2} = \frac{1}{3} \;;\quad x^2 = 3 \rightarrow x = \boxed{\sqrt{3}}$, $\cancel{-\sqrt{3}}$ No existen logarit. con bases negativas.

$\log_2 x = 3 \rightarrow 2^3 = x \rightarrow \boxed{x=8}$

$\log_{10} x = -1 \rightarrow 10^{-1} = x \rightarrow \boxed{x = \frac{1}{10} = 0'1}$

$\log_8 2 = x \rightarrow 8^x = 2 \rightarrow (2^3)^x = 2 \;;\quad 2^{3x} = 2 \rightarrow 3x = 1 \;;\quad \boxed{x = \frac{1}{3}}$

6) a) $5^{2x} - 30\cdot 5^{x} + 125 = 0$

$t = 5^x$ $\quad t^2 - 30t + 125 = 0$; $t = \begin{cases} 25 \to 5^x = 25 & \boxed{x=2} \\ 5 \to 5^x = 5 & \boxed{x=1} \end{cases}$

b) $9^x - 2\cdot 3^{x+2} + 81 = 0$

$3^{2x} - 2\cdot 3^x \cdot 3^2 + 81 = 0$

$t = 3^x$ $\quad t^2 - 18t + 81 = 0$; $t = \begin{cases} 9 \\ 9 \end{cases} \to 3^x = 9$ $\boxed{x=2}$

c) $2^{x-1} + 2^x + 2^{x+1} = 7$

$\frac{2^x}{2} + 2^x + 2\cdot 2^x = 7$

$t = 2^x$ $\quad \frac{t}{2} + t + 2t = 7$; $t + 2t + 4t = 14$; $7t = 14$; $t = 2 \to 2^x = 2$ $\boxed{x=1}$

7) a) $\log_{10} x = 3 - \log_{10} 50$; $\log_{10} x + \log_{10} 50 = 3$; $\log_{10}(50x) = 3$; $50x = 10^3$ $\boxed{x=20}$

b) $\log(3-5x) - \log 4 = \frac{1}{2}\log(3-x)$

$\log \frac{3-5x}{4} = \frac{1}{2}\log(3-x)$

$2\log \frac{3-5x}{4} = \log(3-x)$

$\log\left(\frac{3-5x}{4}\right)^2 = 3-x \longrightarrow \left(\frac{3-5x}{4}\right)^2 = 3-x$; $\frac{9-30x+25x^2}{16} = 3-x$;

$9 - 30x + 25x^2 = 48 - 16x$; $25x^2 - 14x - 39 = 0$ $\quad x = \begin{cases} \cancel{1'56} & \text{No es solución.} \\ \boxed{-1} \end{cases}$

c) $\log x^3 = \log 6 + 2\log x$

$\log x^3 = \log 6 + \log x^2$

$\log x^3 = \log(6x^2) \longrightarrow x^3 = 6x^2$; $x^2(x-6) = 0 \to \begin{cases} x = \cancel{0} & \text{No está definido log 0} \\ \boxed{x=6} \end{cases}$

8) $\log_x 100 = 2 + \log_x 25$

$\log_x 100 - \log_{10} 25 = 2$

$\log_x\left(\frac{100}{25}\right) = 2$

$\log_x 4 = 2 \longrightarrow x^2 = 4 \longrightarrow x = \begin{cases} 2 \\ \cancel{-2} & \text{La base no puede ser negativa.} \end{cases}$

9) $B = 10 \log\left(\frac{I}{10^{-16}}\right)$

a) $I = 10^{-14}\ W/m^2 \longrightarrow B = 10 \log \frac{10^{-14}}{10^{-16}} = 10 \log 10^2 = \boxed{20 \text{ decibelios}}$

b) $B = 110$ decibelios $\longrightarrow 110 = 10 \log\left(\frac{I}{10^{-16}}\right)$; $11 = \log\left(\frac{I}{10^{-16}}\right) \longrightarrow \frac{I}{10^{-16}} = 10^{11}$

$\boxed{I = 10^{-5}\ W/m^2}$

10) $M_n = M_0 \left(1 + \frac{3'5}{100}\right)^n$

$M_n = M_0 \cdot 1'035^n$

$M_0 = 24.000$
$M_n = 2 \cdot 24000 = 48000$ $\Big\} \longrightarrow 48000 = 24000 \cdot 1'035^n$; $2 = 1'035^n \longrightarrow n = \frac{\ln 2}{\ln 1'035} = \boxed{20,149 \text{ años}}$

$M_0 = 50.000$
$M_n = 2 \cdot 50.000 = 100.000 \longrightarrow 100000 = 50000 \cdot 1'035^n$; $2 = 1'035^n$

M_0
$M_n = 2M_0$ $\Big\} \longrightarrow 2\cancel{M_0} = \cancel{M_0} \cdot 1'035^n$; $2 = 1'035^n$

El tiempo de duplicación será siempre el mismo.

11) $P_n = P_0 \cdot 1'04^n$

$P_{10} = 200 \cdot 1'04^{10} = \boxed{296'05\ €}$

12) $C \xrightarrow{4\%} 1'04 \cdot C \xrightarrow{5\%} 1'05 \cdot 1'04 \cdot C \xrightarrow{2,5\%} 1'025 \cdot 1'05 \cdot 1'04\, C$

$1'025 \cdot 1'05 \cdot 1'04\, C = 1'1193C$, que significa una subida del $\boxed{11,93\%}$