M03 P1

$$3^x \cdot 4^{2x+1} = 6^{x+2}$$

$\ln 3^x \cdot 4^{2x+1} = \ln 6^{x+2}$; $x\ln 3 + (2x+1)\ln 4 = (x+2)\ln 6$; $x\ln 3 + 2x\ln 4 + \ln 4 = x\ln 6 + 2\ln 6$;

$x(\ln 3 + 2\ln 4 - \ln 6) = 2\ln 6 - \ln 4$; $x = \dfrac{2\ln 6 - \ln 4}{\ln 3 + 2\ln 4 - \ln 6} = \dfrac{\ln(6^2/4)}{\ln\left(\frac{3\cdot 4^2}{6}\right)} = \boxed{\dfrac{\ln 9}{\ln 8}}$ $a=9$, $b=8$

N03 P1

$$2\cdot 5^{x+1} = 1 + \frac{3}{5^x}$$

$$2\cdot 5^x \cdot 5 = 1 + \frac{3}{5^x}$$

$t = 5^x \longrightarrow 10t = 1 + \frac{3}{t}$; $10t^2 = t + 3$; $10t^2 - t - 3 = 0$

$t = \dfrac{1 \pm \sqrt{1+120}}{20} = \dfrac{1 \pm 11}{20}$ → $\frac{12}{20} = \frac{3}{5}$; $-\frac{10}{20} = -\frac{1}{2}$

$t = \frac{3}{5} \Rightarrow 5^x = \frac{3}{5}$; $x = \log_5 \frac{3}{5} = \log_5 3 - \log_5 5 = \boxed{\log_5 3 - 1}$ $a=-1$, $b=3$

$t = -\frac{1}{2} \Rightarrow 5^x = -\frac{1}{2}$; Absurdo

N03 P1

$$\log_{16} \sqrt[3]{100 - x^2} = \frac{1}{2}$$

$\sqrt[3]{100-x^2} = 16^{1/2}$; $\sqrt[3]{100-x^2} = 4$; $100 - x^2 = 64$; $x^2 = 36$; $\boxed{x = \pm 6}$

M05 P1

$$2\log_3(x-3) + \log_{1/3}(x+1) = 2$$

$2\log_3(x-3) + \dfrac{\log_3(x+1)}{\log_3(1/3)} = 2$; $2\log_3(x-3) + \dfrac{\log_3(x+1)}{-1} = 2$; $\log_3(x-3)^2 - \log_3(x+1) = 2$;

$\log_3 \dfrac{(x-3)^2}{x+1} = 2$; $\dfrac{(x-3)^2}{x+1} = 3^2$; $(x-3)^2 = 9(x+1)$; $x^2 - 6x + 9 = 9x + 9$;

$x^2 - 15x = 0$; $x(x-15) = 0 \Rightarrow x =$ $\cancel{0}$ No estaría definido $\log_3(x-3)$; $\boxed{15}$

M06 P1

$$|\ln(x+3)| = 1$$

$\ln(x+3) = 1 \Rightarrow x + 3 = e^1$; $\boxed{x = e - 3}$

$\ln(x+3) = -1 \Rightarrow x + 3 = e^{-1}$; $\boxed{x = e^{-1} - 3}$

N06 P1

$$9\log_5 x = 25 \log_x 5$$

$9\log_5 x = 25 \dfrac{\log_5 5}{\log_5 x}$; $(\log_5 x)^2 = \dfrac{25\cdot \log_5 5}{9}$; $(\log_5 x)^2 = \dfrac{25}{9}$; $\log_5 x = \pm\sqrt{\dfrac{25}{9}}$;

$\log_5 x = 5/3 \Rightarrow \boxed{x = 5^{5/3}}$

$\log_5 x = -5/3 \Rightarrow \boxed{x = 5^{-5/3}}$

M07 P1

$$2^{2x+3} = 2^{x+1} + 3$$

$$2^{2x}\cdot 2^3 = 2^x \cdot 2 + 3$$

$t = 2^x \longrightarrow t^2 \cdot 8 = t\cdot 2 + 3$; $8t^2 - 2t - 3 = 0$; $t = \dfrac{2 \pm \sqrt{4+96}}{16} = \dfrac{2 \pm 10}{16}$ → $\frac{12}{16} = \frac{3}{4}$; $-\frac{8}{16} = -\frac{1}{2}$

$t = \frac{3}{4} \Rightarrow 2^x = \frac{3}{4}$; $x = \log_2\left(\frac{3}{4}\right) = \log_2 3 - \log_2 4 = \boxed{\log_2 3 - 2}$ $a=-2$, $b=3$

$t = -1/2 \Rightarrow 2^x = -\frac{1}{2}$; Absurdo

M07 P1

$$2(\ln x)^2 = 3\ln x - 1$$

$t = \ln x \longrightarrow 2t^2 = 3t - 1 \;;\; 2t^2 - 3t + 1 = 0 \;;\; t = \dfrac{3 \pm \sqrt{9-8}}{4} = \dfrac{3 \pm 1}{4} \begin{cases} 1 \\ 1/2 \end{cases}$

$t = 1 \Rightarrow \ln x = 1 \;;\; \boxed{x = e}$

$t = 1/2 \Rightarrow \ln x = \frac{1}{2} \;;\; \boxed{x = e^{1/2}}$

N07 P1

a) $2 \cdot 4^x + 4^{-x} = 3 \;;\; 2 \cdot 4^x + \dfrac{1}{4^x} = 3$

$t = 4^x \longrightarrow 2t + \dfrac{1}{t} = 3 \;;\; 2t^2 + 1 = 3t \;;\; 2t^2 - 3t + 1 = 0 \;;$

$t = \dfrac{3 \pm \sqrt{9-8}}{4} = \dfrac{3 \pm 1}{4} \begin{cases} 1 \\ 1/2 \end{cases}$

$t = 1 \Rightarrow 4^x = 1 \;;\; \boxed{x = 0}$

$t = 1/2 \Rightarrow 4^x = \frac{1}{2} \;;\; (2^2)^x = 2^{-1} \;;\; 2^{2x} = 2^{-1} \;;\; 2x = -1 \;;\; \boxed{x = -1/2}$

b) $a^x = e^{2x+1}$

$\ln a^x = \ln e^{2x+1} \;;\; x \cdot \ln a = (2x+1) \cdot \ln e \;;\; x \ln a = 2x + 1 \;;\; x(\ln a - 2) = 1$

$\boxed{x = \dfrac{1}{\ln a - 2}}$

No tiene solución si $\ln a - 2 = 0 \;;\; \ln a = 2 \;;\; \boxed{a = e^2}$

Muestra 08 P1

$$2^{2x+2} - 10 \cdot 2^x + 4 = 0$$

$$2^{2x} \cdot 2^2 - 10 \cdot 2^x + 4 = 0$$

$t = 2^x \longrightarrow t^2 \cdot 4 - 10 \cdot t + 4 = 0 \;;\; 4t^2 - 10t + 4 = 0 \;;\; 2t^2 - 5t + 2 = 0$

$t = \dfrac{5 \pm \sqrt{25-16}}{4} = \dfrac{5 \pm 3}{4} \begin{cases} 2 \\ 1/2 \end{cases}$

$t = 2 \Rightarrow 2^x = 2 \;;\; \boxed{x = 1}$

$t = \frac{1}{2} \Rightarrow 2^x = \frac{1}{2} \;;\; 2^x = 2^{-1} \;;\; \boxed{x = -1}$

Muestra 08 P1

$$\log_3(x+17) - 2 = \log_3 2x$$

$\log_3(x+17) - \log_3 2x = 2 \;;\; \log_3 \dfrac{x+17}{2x} = 2 \;;\; \dfrac{x+17}{2x} = 3^2 \;;\; x + 17 = 18x \;;\; -17x = -17$

$\boxed{x = 1}$

Muestra 08 P1

$$4\ln 2 - 3\ln 4 = -\ln K$$

$$\ln 2^4 - \ln 4^3 = \ln K^{-1}$$

$\ln \dfrac{2^4}{4^3} = \ln \dfrac{1}{K} \longrightarrow \dfrac{2^4}{4^3} = \dfrac{1}{K} \;;\; \boxed{K = 4}$

N08 P1

$\ln(x^2-1) - 2\ln(x+1) + \ln(x^2+x) = \ln(x^2-1) - \ln(x+1)^2 + \ln(x^2+x) = \ln \dfrac{(x^2-1)(x^2+x)}{(x+1)^2} =$

$= \ln \dfrac{\cancel{(x+1)}(x-1)\,x\,\cancel{(x+1)}}{\cancel{(x+1)^2}} = \boxed{\ln x(x-1)}$

M09 P1

$$g(x) = \log_5 |2\log_3 x|$$

$g(x) = 0 \Rightarrow \log_5 |2\log_3 x| = 0 \Rightarrow |2\log_3 x| = 1 \Rightarrow 2\log_3 x = \pm 1 \;;$

$\log_3 x = \pm 1/2 \longrightarrow x = 3^{\pm 1/2}$ $\quad x_1 = \sqrt{3}$, $x_2 = \dfrac{1}{\sqrt{3}}$ $\longrightarrow x_1 \cdot x_2 = \sqrt{3} \cdot \dfrac{1}{\sqrt{3}} = \boxed{1}$

M10 P2

a)
$$\ln 2^{4x-1} = \ln 8^{x+5} + \log_2 16^{1-2x}$$
$$\ln 2^{4x-1} = \ln (2^3)^{x+5} + \frac{\ln (2^4)^{1-2x}}{\ln 2}$$
$$(4x-1)\ln 2 = 3(x+5)\ln 2 + \frac{4(1-2x)\cancel{\ln 2}}{\cancel{\ln 2}}$$
$$4x\ln 2 - \ln 2 = 3x\ln 2 + 15\ln 2 + 4 - 8x$$
$$x(4\ln 2 - 3\ln 2 + 8) = 15\ln 2 + 4 + \ln 2 \;;\; x\cdot(\ln 2 + 8) = 16\ln 2 + 4 \;;\; \boxed{x = \frac{16\ln 2 + 4}{\ln 2 + 8}}$$

b)
$$\log_a \frac{16\ln 2 + 4}{\ln 2 + 8} = 2 \rightarrow a^2 = \frac{16\ln 2 + 4}{\ln 2 + 8}$$
$$a = \pm\sqrt{\frac{16\ln 2 + 4}{\ln 2 + 8}} = \begin{cases} \boxed{1'318} \\ \cancel{-1'318} \end{cases}$$

La base de un logaritmo no puede ser negativa

M10 P1

$$4^{x-1} = 2^x + 8$$
$$\frac{4^x}{4} = 2^x + 8$$
$$t = 2^x \longrightarrow \frac{t^2}{4} = t + 8 \;;\; t^2 = 4t + 32 \;;\; t^2 - 4t - 32 = 0 \;;\; t = \frac{4 \pm \sqrt{16+128}}{2} = \frac{4 \pm 12}{2} \begin{cases} 8 \\ -4 \end{cases}$$
$$t = 8 \Rightarrow 2^x = 8 \;;\; \boxed{x = 3}$$
$$t = -4 \Rightarrow 2^x = -4 \quad \underline{\text{Absurdo}}$$

N10 P2

$$\left.\begin{array}{l} \ln \frac{x}{y} = 1 \\ \ln x^3 + \ln y^2 = 5 \end{array}\right\} \rightarrow \left.\begin{array}{l} \ln x - \ln y = 1 \\ 3\ln x + 2\ln y = 5 \end{array}\right\} \quad \begin{array}{l} a = \ln x \\ b = \ln y \end{array}$$

$$\left.\begin{array}{l} a - b = 1 \\ 3a + 2b = 5 \end{array}\right\} \rightarrow \left.\begin{array}{l} 2a - 2b = 2 \\ 3a + 2b = 5 \end{array}\right\} \quad 5a = 7 \quad a = 7/5 \downarrow \ln x = 7/5 \quad \boxed{x = e^{7/5}}$$

$$\downarrow \left.\begin{array}{l} -3a + 3b = -3 \\ 3a + 2b = 5 \end{array}\right\} \quad 5b = 2 \quad b = 2/5 \downarrow \ln y = 2/5 \rightarrow \boxed{y = e^{2/5}}$$

M11 P2

$$\left.\begin{array}{l} \log_{x+1} y = 2 \\ \log_{y+1} x = \frac{1}{4} \end{array}\right\} \rightarrow \left.\begin{array}{l} (x+1)^2 = y \\ (y+1)^{1/4} = x \end{array}\right\} \;;\; \left.\begin{array}{l} x^2 + 2x + 1 = y \\ y + 1 = x^4 \end{array}\right\} \rightarrow y = x^4 - 1 \;;\; x^2 + 2x + 1 = x^4 - 1 \;;$$

$$0 = x^4 - x^2 - 2x - 2$$

$$\begin{array}{r|rrrrr} & 1 & 0 & -1 & -2 & -2 \\ -1 & & -1 & 1 & 0 & 2 \\ \hline & 1 & -1 & 0 & -2 & \boxed{0} \end{array} \rightarrow \boxed{x = -1} \Rightarrow \boxed{y = 0}$$

$$x^3 - x^2 - 2 = 0 \longrightarrow \boxed{x = 1'6956}$$ (resuelto con calculadora gráfica)

$$\Rightarrow \boxed{y = 7'2660}$$

M12 P1 TZ1 #8

$$2-\log_3(x+7)=\log_{1/3}2x$$

$$2-\log_3(x+7)=\frac{\log_3 2x}{\log_3 1/3}$$

$$2-\log_3(x+7)=\frac{\log_3 2x}{-1}$$

$$2=\log_3(x+7)-\log_3 2x$$

$$2=\log_3\frac{x+7}{2x} \Rightarrow \frac{x+7}{2x}=3^2 \ ; \ \frac{x+7}{2x}=9 \ ; \ x+7=18x \ ; \ \boxed{x=\frac{7}{17}}$$

N13 P1#9

a)

$$\log_2(x-2)=\log_4(x^2-6x+12)$$

$$\log_2(x-2)=\frac{\log_2(x^2-6x+12)}{\log_2 4}$$

$$\log_2(x-2)=\frac{\log_2(x^2-6x+12)}{2}$$

$$2\log_2(x-2)=\log_2(x^2-6x+12)$$

$$\cancel{\log_2}(x-2)^2=\cancel{\log_2}(x^2-6x+12)$$

$$\cancel{x^2}-4x+4=\cancel{x^2}-6x+12 \ ; \ 2x=8 \ ; \ \boxed{x=4}$$

b)

$$x^{\ln x}=e^{(\ln x)^3}$$

$$\ln x^{\ln x}=(\ln x)^3$$

$$\ln x\cdot\ln x=(\ln x)^3$$

$$(\ln x)^2=(\ln x)^3$$

$$0=(\ln x)^3-(\ln x)^2$$

$$0=(\ln x)^2\cdot[\ln x-1] \to \ln x=0 \Rightarrow \boxed{x=1}$$

$$\to \ln x=1 \Rightarrow \boxed{x=e}$$

M14 P1 TZ1 #3

$$a=\log_2 3\cdot\log_3 4\cdot\log_4 5\cdot\ldots\cdot\log_{31}32=$$

$$=\cancel{\log_2 3}\cdot\frac{\cancel{\log_2 4}}{\cancel{\log_2 3}}\cdot\frac{\cancel{\log_2 5}}{\cancel{\log_2 4}}\cdot\ldots\cdot\frac{\log_2 32}{\cancel{\log_2 31}}=\log_2 32=\boxed{5}$$

M14 P1 TZ2 #2

$$8^{x-1}=6^{3x}$$

$$\ln 8^{x-1}=\ln 6^{3x} \ ; \ (x-1)\ln 8=3x\ln 6 \ ; \ x\ln 8-3x\ln 6=\ln 8 \ ;$$

$$x(\ln 8-3\ln 6)=\ln 8 \ ; \ x=\frac{\ln 8}{\ln 8-3\ln 6}=\frac{\ln 8}{\ln\frac{8}{6^3}}=\frac{\ln 2^3}{\ln\frac{1}{3^3}}=\frac{\cancel{3}\ln 2}{-\cancel{3}\ln 3}=\boxed{-\frac{\ln 2}{\ln 3}}$$

M15
TZ2
P1 #9b

$\log_x y = 4\log_y x$

$\frac{\ln y}{\ln x} = 4\frac{\ln x}{\ln y}$; $(\ln y)^2 = 4(\ln x)^2$; $\ln y = \pm 2\ln x$;

$\cancel{\ln} y = \cancel{\ln} x^{\pm 2}$; $y = x^{\pm 2}$

$\boxed{\begin{array}{l} y = x^2 \\ y = \frac{1}{x^2} \end{array}}$