

INTEGRACIÓN POR PARTES

Las integrales que podemos resolver con este método son:

- Integrales de la forma: $\int x^n \sin x dx$ $\int x^n \cos x dx$ $n \in \mathbb{N}, n \geq 1$
- Integrales de la forma: $\int e^x \cos x dx$ $\int e^x \sin x dx$ $\int x^n e^x dx$
- Integrales de la forma: $\int x^n \ln x dx$ $n \in \mathbb{N}$
- Integrales de la forma: $\int \sin^n x dx$ $\int \cos^n x dx$ $n \in \mathbb{N}, n > 1$
- Integrales de la forma: $\int \arcsin x dx$ $\int \arccos x dx$ $\int \arctan x dx$ $\int \operatorname{arccot} x dx$

En el ejercicio nº8, antes de integrar por partes considera que $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

En el ejercicio nº9 haz primero el cambio de variable:

$$y = \arcsin x \Rightarrow \begin{cases} x = \sin y \\ dx = \cos y dy \end{cases}$$

- $\int \arcsin x dx = (\arcsin x)x - \int \frac{1}{\sqrt{1-x^2}} x dx = (\arcsin x)x + \sqrt{1-x^2} + C$
- $\int x^2 \arctan x dx = \frac{1}{3}(\arctan x)x^3 - \int \frac{1}{3(1+x^2)} x^3 dx = \frac{1}{3}(\arctan x)x^3 - \frac{1}{6}x^2 + \frac{1}{6} \ln(3+3x^2) + C$
- $\int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x + C$
- $\int \cos x \ln(\sin x) dx = \ln(\sin x) \sin x - \int \cos x dx = \ln(\sin x) \sin x - \sin x + C$
- $\int \arcsin 2x dx = (\arcsin 2x)x - \int \frac{2}{\sqrt{1-4x^2}} x dx = (\arcsin 2x)x + \frac{1}{2} \sqrt{1-4x^2} + C$
- $\int x \sqrt{1+x} dx = \frac{2}{3}x(\sqrt{1+x})^3 - \int \frac{2}{3}(\sqrt{1+x})^3 dx = \frac{2}{3}x(\sqrt{1+x})^3 - \frac{4}{15}(\sqrt{1+x})^5 + C$
- $\int x^2 e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \int \frac{2}{3}x e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \int \frac{2}{9}e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + C$
- $\int e^{2x} \cos^2 x dx = \int \frac{1}{2}e^{2x}(1 + \cos 2x) dx = \int \frac{1}{2}e^{2x} dx + \int \frac{1}{2}e^{2x} \cos 2x dx = \frac{1}{4}e^{2x} + \frac{1}{8}e^{2x} \cos 2x + \frac{1}{8}e^{2x} \sin 2x + C$
- $\int e^{\arcsin x} dx = \int e^y \cos y dy = \frac{1}{2}e^y \cos y + \frac{1}{2}e^y \sin y = \frac{1}{2}e^{\arcsin x} \sqrt{1-x^2} + \frac{1}{2}e^{\arcsin x} x + C$
- $\int x \sin x dx = -x \cos x - \int (-\cos x) dx = -x \cos x + \sin x + C$
- $\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C$
- $\int x^2 \ln x dx = \frac{1}{3}(\ln x)x^3 - \int \frac{1}{3}x^2 dx = \frac{1}{3}(\ln x)x^3 - \frac{1}{9}x^3 + C$
- $\int x^2 \sin x dx = -x^2 \cos x - \int (-2x \cos x) dx = -x^2 \cos x + 2x \sin x + \int (-2 \sin x) dx = -x^2 \cos x + 2x \sin x + 2 \cos x + C$
- $\int x^3 e^{2x} dx = \frac{1}{2}x^3 e^{2x} - \int \frac{3}{2}x^2 e^{2x} dx = \frac{1}{2}x^3 e^{2x} - \frac{3}{4}x^2 e^{2x} + \int \frac{3}{2}x e^{2x} dx = \frac{1}{2}x^3 e^{2x} - \frac{3}{4}x^2 e^{2x} + \frac{3}{4}x e^{2x} - \frac{3}{8}e^{2x} + C$
- $\int x \arctan x dx = \frac{1}{2}(\arctan x)x^2 - \int \frac{x^2}{2(1+x^2)} dx = \frac{1}{2}(\arctan x)x^2 - \frac{1}{2}x + \frac{1}{2} \arctan x + C$
- $\int \sin x \sin 3x dx = -\frac{1}{3} \sin x \cos 3x - \int \left(-\frac{1}{3} \cos x \cos 3x\right) dx = -\frac{1}{3} \sin x \cos 3x + \frac{1}{9} \cos x \sin 3x + \int \frac{1}{9} \sin x \sin 3x dx$
Obtenemos la siguiente igualdad:
 $\int \sin x \sin 3x dx = -\frac{1}{3} \sin x \cos 3x + \frac{1}{9} \cos x \sin 3x + \frac{1}{9} \int \sin x \sin 3x dx$
 $\frac{8}{9} \int \sin x \sin 3x dx = -\frac{1}{3} \sin x \cos 3x + \frac{1}{9} \cos x \sin 3x$

$$\int \sin x \sin 3x dx = -\frac{9}{24} \sin x \cos 3x + \frac{1}{8} \cos x \sin 3x = -\frac{3}{8} \sin x \cos 3x + \frac{1}{8} \cos x \sin 3x$$

$$17. \int x \arcsin x^2 dx = \frac{1}{2} (\arcsin x^2) x^2 - \int \frac{x^3}{\sqrt{(1-x^4)}} dx = \frac{1}{2} (\arcsin x^2) x^2 + \frac{1}{2} \sqrt{(1-x^4)} + C$$

$$18. \int e^{ax} \sin bx dx = -\frac{b}{a^2 + b^2} e^{ax} \cos bx + \frac{a}{a^2 + b^2} e^{ax} \sin bx + C$$

$$19. \int \sin 3x \cos 2x dx = -\frac{1}{10} \cos 5x - \frac{1}{2} \cos x + C$$

$$20. \int \sin^4 x dx = -\frac{1}{4} \sin^3 x \cos x - \frac{3}{8} \cos x \sin x + \frac{3}{8} x + C$$

$$21. \int \cos^5 x dx = \frac{1}{5} \cos^4 x \sin x + \frac{4}{15} \cos^2 x \sin x + \frac{8}{15} \sin x + C$$

$$22. \int \sin^3 x \cos^2 x dx = -\frac{1}{5} \cos^3 x \sin^2 x - \frac{2}{15} \cos^3 x + C$$