$$\lim_{x\to\left(\frac{\pi}{2}\right)^-} \cos x\cdot \ln(\operatorname{tg} x) = (0\cdot\infty) = \lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{\ln(\operatorname{tg}(x))}{\frac{1}{\cos x}} = \left(\frac{\infty}{\infty}\right) \overset{\text{L'Hôpital}}{=}$$

$$= \lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{\frac{1}{\operatorname{tg} x}\cdot\frac{1}{\cancel{\cos^2 x}}}{\frac{\operatorname{sen} x}{\cancel{\cos^2 x}}} = \lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{1}{\frac{\operatorname{sen} x}{\cos x}\cdot \operatorname{sen} x} = \lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{\cos x}{\operatorname{sen}^2 x} = \frac{0}{1} = 0$$

$$\lim_{x\to\frac{\pi}{4}} \left(\frac{1}{\cos 2x} - \frac{\operatorname{tg} x}{1-\left(\frac{4x}{\pi}\right)}\right) = \frac{1}{\cos 2\left(\frac{\pi}{4}\right)} - \frac{\operatorname{tg}\pi/4}{1-\frac{4\left(\frac{\pi}{4}\right)}{\pi}} = \frac{1}{0} - \frac{1}{0} = (\infty-\infty) =$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{1-\frac{4x}{\pi} - \cos 2x\cdot \operatorname{tg} x}{\cos 2x\left(1-\frac{4x}{\pi}\right)} = \frac{1-\frac{4}{\pi}\cdot\frac{\pi}{4} - \cos\left(2\cdot\frac{\pi}{4}\right)\cdot \operatorname{tg}\frac{\pi}{4}}{\cos 2\left(\frac{\pi}{4}\right)\left(1-\frac{4}{\pi}\cdot\frac{\pi}{4}\right)} = \left(\frac{0}{0}\right) \overset{\text{L'Hôpital}}{=}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{-\frac{4}{\pi} - (-\operatorname{sen} 2x)\cdot 2\cdot \operatorname{tg} x + \cos 2x\cdot\frac{1}{\cos^2 x}}{-\operatorname{sen} 2x\cdot 2\left(1-\frac{4x}{\pi}\right) + \cos 2x\cdot\left(-\frac{4}{\pi}\right)} = \lim_{x\to\frac{\pi}{4}} \frac{-\frac{4}{\pi} + 2\operatorname{sen} 2x\operatorname{tg} x + \frac{\cos 2x}{\cos^2 x}}{-2\operatorname{sen} 2x\left(1-\frac{4x}{\pi}\right) - \frac{4}{\pi}\cos 2x} =$$

$$= \frac{-\frac{4}{\pi} + 2\cdot1\cdot1 + \cancel{\frac{0}{(\sqrt{2}/2)^2}}}{-\cancel{2\cdot1\cdot0} - \cancel{\frac{4}{\pi}\cdot0}} = \frac{-\frac{4}{\pi}+2}{0}$$ (Indeterminación $k/0$) Planteamos límites laterales:

$$\lim_{x\to\left(\frac{\pi}{4}\right)^-} \left(\frac{1}{\cos 2x} - \frac{\operatorname{tg} x}{1-\left(\frac{4x}{\pi}\right)}\right) = -\infty \qquad \lim_{x\to\left(\frac{\pi}{4}\right)^+} \left(\frac{1}{\cos 2x} - \frac{\operatorname{tg} x}{1-\frac{4x}{\pi}}\right) = +\infty$$

luego no existe el límite.

$$\lim_{x\to1} \left(\frac{e}{e^x-e} - \frac{1}{x-1}\right) = \frac{e}{e^1-e} - \frac{1}{0} = (\infty-\infty) = \lim_{x\to1} \frac{e(x-1)-(e^x-e)}{(e^x-e)(x-1)} =$$

$$= \lim_{x\to1} \frac{ex-\cancel{e}-e^x+\cancel{e}}{(e^x-e)\cdot(x-1)} = \frac{e\cdot1-e^1}{(e^1-e)(1-1)} = \left(\frac{0}{0}\right) \overset{\text{L'Hôpital}}{=}$$

$$= \lim_{x\to1} \frac{e-e^x}{e^x(x-1)+(e^x-e)\cdot1} = \left(\frac{0}{0}\right) \overset{\text{L'Hôpital}}{=} \lim_{x\to1} \frac{-e^x}{e^x(x-1)+e^x+e^x} = \frac{-e}{2e} =$$

$$= \boxed{-\frac{1}{2}}$$