

1. Estudiar la continuidad y derivabilidad de $f(x) = \begin{cases} \cos x & \text{si } x < 0 \\ e^{x^2} & \text{si } 0 \leq x < 1 \\ e+1-\frac{1}{x} & \text{si } x \geq 1 \end{cases}$ (1,5 puntos)

3) CONTINUIDAD: Cada rama es continua en su intervalo de definición.

¿continua en $x=0$? $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \cos x = 1$
 $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} e^{x^2} = 1$ $\Rightarrow f(x)$ continua en $x=0$

¿continua en $x=1$? $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} e^{x^2} = e$
 $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (e+1-\frac{1}{x}) = e$ $\Rightarrow f(x)$ continua en $x=1$
 Soluc: $f(x)$ continua $\forall x \in \mathbb{R}$

1,5
(0,75+0,75)

(C) DERIVABILIDAD:

¿derivable en $x=0$? $f'(0^-) = \cos x \Big|_{x=0} = -\sin x \Big|_{x=0} = 0$
 $f'(0^+) = e^{x^2} \Big|_{x=0} = 2x \cdot e^{x^2} \Big|_{x=0} = 0$ $\Rightarrow f(x)$ derivable en $x=0$

¿derivable en $x=1$? $f'(1^-) = e^{x^2} \Big|_{x=1} = 2x \cdot e^{x^2} \Big|_{x=1} = 2e$
 $f'(1^+) = e+1-\frac{1}{x} \Big|_{x=1} = \frac{1}{x^2} \Big|_{x=1} = 1$ $\Rightarrow f(x)$ no es derivable en $x=1$
 Soluc: $f(x)$ derivable $\forall x \in \mathbb{R} - \{1\}$

2. Calcular $\lim_{x \rightarrow 0^+} (1+x)^{1/x} = 1^\infty = \text{indeterminado} = e^{\frac{1}{x} \ln(1+x)}$ (1 punto)

$\lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x} \stackrel{0/0}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x}}{1} = 1$

Sustituimos en (*): Soluc: $e^1 = e$

1

otra forma: $\lim_{x \rightarrow 0^+} (1+x)^{1/x} = 1^{\frac{1}{x}} = e^{\lim_{x \rightarrow 0^+} (1+x-1) \cdot \frac{1}{x}} = e^{\lim_{x \rightarrow 0^+} \frac{x}{x}} = e^1 = e$

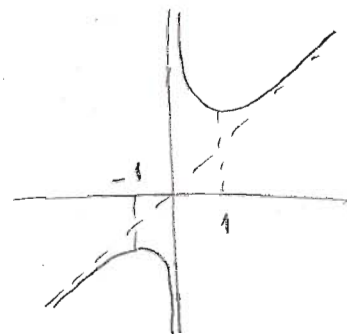
3. Dada $f(x) = \frac{x^2+1}{x}$ se pide: $\rightarrow \text{Dom}(f) = \mathbb{R} - \{0\}$; se ve que es simétrica impar (1,5 puntos)

a) M y m relativos, e intervalos de monotonía.

$$f'(x) = \frac{2x \cdot x - (x^2+1)}{x^2} = \frac{x^2-1}{x^2} = 0 \Rightarrow x^2-1=0; x^2=1; x=\pm 1 \text{ posibles Hom}$$

$$f''(x) = \frac{2x \cdot x^2 - (x^2-1) \cdot 2x}{x^4} = \frac{2x^3 - 2x^3 + 2x}{x^4} = \frac{2}{x^3}$$

$$\left. \begin{array}{l} f''(1) = \frac{2}{1} > 0 \Rightarrow \boxed{m(1,2)} \\ f''(-1) = \frac{2}{-1} < 0 \Rightarrow \boxed{M(-1,-2)} \end{array} \right\} \Rightarrow \left. \begin{array}{l} f(x) \nearrow \forall x \in (-\infty, -1) \cup (1, \infty) \\ f(x) \searrow \forall x \in (-1, 0) \cup (0, 1) \end{array} \right\}$$



b) P.I. e intervalos de curvatura.

$$f''(x) = \frac{2}{x^3} = 0 \Rightarrow 2=0 \Rightarrow \boxed{\text{A P.I.}}$$

viendo el signo de $f''(x)$ vemos que es positiva si $x > 0$ y negativa si $x < 0 \Rightarrow$

$$\Rightarrow \left. \begin{array}{l} f(x) \nearrow \forall x \in (-\infty, 0) \\ f(x) \searrow \forall x \in (0, \infty) \end{array} \right\}$$

c) Hallar las posibles asíntotas.

$$\text{¿A.N.?} \quad \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x^2+1}{x} \approx \lim_{x \rightarrow \infty} \frac{x^2}{x} = \lim_{x \rightarrow \infty} x = \infty \Rightarrow \boxed{\text{A.N.}}$$

$$\text{¿A.V.?} \quad \lim_{x \rightarrow 0} f(x) = \frac{1}{0} = \infty \Rightarrow \boxed{x=0 \text{ A.V.}}$$

¿A.O.?

$$\begin{aligned} m &= \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x^2+1}{x^2} \approx \lim_{x \rightarrow \infty} \frac{x^2}{x^2} = \lim_{x \rightarrow \infty} 1 = 1 \\ n &= \lim_{x \rightarrow \infty} [f(x) - mx] = \lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x} - x \right) = \lim_{x \rightarrow \infty} \frac{x^2+1-x^2}{x} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0 \end{aligned} \Rightarrow \boxed{y=x \text{ A.O.}}$$

4. Calcular:

(1,5 puntos)

a) $\int \frac{3x-15}{x^3-3x-2} dx$

↪ raíces -1 doble y 2 (por Ruffini...)

$$\frac{3x-15}{(x+1)^2(x-2)} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \xrightarrow{\text{multiplicamos por } (x+1)^2(x-2)} 3x-15 = A(x+1)^2 + B(x+1)(x-2) + C(x-2)$$

$x = -1 \rightarrow -18 = -3C ; \boxed{C = 6}$

$x = 2 \rightarrow -9 = 9A ; \boxed{A = -1}$

$x = 1 \rightarrow -12 = 4A - 2B - C ; -12 = -4 - 2B - 6 ; 2B = 2 ; \boxed{B = 1}$

$$I = \int \frac{-1}{x-2} dx + \int \frac{1}{x+1} dx + \int \frac{6}{(x+1)^2} dx = -\int \frac{1}{x-2} dx + \int \frac{1}{x+1} dx + 6 \int (x+1)^{-2} dx =$$

$$= -\ln|x-2| + \ln|x+1| + 6 \frac{(x+1)^{-1}}{-1} = \boxed{\ln|x+1| - \ln|x-2| - \frac{6}{x+1} + C_1}$$

$\boxed{1,5}$
(0,75 + 0,75)

b) $\int x^2 \cdot e^x \cdot dx = x^2 e^x - \int e^x \cdot 2x dx = x^2 e^x - 2 \int x e^x dx =$

$u = x^2 \rightarrow du = 2x dx$

$dv = e^x dx \rightarrow v = \int e^x dx = e^x$

$u = x \rightarrow du = dx$

$dv = e^x dx \rightarrow v = e^x$

$$= x^2 e^x - 2 \left(x e^x - \int e^x dx \right) = x^2 e^x - 2x e^x + 2e^x = \boxed{(x^2 - 2x + 2) e^x}$$

5. Dibujar, de forma aproximada, el recinto limitado por $f(x)=5x^3-4x$ y $g(x)=x$. Hallar su área. (1,5 puntos)

$$5x^3 - 4x = x$$

$$5x^3 - 5x = 0$$

$$5x(x^2 - 1) = 0 \quad \begin{matrix} \nearrow x=0 \\ \searrow x^2-1=0; x^2=1; x=\pm 1 \end{matrix}$$

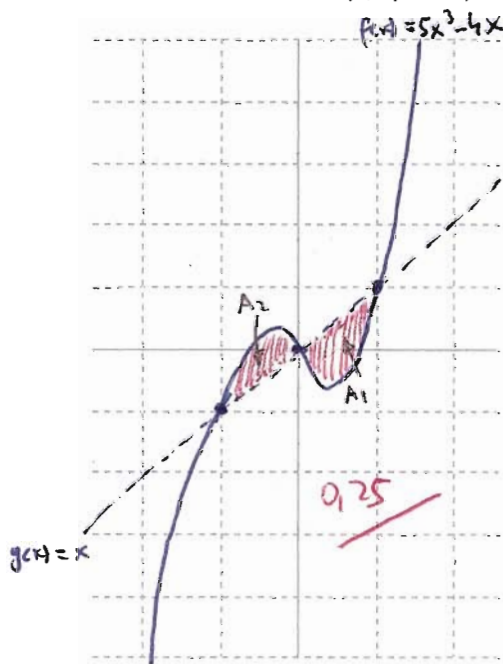
x	-∞	...	-2	-1	0	1	2	...	∞
y=5x ³ -4x	-∞	...	-32	-1	0	1	32	...	∞

Por simetría, se ve que $A_T = A_1 + A_2 = 2A_1$

$$A_1 = \int_0^1 [g(x) - f(x)] dx = \int_0^1 [x - (5x^3 - 4x)] dx = \int_0^1 (x - 5x^3 + 4x) dx =$$

$$= \int_0^1 (5x - 5x^3) dx = 5 \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{5}{2} - \frac{5}{4} = \frac{5}{4}$$

$$\Rightarrow A_T = \frac{5}{2} u^2$$



1,5
(0,5 + 1)

6. Resolver la ecuación matricial $AX + B = C$, siendo $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ y $C = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 5 & 2 \\ 0 & 1 & 3 \end{pmatrix}$ (1,5 puntos)

$$AX + B = C; \quad AX = C - B; \quad A^{-1} \cdot AX = A^{-1} \cdot (C - B); \quad \boxed{X = A^{-1} \cdot (C - B)} \quad (*)$$

$$C - B = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 5 & 2 \\ 0 & 1 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 4 & 2 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 4 \end{vmatrix} = 8 \Rightarrow \exists A^{-1}$$

$$A_{11} = \begin{vmatrix} 2 & 0 \\ 2 & 4 \end{vmatrix} = 8 \quad A_{12} = -\begin{vmatrix} 1 & 0 \\ 1 & 4 \end{vmatrix} = -4 \quad A_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$$

$$A_{21} = -\begin{vmatrix} 0 & 0 \\ 2 & 4 \end{vmatrix} = 0 \quad A_{22} = \begin{vmatrix} 1 & 0 \\ 1 & 4 \end{vmatrix} = 4 \quad A_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = -2$$

$$A_{31} = \begin{vmatrix} 0 & 0 \\ 2 & 0 \end{vmatrix} = 0 \quad A_{32} = -\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0 \quad A_{33} = \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = 2$$

$$Adj(A) = \begin{pmatrix} 8 & -4 & 0 \\ 0 & 4 & -2 \\ 0 & 0 & 2 \end{pmatrix} \rightarrow Adj(A) = \begin{pmatrix} 8 & 0 & 0 \\ -4 & 4 & 0 \\ 0 & -2 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} \cdot Adj(A) = \frac{1}{8} \begin{pmatrix} 8 & 0 & 0 \\ -4 & 4 & 0 \\ 0 & -2 & 2 \end{pmatrix}$$

Substituir en (*):

$$\bar{X} = \frac{1}{8} \begin{pmatrix} 8 & 0 & 0 \\ -4 & 4 & 0 \\ 0 & -2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 0 \\ 2 & 4 & 2 \\ 0 & 1 & 2 \end{pmatrix} = \frac{1}{8} \begin{pmatrix} 16 & 0 & 0 \\ 0 & 16 & 8 \\ -4 & -6 & 0 \end{pmatrix}$$

$$\bar{X} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ -1/2 & -3/4 & 0 \end{pmatrix}$$

7. Dado el sistema

$$\begin{cases} x + y + z = 1 \\ mx + y + z = 1 \\ x + my + 3z = m \end{cases}$$

$$\begin{array}{ccc|c} & & & M^* \\ \hline 1 & 1 & 1 & 1 \\ m & 1 & 1 & 1 \\ 1 & m & 3 & m \\ \hline & & & M \end{array}$$

(1,5 puntos)

a) Discutirlo según los valores del parámetro m

$$|M| = \begin{vmatrix} 1 & 1 & 1 \\ m & 1 & 1 \\ 1 & m & 3 \end{vmatrix} = 3 + 1 + m^2 - 1 - m - 3m = m^2 - 4m + 3 = 0 \quad \begin{matrix} \nearrow m=1 \\ \searrow m=3 \end{matrix} \quad 0,1/$$

$$m=1 \Rightarrow M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 3 \end{pmatrix}; \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} \neq 0 \Rightarrow r_M = 2 \quad 0,1/$$

$$M^* = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 \end{pmatrix}; \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} \neq 0 \Rightarrow r_{M^*} = 2 \quad 0,1/$$

$$m=3 \Rightarrow M = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 1 & 3 & 3 \end{pmatrix}; \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} \neq 0 \Rightarrow r_M = 2 \quad 0,1/$$

1,5
(0,75 + 0,75)

$$M^* = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 1 \\ 1 & 3 & 3 & 3 \end{pmatrix}; \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} \neq 0 \Rightarrow r_{M^*} = 2 \quad 0,1/$$

0,25/

Soluc: I) $m \neq 1$ y $m \neq 3 \Rightarrow r_M = r_{M^*} = 3 = n = \text{incógnitas} \Rightarrow \text{sist. compat. d. tdo. (soluc. única)}$
II) $m = 1$ o $m = 3 \Rightarrow r_M = r_{M^*} = 2 < 3 = n = \text{incógnitas} \Rightarrow \text{sist. compat. limitado uniparamétrico (20 soluciones)}$

b) Resolverlo para $m=3$

$m=3 \Rightarrow \text{sist. compat. limitado uniparamétrico} \Rightarrow \text{quitamos la 3ª ecuación y paramos } z \text{ como parámetro:}$

$$\begin{cases} x + y = 1 - \lambda \\ 3x + y = 1 - \lambda \end{cases} \quad x = \frac{\begin{vmatrix} 1-\lambda & 1 \\ 1-\lambda & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}} = \frac{1-\lambda - (1-\lambda)}{-2} = \frac{0}{-2} = 0 \quad 0,25/$$

$$y = \frac{\begin{vmatrix} 1 & 1-\lambda \\ 3 & 1-\lambda \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}} = \frac{1-\lambda - 3(1-\lambda)}{-2} = \frac{1-\lambda - 3 + 3\lambda}{-2} = \frac{-2 + 2\lambda}{-2} = 1 - \lambda \quad 0,25/$$

$$z = \lambda \quad 0,25/$$

Soluc: $x = 0; y = 1 - \lambda; z = \lambda$