

Problema 1 Calcular el rango de la matriz

$$A = \begin{pmatrix} 1 & -1 & 2 & 5 \\ 2 & 2 & -1 & 3 \\ 4 & 8 & -7 & 1 \end{pmatrix}$$

Solución:

$$|A_1| = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 2 & -1 \\ 4 & 8 & -7 \end{vmatrix} = 0, \quad |A_2| = \begin{vmatrix} 1 & -1 & 5 \\ 2 & 2 & 3 \\ 4 & 8 & 1 \end{vmatrix} = 8$$

Luego $\text{Rango}(A) = 3$.

Problema 2 Sea la matriz

$$A = \begin{pmatrix} m & -2 & 3 \\ 2 & m & m \\ 3 & -1 & 4 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} m & -2 & 3 \\ 2 & m & m \\ 3 & -1 & 4 \end{vmatrix} = 5(m^2 - 3m + 2) = 0 \implies m = 1, \quad m = 2$$

Si $m = 1$ o $m = 2 \implies |A| = 0 \implies$ no existe A^{-1} .

Si $m \neq 1$ y $m \neq 2 \implies |A| \neq 0 \implies$ existe A^{-1} .

2.

$$A = \begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & 0 \\ 3 & -1 & 4 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} 0 & 1/2 & 0 \\ -4/5 & -9/10 & 3/5 \\ -1/5 & -3/5 & 2/5 \end{pmatrix}$$

Problema 3 Dada la matriz

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Calcular A^n y en particular A^{1000}

Solución:

$$A^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ n & 0 & 1 \end{pmatrix}$$

$$A^{1000} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1000 & 0 & 1 \end{pmatrix}$$

Problema 4 Calcular todas las matrices X que cumplan $AX = XA$ donde

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$$

Solución:

Llamamos $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$AX = XA \implies \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} \implies$$

$$\begin{pmatrix} c & d \\ a+2c & b+2d \end{pmatrix} = \begin{pmatrix} b & a+2b \\ d & c+2d \end{pmatrix} \implies \begin{cases} c = b \implies c = b \\ d = a+2b \implies d = a-2b \\ a+2c = d \implies d = a-2b \\ b+2d = c+2d \implies c = b \end{cases}$$

$$\text{Luego } X = \begin{pmatrix} a & b \\ b & a+2b \end{pmatrix}$$