

Problema 1 Calcular el rango de la matriz

$$A = \begin{pmatrix} 2 & 1 & -1 & 3 \\ -1 & 2 & 2 & 1 \\ 1 & 8 & 4 & 9 \end{pmatrix}$$

Solución:

$$|A_1| = \begin{vmatrix} 2 & 1 & -1 \\ -1 & 2 & 2 \\ 1 & 8 & 4 \end{vmatrix} = 0, \quad |A_2| = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 1 & 8 & 9 \end{vmatrix} = 0$$

$$|A_3| = \begin{vmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 1 & 4 & 9 \end{vmatrix} = 0, \quad |A_4| = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 2 & 1 \\ 8 & 4 & 9 \end{vmatrix} = 0$$

Como

$$\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 5 \neq 0 \implies \text{Rango}(A) = 2$$

Problema 2 Sea la matriz

$$A = \begin{pmatrix} 2 & m & -m \\ m & 2 & 3 \\ 3 & 3 & 2 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} 2 & m & -m \\ m & 2 & 3 \\ 3 & 3 & 2 \end{vmatrix} = -5m^2 + 15m - 10 = 0 \implies m = 1, \quad m = 2$$

Si $m = 1$ o $m = 2 \implies |A| = 0 \implies$ no existe A^{-1} .

Si $m \neq 1$ y $m \neq 2 \implies |A| \neq 0 \implies$ existe A^{-1} .

2.

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 3 \\ 3 & 3 & 2 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} 1/2 & 0 & 0 \\ -9/10 & -2/5 & 3/5 \\ 3/5 & 3/5 & -2/5 \end{pmatrix}$$

Problema 3 Dada la matriz

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Calcular A^n y en particular A^{1000}

Solución:

$$A^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{1000} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1000 \\ 0 & 0 & 1 \end{pmatrix}$$

Problema 4 Calcular todas las matrices X que cumplan $AX = XA$ donde

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

Solución:

Llamamos $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$AX = XA \implies \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \implies$$

$$\begin{pmatrix} a & b \\ 2a+c & 2b+d \end{pmatrix} = \begin{pmatrix} a+2b & b \\ c+2d & d \end{pmatrix} \implies \begin{cases} a = a+2b \implies b=0 \\ b = b \implies b=b \\ 2a+c = c+2d \implies a=d \\ 2b+d = d \implies b=0 \end{cases}$$

$$\text{Luego } X = \begin{pmatrix} d & 0 \\ c & d \end{pmatrix}$$