

Problema 1 Calcular el rango de la matriz

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 1 & 8 & -7 & 5 \\ 1 & -1 & 2 & 2 \end{pmatrix}$$

Solución:

$$|A_1| = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 8 & -7 \\ 1 & -1 & 2 \end{vmatrix} = 0, \quad |A_2| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 8 & 5 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

$$|A_3| = \begin{vmatrix} 1 & -1 & 3 \\ 1 & -7 & 5 \\ 1 & 2 & 2 \end{vmatrix} = 0, \quad |A_4| = \begin{vmatrix} 2 & -1 & 3 \\ 8 & -7 & 5 \\ -1 & 2 & 2 \end{vmatrix} = 0$$

Como

$$\begin{vmatrix} 1 & 2 \\ 1 & 8 \end{vmatrix} = 6 \neq 0 \implies \text{Rango}(A) = 2$$

Problema 2 Sea la matriz

$$A = \begin{pmatrix} 2 & m & -1 \\ m & -2 & 1 \\ 3 & -m & 0 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} 2 & m & -1 \\ m & -2 & 1 \\ 3 & -m & 0 \end{vmatrix} = m^2 + 5m - 6 = 0 \implies m = 1, \quad m = -6$$

Si $m = 1$ o $m = -6 \implies |A| = 0 \implies$ no existe A^{-1} .

Si $m \neq 1$ y $m \neq -6 \implies |A| \neq 0 \implies$ existe A^{-1} .

2.

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 0 & -2 & 1 \\ 3 & 0 & 0 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} 0 & 0 & 1/3 \\ -1/2 & -1/2 & 1/3 \\ -1 & 0 & 2/3 \end{pmatrix}$$

Problema 3 Dada la matriz

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Calcular A^n y en particular A^{1000}

Solución:

$$A^1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{1000} = \begin{pmatrix} 1 & 0 & 0 \\ 1000 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Problema 4 Calcular todas las matrices X que cumplan $AX = XA$ donde

$$A = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

Solución:

LLamamos $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$AX = XA \implies \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \implies$$

$$\begin{pmatrix} a & b \\ -2a + c & -2b + d \end{pmatrix} = \begin{pmatrix} a - 2b & b \\ c - 2d & d \end{pmatrix} \implies \begin{cases} a = a - 2b \implies b = 0 \\ b = b \implies b = b \\ -2a + c = c - 2d \implies a = d \\ -2b + d = d \implies b = 0 \end{cases}$$

$$\text{Luego } X = \begin{pmatrix} d & 0 \\ c & d \end{pmatrix}.$$