

Problema 1 Calcular el rango de la matriz

$$A = \begin{pmatrix} 1 & 3 & -1 & 5 \\ 2 & 1 & 2 & 2 \\ -1 & 7 & -7 & 11 \end{pmatrix}$$

Solución:

$$|A_1| = \begin{vmatrix} 1 & 3 & -1 \\ 2 & 1 & 2 \\ -1 & 7 & -7 \end{vmatrix} = 0, \quad |A_2| = \begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 2 \\ -1 & 7 & 11 \end{vmatrix} = 0$$

$$|A_3| = \begin{vmatrix} 1 & -1 & 5 \\ 2 & 2 & 2 \\ -1 & -7 & 11 \end{vmatrix} = 0, \quad |A_4| = \begin{vmatrix} 3 & -1 & 5 \\ 1 & 2 & 2 \\ 7 & -7 & 11 \end{vmatrix} = 0$$

Como

$$\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -5 \neq 0 \implies \text{Rango}(A) = 2$$

Problema 2 Sea la matriz

$$A = \begin{pmatrix} m & 2 & -3 \\ m-1 & m & 2 \\ 2 & 7 & 0 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} m & 2 & -3 \\ m-1 & m & 2 \\ 2 & 7 & 0 \end{vmatrix} = 29 - 29m = 0 \implies m = 1$$

Si $m = 1 \implies |A| = 0 \implies$ no existe A^{-1} .

Si $m \neq 1 \implies |A| \neq 0 \implies$ existe A^{-1} .

2.

$$A = \begin{pmatrix} 0 & 2 & -3 \\ -1 & 0 & 2 \\ 2 & 7 & 0 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} -14/29 & -21/29 & 4/29 \\ 4/29 & 6/29 & 3/29 \\ -7/29 & 4/29 & 2/29 \end{pmatrix}$$

Problema 3 Dada la matriz

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Calcular A^n y en particular A^{503}

Solución:

$$A^1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ n & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{503} = \begin{pmatrix} 1 & 0 & 0 \\ 503 & 1 & 503 \\ 0 & 0 & 1 \end{pmatrix}$$

Problema 4 Calcular todas las matrices X que cumplan $AX = XA$ donde

$$A = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$$

Solución:

LLamamos $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$AX = XA \implies \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} \implies$$

$$\begin{pmatrix} 3a & 3b \\ a+2c & b+2d \end{pmatrix} = \begin{pmatrix} 3a+b & 2b \\ 3c+d & 2d \end{pmatrix} \implies \begin{cases} 3a = 3a+b \implies b=0 \\ 3b = 2b \implies b=0 \\ a+2c = 3c+d \implies a=c+d \\ b+2d = 2d \implies b=0 \end{cases}$$

$$\text{Luego } X = \begin{pmatrix} c+d & 0 \\ c & d \end{pmatrix}$$