

Problema 1 Sea la matriz

$$A = \begin{pmatrix} 1 & -m & 2 \\ 2 & 2 & m \\ 3 & m & 3 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} 1 & -m & 2 \\ 2 & 2 & m \\ 3 & m & 3 \end{vmatrix} = -4m^2 + 10m - 6 = 0 \implies m = 1, \quad m = \frac{3}{2}$$

Si $m = 1$ o $m = \frac{3}{2} \implies |A| = 0 \implies$ no existe A^{-1} .

Si $m \neq 1$ y $m \neq \frac{3}{2} \implies |A| \neq 0 \implies$ existe A^{-1} .

2.

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 0 \\ 3 & 0 & 3 \end{pmatrix} \implies A^{-1} = \begin{pmatrix} -1 & 0 & 2/3 \\ 1 & 1/2 & -2/3 \\ 1 & 0 & -1/3 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $XA + XB = C + I$. Donde

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}; \quad B = \begin{pmatrix} 3 & -1 \\ 1 & -1 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

Solución:

$$XA + XB = C + I \implies X(A + B) = C + I \implies X = (C + I)(A + B)^{-1}$$

$$(A + B)^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & -4 \end{pmatrix}, \quad C + I = \begin{pmatrix} 2 & 0 \\ 2 & 2 \end{pmatrix}$$

$$X = (C + I)(A + B)^{-1} = \begin{pmatrix} 0 & 1 \\ 2 & 6 \end{pmatrix}$$

Problema 3 Resolver utilizando las propiedades de los determinantes:

$$\begin{vmatrix} x & 1 & 1 & 0 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix}$$

Solución:

$$\begin{aligned} \begin{vmatrix} x & 1 & 1 & 0 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} &= \begin{bmatrix} F_1 + F_2 + F_3 + F_4 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{vmatrix} x+2 & x+2 & x+2 & x+2 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} = \\ (x+2) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} &= \begin{bmatrix} F_1 \\ F_2 - F_1 \\ F_3 - F_1 \\ F_4 \end{bmatrix} = (x+2) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x-1 & -1 & 0 \\ 0 & -1 & x-1 & 0 \\ 0 & 1 & 1 & x \end{vmatrix} = \\ (x+2) \begin{vmatrix} x-1 & -1 & 0 \\ -1 & x-1 & 0 \\ 1 & 1 & x \end{vmatrix} &= x(x+2) \begin{vmatrix} x-1 & -1 \\ -1 & x-1 \end{vmatrix} = x^2(x^2 - 4) \end{aligned}$$